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NOTE

Estimation of flood peak discharge by the slope-area method

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Abstract

The magnitude of large flood peaks in natural rivers is normally estimated in New Zealand by the slope-area method when conditions prevent direct flow gauging using, for example, a current meter.

Employment of the slope-area method is nowadays less frequent, owing mainly to a perceived lack of need, resource constraints, adjudged higher priorities and improved flood gauging instrumentation.

The calculation procedures of the method are reviewed and updated given that field procedures are well known and understood. In particular, commentary is provided about the Manning resistance coefficient, velocity and energy loss coefficients, loop ratings, compound sections, Froude number and mean velocity, scour and fill and standard errors. Although the standard errors can be large, this is now of less concern as a range of peak values can be readily incorporated in modern improved flood frequency analysis.

Further regular use of the slope-area method is recommended to obtain new measurements and revisit historical ones because knowledge of a peak size and occurrence is vital for understanding the flood hydrology of a catchment.

Keywords

slope-area method; indirect flow measurement; flood peak; flood estimation; river floods

Introduction

The slope-area method is an indirect method of estimating a flow rate in an open channel. In New Zealand, its use is almost exclusively confined to estimating flood peak discharges in natural channels. The method is employed when flow velocity is too high or there are problems with sediment transport, or access to the site, that preclude the use of conventional direct flow gauging techniques using, for instance, current meters.

Data obtained from peak flow estimation techniques, such as slope-area measurements, is of the first importance in understanding the hydrology of flooding in gauged or ungauged catchments and has direct application in flood frequency analysis, flood plain management, hazard analysis and the design of flood protection systems and bridges. Unfortunately, use of the technique has declined in New Zealand for a number of reasons, including: a perception that enough measurements have been obtained to define the upper end of rating curves, which at many hydrological recording stations or sites are stable; resource constraints, as considerable field time is involved in each measurement; few requests from users of the information to obtain more measurements;

and the availability of instruments such as the Acoustic Doppler Current Profiler (ADCP) capable of carrying out high stage gaugings in situations where a slope-area approach would previously have been employed (J. Fenwick, *pers. comm.*).

Field procedures for setting up slope-area reaches and undertaking field measurements are well understood and documented (Arnold *et al.*, 1988; Fenwick, 1994; Hicks and Mason, 1998). A typical reach is more or less straight with a stable control, is not compound (no berms), has a bed slope of less than 0.02, and there may or may not be a site with a water level or stage recorder within the reach or nearby. Commonly, two or three cross-sections are used for slope-area measurements. Where these conditions cannot be met, or where greater accuracy is required, then a calibrated hydraulic model such as HEC-RAS (US Army Corps of Engineers, 2010) should be employed (Domeneghetti *et al.*, 2012).

The purposes of this study are to: (1) review and update slope-area calculation (not field) procedures; and (2) provide practical comments about: estimation of various parameters; loop ratings; compound cross sections; Froude number and mean velocity checks; reach scour and fill; and approximate standard error of the calculated water discharge. The aim is to encourage more frequent use of the slope-area method to obtain estimates of the magnitude of very large flood peaks in New Zealand rivers and streams, where these cannot be directly measured.

Theory

The slope-area method of discharge estimation is essentially a variant of the standard step method of nonuniform flow computation as detailed, for example, by Henderson (1966). Within New Zealand the empirical Manning Equation given by:

$$Q = (1/n)\, A\, R^{2/3}\, S^{1/2} \qquad (1)$$

in which Q is water discharge, n is the Manning roughness coefficient, A is cross-section area, R hydraulic radius and S is friction slope, is usually employed to describe flow resistance (Fenwick, 1994). In Equation 1, S is defined from energy considerations to be:

$$S = [\Delta h + \Delta h_v - k(\Delta h_v)]/L \qquad (2)$$

where Δh is the difference in water surface elevation between the upstream and downstream cross-sections of a reach, Δh_v is the difference in velocity head between cross-sections, k is an energy loss coefficient and L is reach length. Here, velocity head is given by $\alpha V^2/2g$ in which α is a velocity head co-efficient, V is mean velocity in a section and g is the acceleration of gravity. Substitution of Equation 1 into Equation 2 leads to the well-known discharge formulae listed in the Appendix (Equations 7 and 8). In what follows we comment upon: selecting values for n, α and k; loop ratings; compound cross-sections; Froude number and mean velocity; and scour and fill within the slope-area reach. Finally, we provide a formula for estimating the approximate standard error in Q.

