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EDITORIAL
UNITS, COEFFICIENTS AND DIMENSIONS

Metrication rules aim to standardise the physical units we use, and strict application should ideally admit only one unit for each purpose so that few units are required. However the rules allow alternatives, presumably to obtain acceptance in diverse disciplines. These alternatives can vitiate the desired standardisation unless the practitioners of each discipline agree to use a particular subset of the legitimate units.

Some cumbersome units have been introduced, which were derived by algebraically combining other derived units, and new words have been coined for these cumbersome units. Thus in New Zealand, but nowhere else to my knowledge, we express water volumes in cumec-days. My view is that we should avoid this kind of jargon, follow the British Standard, and use reduced forms because they can be more concisely written and therefore more easily remembered. For example, specific discharges commonly expressed in $m^3 s^{-1} km^{-2}$ can more succinctly be expressed in um/s, i.e., micrometres per second which is the same unit. Likewise $l\, s^{-1} km^{-2}$ is better written nm/s, i.e., nànometres per second. In these two examples the prefix u = micro = 10^{-6} and the prefix n = nano = 10^{-9}. It seems a good idea to use the Roman letter u in place of the Greek letter μ when the latter is not available, and to avoid the need to represent litres by a symbol when the script l is not available.

Reduced forms have only one prefix, and only one oblique stroke to separate the "numerator" from the "denominator". All of the following units can be understood, and mean the same thing, but only the first is a truly reduced form:

$$g/m^2a = g\,m^{-2}\,a^{-1} = g/m^2/a = t\,km^{-2}\,a^{-1}.$$

I believe superscripts are so important that they should be printed as full-size numbers and freed from the need for signs, although many journals disagree with me, including this one.

The number of coefficients used in hydrological formulae can also be reduced by using appropriate prefixes, for example, the following formula requires no coefficient at all with the given units: Q = IA relates flow Q in m^3/s to rainfall intensity, I in um/s and catchment area A in km^2. Furthermore, when these units are used the values of Q, I and A are often in the convenient range 0.1 to 99. Someone is sure to point out that there is nothing new in this because the formula

$Q = IA$ may be used with Q in cubic feet per second, I in inches per hour and A in acres; but that is just coincidence! There are many other examples such as:

$G = CQ$ relating load G in g/s and concentration C in g/m^3;

$X = PA$ relating yield X in hm^3/a and intensity P in m/a;

$Y = LA$ relating yield Y in t/a and intensity L in g/m^2a.

Note that the prefix h = hecto = 10^2, so that hm^3 represents a million cubic metres.

With these particular units one numerical constant cannot be avoided and that is the number of seconds in a year which is 31.6×10^6, to three figure accuracy. Thus we get the formulae:

$$P = 31.6I, \quad X = 31.6Q \quad \text{and} \quad Y = 31.6G.$$

If we agree to avoid using the ubiquitous time units—day, hour and minute—then we can settle for only the one number 31.6 in our formulae. This is not too difficult given that we must learn to use new units anyway. For example, the flows 166 m^3/a, 454 litres per day, and 100 gallons per day are equivalent, and the first metric form seems just as easy to use as the second metric form, so that avoidance of days in favour of years can easily be achieved.

I suspect that dimensional analysis seems like witchcraft to many hydrologists, and raise the subject now because metrication ought to make this craft less mysterious because it is easier to do arithmetic. Consider, for example, Beable and McKerchar's formula for estimating a mean annual flood in the Hawkes Bay region:

$$Q = 8.84 \times 10^{-5} A^{0.76} (86.4\, I)^{2.24} \qquad (1)$$

This is the same as in their text except that the expression (86.4 I) with I in m/s appears in the place of their I_{224} in mm/day.

If we rearrange equation (1) as equation (2):

$$\frac{Q/A}{I} = 1.92 \frac{I^{1.24}}{A^{0.24}} \qquad (2)$$

then both sides will be unity if the return period of the flood is 2 years, the same as I, and the time of concentration of the catchment is one day, the same as the duration of I. Reasonable values might be $Q = 2250\, m^3/s$, $A = 1000\, km^2$ and $I = 2.25$ um/s. In fact the return period of the annual flood is about 2.3 years which is close enough (?) to 2.0 years, but at many sites the time of concentration differs significantly from a day. In this context the time of concentration, t_c seconds, is "defined" as the rainfall duration which at the return period of Q has the intensity Q/A. Now we know that for maximum rainfalls of given return period the intensity decreases approximately as the square root of the duration as in equation (4):

$$\frac{Q/A}{I} = \left(\frac{t_{24}}{t_c}\right)^e \qquad (4)$$

in which e is approximately a half. At last we have a dimensionally consistent formula!

Following the principle of dimensional analysis we assume a formula for t_c:

$$t_c = = \frac{L}{Q/A} \qquad (5)$$

in which L is a length scale representing both the catchment size and the roughness which retards flow, (and is Snyder's coefficient Cp). Substituting (5) into (4) and rearranging we obtain:

$$\frac{Q/A}{I} = \left(\frac{t_{24}\, Q/A}{L}\right)^e$$

$$\frac{(Q/A)\ 1{-}e}{I} = \left(\frac{t_{24}}{L}\right)^e$$

$$\left(\frac{Q/A}{I}\right)^{1-e} = \left(\frac{t_{24}\, I}{L}\right)^e$$

$$\frac{Q/A}{I} = \left(\frac{t_{24}\, I}{L}\right)^{\frac{e}{1-e}} \qquad (6)$$

Comparing equations (2) and (6) we may equate the exponent of I by setting:

$$\frac{e}{1-e} = 1.24 \qquad \text{so } e = 0.554 \qquad (7)$$

Given this value of e, which is close to the anticipated value, we may express our arbitrary length as follows:

$$\left(\frac{t_{24}}{L}\right)^{1 \cdot 24} = \frac{1.92]}{A^{0 \cdot 24}}$$

$$\therefore L = t_{24}\left(\frac{A^{0 \cdot 24}}{1.92}\right)^{\frac{1}{1.24}} \qquad (t_{24} = 86\,400 \text{ seconds})$$

$$\simeq 50\,000\ A^{0 \cdot 2} \qquad (8)$$

The number 50 000 in equation (8) has the dimension $\text{metre}^{0.6}$ and presumably represents the roughness of the catchments. Substitution of $Q = 2250$ and $A = 1000$ into equations (5) and (8) gives $t_c = 86\,400$, the number of seconds in a day, as it ought to.

Other regional flood formulae can be analysed in the same way to find out what values of e and L are implied. The next step is to see whether the rainfall data show regional variations in e, or the catchment responses show regional variations of L. Thus we may use dimensional analysis to guide our investigations.

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