Estimation of Manning *n*

The value of n normally varies with increasing water stage, y_s, or with R. For flow gaugings at a hydrological recording station or site where water surface slope S_w has been measured, n can be calculated and a y_s, or preferably R, versus n curve can be constructed. This may be applied where the site cross-section lies within or near to a slope-area reach. If there are insufficient data for this approach, reference may be made to the dataset in Hicks and Mason (1998) for visual estimation of n values in New Zealand.

As a check, for perhaps a minimum value of n in gravel bed channels, Griffiths (1981) gives

$$n = 0.354\, R^{0.617}/g^{0.5}[0.76 + 1.98 \log_{10}(R/d_{50})]$$

for a rigid bed where d_{50} is the median size of the surface bed material, and

$$n = 0.16\,R^{0.167}/g^{0.33}(V/d_{50}{}^{0.5})^{0.34}$$

for a mobile bed, that is, when active bedload transport has probably occurred at the time of the flood peak. If so, then using the Shields entrainment criterion (Garcia, 2008) $RS_w/(S_s - 1)d_{50}$ will exceed 0.056, where S_s is the specific gravity of the bed material. In applying this mobile bed formula V can be estimated from a y_s or R versus V curve constructed at the recorder site.

After discussing the Froude number we note another approach to help determine n.

Bedforms influence the value of n. Little is known about their development in gravel bed rivers; however, the transition from dunes to flatbed in subcritical flow probably occurs when the Froude number $Fr = V/(gR)^{1/2}$ or $Q\,/A(gR)^{1/2}$ exceeds about 0.84 (Griffiths, 1989; Dinehart, 1992; Carling, 1999). For sand bed rivers the transition occurs at about $Fr = 0.80$ (Garcia, 2008). A flat bed offers less resistance than a dune bed so for $Fr \geq 0.8$ the value of n will be less than that for $Fr < 0.8$.

If the channel slope exceeds 0.02, the formula:

$$n = 0.32\,S_w{}^{0.38}\,R^{-0.16} \quad (3)$$

of Jarrett (1984) can be used.

Estimation of velocity head coefficients

In a channel viscous drag makes the velocity lower near the boundary, so the true mean velocity head for a cross-section will not, in practice, be equal to simply $V^2/2g$. Hulsing *et al.* (1966) supply an equation for α based on regression given by:

$$\alpha = 14.8n + 0.884 \quad (4)$$

where $0.012 \leq n \leq 0.070$, $\alpha \leq 2.0$ and where skewed currents occur in less than 10% of a cross-section.

Estimation of energy loss coefficients

Energy loss due to contraction or expansion of the channel in a reach has traditionally been taken as zero for contracting reaches and 0.5 for expanding reaches (Darymple and Benson, 1989; Hicks and Mason, 1998). However, more recent work suggests values of $k = 0.1$ for contracting reaches and $k = 0.4$ for expanding reaches (USACE, 2010).

Loop ratings

Flood waves show kinematic behaviour when the channel bed slope exceeds about 0.001. Otherwise, variable energy slopes associated with dynamic, inertia and pressure forces leads to the formation of a loop-rating so that the steady flow curve no longer correctly describes the actual stage-discharge relationship. The slope-area method is inapplicable in these circumstances: instead, the stage-fall-discharge method described in detail in Herschy (1995), or a similar approach given Dottori *et al.* (2009), may be employed.

It is rare in New Zealand to find evidence of loop ratings in y_s versus Q relations because high stage gaugings are usually undertaken only after a flood has peaked at a site.

Compound sections

Compound sections, where for instance over-bank flow from the channel occurs, should be avoided if possible. If this is not possible a compound section can be used as long as it is properly subdivided to reflect distinct regions with distinct flow characteristics and thus different n values. The assumption of uniform total head across a compound section gives results that are accurate enough for practical purposes (Henderson, 1966).

Froude number and mean velocity

After Q has been computed by the slope-area method, Fr should be computed. If the flow has no surface waves moving downstream, standing waves or hydraulic jumps, or is otherwise known to be subcritical, then $Fr < 1$. It is useful to construct y_s or R versus Fr relationships from gaugings at a site to provide an approximate value of expected *Fr*. A further check on *V* at the recorder cross-section may be supplied from a plot of y_s or *R* versus *V* at that section (McKerchar and Henderson, 1987).

Scour and fill

During the passage of a flood wave through a reach, scour or fill, or both, may occur if the bed of the reach is unstable (Griffiths, 1993). To investigate such behaviour it is useful to construct an apparent bed level relationship (Ibbitt, 1979; McKerchar and Henderson, 1987) showing shifts in rating for a prescribed constant flow. If significant shifts in rating consistently occur from either scour or fill effects, an estimate can be made of change in bed level and an allowance made for the values of *A* and *R* used in the slope-area calculation. It is preferable to select stable reaches for slope-area work, but even with these, scour and fill may occur during a flood without any obvious evidence left after the event. This is one reason why slope-area estimates can exhibit significant standard errors when compared with direct flow gauging measurements.

The matter is complex and substantive understanding of sediment transport behaviour at a site requires employment of a numerical mobile-bed hydrodynamic model (Spasogevic and Holly, 2007).

Errors

If the variables in Equation 1 are assumed to be independent, then the variance formula (Ku, 1966):

$$\sigma(Q) = [(\partial Q/\partial n)^2\, \sigma^2(n) + (\partial Q/\partial A)^2\, \sigma^2(A) + (\partial Q/\partial R)^2\, \sigma^2(R) + (\partial Q/\partial S)^2\, \sigma^2(S)]^{1/2} \quad (5)$$

may be applied to calculate the approximate standard error, σ, in *Q*.

From Equation 1 and Equation 2 we may write:

$$\sigma(Q) = [(-A\, R^{2/3}\, S^{1/2}/n^2)^2\, \sigma^2(n) + (R^{2/3}\, S^{1/2}/n)^2\, \sigma^2(A) + ((2/3)\, A\, S^{1/2}/n\, R^{1/3})^2\, \sigma^2(R) + ((1/2)\, A\, R^{2/3}\, /n\, S^{1/2})^2\, \sigma^2(S)]^{1/2} \quad (6)$$

To use Equation 6, the standard errors of *A, R* and *S* must be estimated based on experience in measuring them in the field (where the highest flood level may be difficult to define owing, for example, to the effects of floating wood and its subsequent deposition) and for *n* from an *R* versus *n* curve or from information given in Hicks and Mason (1998).

Applications

Two examples are given below, illustrating application of the slope-area method in a steep gravel-bed channel and in a sand-bed channel. Both are based on conditions pertaining in gauged catchments at actual sites.

Example 1: Gravel-bed river based on Jollie at Mt Cook Station (Site no 71135; Walter, 2000)

The slope-area reach contracts and has two cross-sections, neither of which is compound; the upper one is the recorder cross-section where d_{50} = 30 mm. The bed slope is about 0.01 and, not surprisingly, there is no evidence of loop ratings in the stage-discharge relationship. A plot of apparent bed level at the recorder shows no trend in the period 1990-1998 during which the slope-area measurement was made (Fig. 1). Fluctuations of some ±0.15 m occur but there is no pattern of scour or fill after large floods. *Fr* values computed from current meter gaugings are high, even for moderate stage values, so plane bed conditions are assumed and no increase in *n* is to be expected with increasing stage under subcritical flow.

Relevant data are: A_1(upper) = 41.1 m^2, A_2(lower) = 37.2 m^2, R_1 = 1.41 m, R_2 = 1.73 m, Δh = 1.362 m, *L* = 90 m. There are insufficient data to plot a *R* versus *n* curve but Hicks and Mason (1998) include this site and smooth extrapolation from their limited data, assuming plane bed conditions, suggests a value of *n* = 0.043.

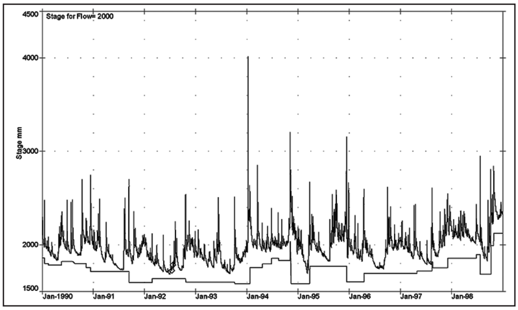

Figure 1 – Plot of apparent bed levels for Jollie at Mt Cook Station: 1990-1998.

From Equation 4, α = 1.52 and k = 0.1 as $A_1 > A_2$. Substitution of all these values in Equation 7 yields (with $n = n_1 = n_2$ and thus $\alpha = \alpha_1 = \alpha_2$) Q = 140 m^3/s. Since $Q = V A$, then V_1 = 3.41 m/s at the recorder section, which is consistent with a predicted value of around 3.5 m/s from a plot of R versus V derived from gauging data (Fig. 2). Fr_1 = 0.92, which is high but is also consistent with measured values.

Note that if Equation 1 had been used to compute the discharge with geometric mean values of A and R (39.1 m^{2}and 1.56 m, respectively) and using $S = S_w$ = 0.0151, then Q = 150 m^3/s, the difference being due to energy losses as from Equation 2, S = 0.0131.

With A = 39.1 m^2, R = 1.56 m, S = 0.0131, n = 0.043, and adopting $\sigma(n)$ = 0.004, $\sigma(A)$ = 2 m^2, $\sigma(R)$ = 0.1 m, and $\sigma(S)$ = 0.003, then from Equation 6 the approximate standard error in Q, $\sigma(Q)$ = 22 m^3/s, where errors in n and S are 34% and 51% of the total standard error, respectively. The estimated discharge is thus 140 ±22 m^3/s.

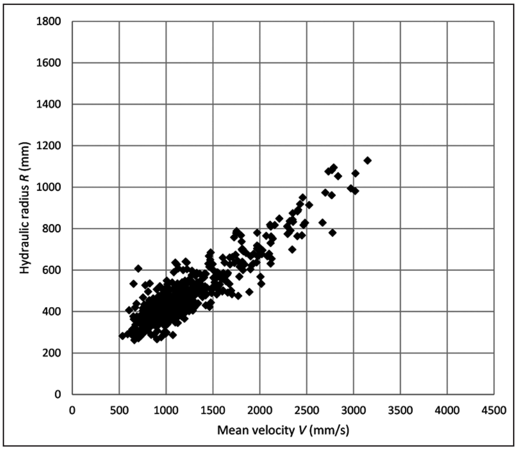

Figure 2 – Hydraulic radius (R) versus mean velocity (V) for Jollie at Mt Cook Station.

Note that the highest gauged flow prior to the time of the slope-area measurement was 64 m^3/s and the discharge predicted by extensive extrapolation of the rating curve for the same stage as in the slope-area was 197 m^3/s. This prediction is unacceptable as it implies the flow was supercritical (Fr > 1.3), which is inconsistent with the R versus Fr relation for the site for which Fr < 1. The difference of 57 m^3/s between the rated and the slope-area estimated discharges demonstrates the value of the slope-area measurement in redefining the top section of the rating curve.

Example 2: Sand-bed river based on Waikato River at Ngaruawahia Cableway (Site 43402; Walter, 2000)

The slope-area reach expands and has three cross-sections, none of which is compound. The middle one is the recorder cross-section where d_{50} = 1 mm. There is no evidence of loop ratings in the stage-discharge relationship. A plot of apparent bed level at the recorder section shows no trend in post-flood event bed levels.

Fr values computed from the current meter gaugings are low, so dune bedforms are expected (Garcia, 2008) and n will increase with increasing stage until Fr = 0.8 approximately.

Relevant values include:

A_1(upper) = 904 m^2,
A_2(middle) = 927 m^2,
A_3(lower) = 950 m^2,
R_1 = 5.28 m^2, R_2 = 4.91 m,
R_3 = 4.73 m, Δh = 0.0319,
L_1 = 75 m, L_2 = 70 m.

There is insufficient data to plot a R versus n curve, but from data in Hicks & Mason

(1998), assuming sand bed conditions, we take n = 0.40 for the whole reach. From Equation 4, α = 1.45 and k = 0.4 as $A_1 < A_2 < A_3$. Substitution of all these values into Equation 1 yields Q = 1090 m^3/s. Then at the recorder section V_2 = 1.18 m/s, which is consistent with a predicted value of 1.1 m/s from a plot of R versus V, and Fr_2 = 0.17, which is also consistent with measured values (Fig. 3). Note that if Equation 1 had been used to compute the discharge with geometric mean values of A = 927 m^2, R = 4.97 m and using S_w = 0.00022, then Q = 1001 m^3/s as from Equation 2, S = 0.00026.

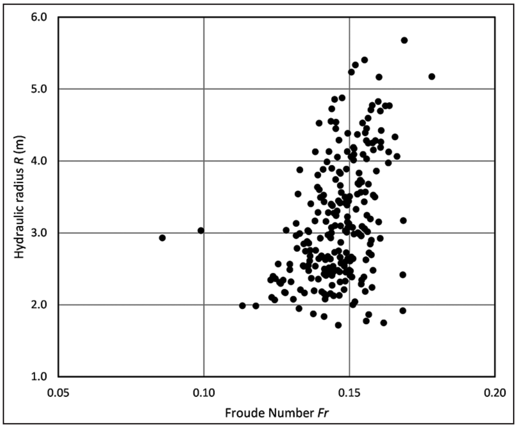

Figure 3 – Hydraulic radius (R) versus Froude number (Fr) for Waikato at Ngaruawahia Cableway.

With A = 927 m^2, R = 4.97 m, S = 0.00026, n = 0.040, $\sigma(n)$ = 0.004, $\sigma(A)$ = 30 m^2, $\sigma(R)$ = 0.2 m and $\sigma(S)$ = 0.00004, then from Equation 6 the approximate standard error in Q, $\sigma(Q)$ = 143 m^3/s, where errors in n and S are 58% and 34% of the total standard error, respectively. The estimated discharge is thus 1090 ±143 m^3/s.

Note that the highest gauged flow at the site prior to the time of the slope-area measurement was 1290 m^3/s. The discharge predicted by interpolation of the rating curve (defined up to 1290 m^3/s) for the same stage as in the slope-area was 1070 m^3/s. The small difference of 20 m^3/s between the rated and the slope-area estimated discharges is perhaps fortuitous but it does demonstrate the value of the slope-area measurement in supporting the definition of the rating curve.

Finally, the most difficult parameter to estimate in using the slope-area method is almost invariably n. One approach to assist in this matter, foreshadowed above, when no R versus n relation is available is as follows. Given R at the recorder site and S_w for the reach, then from the R versus V curve at the site, find V and check this value with the R versus Fr relation at the site. Then from Equation 1, $n = R^{2/3} S_w^{1/2} / V$.

Discussion and recommendations

Energy losses in a slope-area reach can be large so it is important to use the formulae given in the Appendix in calculations rather than the simple Manning formula expressed in terms of water surface slope. There are substantial differences between friction slope and water surface slope at many of the sites treated in Hicks and Mason (1998).

Estimation of the Manning n value for a reach can be difficult. If the water surface slope is measured during, say, current meter or ADCP gaugings, then an approximate relationship between stage or hydraulic radius and n values may be established using the Manning formula to assist in the prediction of n.

It is of vital importance in flood hydrology to obtain estimates of all very large flood peaks for some period in gauged and ungauged catchments. Although the slope-area method often produces a discharge value with a substantial standard error, often of the order of ±20% at a gauged site and of order ±30% at a ungauged site, this is of lesser concern nowadays because, for example, a range in magnitude for a flood peak can readily be incorporated in a flood frequency analysis using Bayesian statistics (Payrastre *et al.*, 2011; Vigilone *et al.*, 2013). Consequently, further, regular employment of the slope-area method is recommended. Moreover, when sufficient data are available, historical slope-area measurements can be revisited to check estimates using information and approaches described in this note.

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References

Arnold, P.E.; Holland, G.; McKerchar, A.I.; Soutter, W.R. 1988: *Water Resources Survey Hydrologists Field Manual*. Department of Scientific and Industrial Research Water Sciences Division, Wellington, New Zealand, 148p.

Carling, P.A. 1999: Subaqueous gravel dunes. *Journal of Sedimentary Research 69(3)*: 534-545.

Dalrymple, T.; Benson, M.A. 1989: *Measurement of peak discharge by the slope-area method.* Techniques of Water-Resources Investigations of the United States Geological Survey, Book 3. US Geological Survey, Denver, Co, 12p.

Dinehart, R.L. 1992: Evolution of coarse gravel bed forms: field measurements at flood stage. *Water Resources Research 28(10)*: 2667-2689.

Dottori, F.; Martina, M.L.V.; Todini, E. 2009: A dynamic rating curve approach to indirect discharge measurement. *Hydrology and Earth System Sciences 13*: 847-863.

Domeneghetti, A.; Castellarin, A.; Brath, A. 2012: Assessing rating-curve uncertainty and its effects on hydraulic model calibration. *Hydrology and Earth System Sciences 16*: 1191-1202.

Fenwick J.K. 1994: *Hydrologists field manual*. NIWA Science and Technology Information Series No. 5. NIWA, Christchurch, NZ.

Garcia, M.H. 2008: Sediment transport and morphodynamics. *In*: Garcia, M.H. (*ed*.), *Sedimentation Engineering*. American Society of Civil Engineers, Reston, VA: 21-146.

Griffiths G.A. 1981: Flow resistance in coarse gravel bed rivers. *Journal of the Hydraulics Divn, ASCE 107 (HY7)*: 899-918.

Griffiths G.A. 1989: Flow resistance in gravel channels with mobile beds. *Journal of Hydraulic Engineering 115(3)*: 340-355.

Griffiths G.A. 1993: Sediment translation waves in braided gravel bed rivers. *Journal of Hydraulic Engineering 119(8)*: 924-937.

Henderson, F.M. 1966: *Open Channel Flow*. Macmillan Co, New York, 522p.

Herschy, R.W. 1995: *Streamflow Measurement* (2nd Edition). E & FN Spon, London.

Hicks, D.M.; Mason, P.D. 1998: *Roughness Characteristics of New Zealand Rivers*. National Institute for Water and Atmospheric Research Ltd, Auckland, New Zealand, 329p.

Hulsing H.; Smith, W.; Cobb, E.D. 1966: *Velocity-head coefficients for open channels*. United States Geological Survey Water Supply Paper 1869-C. US Department of the Interior, Washington, DC, 45p.

Ibbitt, R.P. 1979: Flow estimation in an unstable river illustrated on the Rakaia River for the period 1958-1978. *Journal of Hydrology (NZ) 18(2)*: p88-108.

Jarrett, R.D. 1984: Hydraulics of high-gradient streams. *Journal of the Hydraulics Division of the American Society of Civil Engineers 110(11)*: p1519-1539.

Ku, H.H. 1966: Notes on the use of propagation of error formulas. *Journal of Research of the National Bureau of Standards 70C (4)*: p262.

McKerchar, A.I.; Henderson, R.D. 1987: Drawing and checking stage discharge rating curves. Publication No. 11 of the Hydrology Centre. MWD, Christchurch, NZ. 29 p.

Payrastre, O.; Gaume, E.; Andrieu, H. 2011: Usefulness of historical information for flood frequency analysis: developments based on a case study. *Water Resources Research 47(8)*: W 08511.

Spasojevic, M.; Holly, F.M. 2008: Two and three dimensional numerical simulation of mobile-bed hydrodynamics and sedimentation. *In*: Garcia, M.H. (*ed.*) *Sedimentation Engineering*. American Society of Civil Engineers, Reston, VA. p683-761.

United States Army Corps of Engineers 2010: *HEC-RAS river analysis system – Hydraulic Reference Manual Version 4.1*. USACE, Hydrologic Engineering Center, California.

Viglione, A.; Merz, R.; Salinas, J.L.; Bloschl, G. 2013: Flood frequency hydrology: 3. A Bayesian analysis. *Water Resources Research 49(2)*: 675-692.

Walter, K.M. 2000: *Index to hydrological recording stations in New Zealand*. Technical Report 73, National Institute of Water & Atmospheric Research Ltd, Wellington, 216p.

Appendix: Equations for use in slope-area measurements

(Dalrymple and Benson, 1989)

Two cross sections:

$$Q = K_2 \Delta h^{0.5} / \{(K_2/K_1)L + K_2^2/(2gA_2^2)[-\alpha_1(A_2/A_1)^2(1 - k) + \alpha_2(1-k)]\}^{0.5} \quad (7)$$

Three cross sections:

$$Q = K_3\ \Delta h^{0.5} / \{(K_3/K_2)(K_3 L_{1\text{-}2}/K_1 + L_{2\text{-}3}) + K_3^2/(2gA_3^2)[-\alpha_1(A_3/A_1)^2(1-k_{1\text{-}2}) + \alpha_2(A_3/A_2)^2(k_{2\text{-}3} - k_{1\text{-}2}) + \alpha_3(1-k_{2\text{-}3})]\}^{0.5} \quad (8)$$

where $K = (1/n)AR^{2/3}$

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