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Channel width for maximum bedload transport capacity in gravel-bed rivers, South Island, New Zealand

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Abstract

Annual bedload capacity was determined as a function of channel width for 12 gravel-bed river sites in South Island, New Zealand, based on data for channel slope, bed material size and water discharge. All sites were located in rock-walled gorges. Annual bedload at these sites computed using actual width averaged less than 50% of the theoretical maximum. Channel width of the gorges was more than three times that required for maximum bedload transport. In alluvial reaches away from the gorges, channel width was considerably larger. The notion that alluvial channels can adapt their widths to the maximum-transport capacity condition is thus demonstrated to be false. Similar analysis was undertaken using the dominant discharge rather than the full water discharge record. Maximum bedload transport based on the mean annual flood as the dominant discharge required a much wider channel than that determined from the full flow record, indicating the concept of dominant discharge to be inappropriate in this context.

Introduction

In previous research Carson and Griffiths (1987a) determined that, for a given water discharge, bed slope and bed material size, there is an optimum width of channel that maximises bedload transport capacity. Similar conclusions had been reached previously by Pickup (1976) and White *et al.* (1982).

In contrast, New Zealand river training programs have been designed on the assumption that narrowing a river increases its bedload capacity (Nevins, 1969), a conclusion apparently verified by laboratory models (Davies and Lee, 1988; Warburton and Davies, 1994). Detailed examination of the relationship between channel width and bedload capacity subsequently indicated no contradiction between these two points of view. The fact of the matter is simply that neither the laboratory tests, nor the field river training programs, had taken the narrowing process as far as the (relatively narrow) width that maximises bedload capacity (Carson, 1997). The width/depth ratio at maximum transport capacity is typically less than about 25 (depending upon actual values of slope, water discharge and bed material size) and generally less than 15 for gravel with a median diameter coarser than 25 mm (Table 1).

Table 1 – Sample values of width/depth ratio (A) for maximum transport capacity conditions for given discharge, slope and bed material, based on Equation 5

Discharge Q (m^3/s)	Slope S	S criterion (RHS of Eq. 2)	Bed material d (m)	Width/Depth ratio A	Hydraulic radius R (m)
50	0.0010	0.005	0.010	3	2
50	0.0020	0.006	0.010	7	1
50	0.0035	0.006	0.010	13	1
50	0.0050	0.007	0.010	18	1
50	0.0065	0.007	0.010	23	1
1000	0.0005	0.003	0.010	6	7
1000	0.0020	0.004	0.010	22	3
1000	0.0035	0.005	0.010	38	2
1000	0.0050	0.005	0.010	55	2
1000	0.0060	0.005	0.010	67	2
50	0.0025	0.007	0.025	3	2
50	0.0030	0.008	0.025	4	2
50	0.0040	0.008	0.025	6	1
50	0.0055	0.008	0.025	8	1
50	0.0070	0.009	0.025	11	1
1000	0.0010	0.005	0.025	5	6
1000	0.0025	0.005	0.025	12	4
1000	0.0040	0.006	0.025	19	3
1000	0.0055	0.006	0.025	25	3
1000	0.0070	0.007	0.025	32	2
50	0.0025	0.008	0.04	2	2
50	0.0030	0.009	0.04	2	2
50	0.0045	0.009	0.04	4	1
50	0.0060	0.009	0.04	6	1
50	0.0075	0.010	0.04	7	1
1000	0.0015	0.006	0.04	4	6
1000	0.0030	0.006	0.04	9	4
1000	0.0045	0.007	0.04	14	3
1000	0.006	0.007	0.04	18	3
1000	0.0075	0.007	0.04	22	3

Table 1 – Sample values of width/depth ratio (A) for maximum transport capacity conditions for given discharge, slope and bed material, based on Equation 5 – *contd.*

Discharge Q (m^3/s)	Slope S	S criterion (RHS of Eq. 2)	Bed material d (m)	Width/Depth ratio A	Hydraulic radius R (m)
50	0.0040	0.01	0.055	2	2
50	0.0045	0.01	0.055	3	2
50	0.0060	0.01	0.055	4	1
50	0.0075	0.011	0.055	5	1
50	0.0085	0.011	0.055	6	1
1000	0.0015	0.006	0.055	3	6
1000	0.0030	0.007	0.055	7	5
1000	0.0045	0.007	0.055	10	4
1000	0.0060	0.008	0.055	13	3
1000	0.0075	0.008	0.055	16	3
50	0.0040	0.011	0.07	1	2
50	0.0050	0.011	0.07	2	2
50	0.0060	0.011	0.07	3	1
50	0.0070	0.011	0.07	3	1
50	0.0080	0.012	0.07	4	1
1000	0.0015	0.007	0.07	2	6
1000	0.0030	0.007	0.07	5	5
1000	0.0045	0.008	0.07	8	4
1000	0.0060	0.008	0.07	11	4
1000	0.0075	0.008	0.07	13	3
50	0.0045	0.012	0.085	1	2
50	0.0050	0.011	0.085	2	2
50	0.0060	0.012	0.085	2	2
50	0.0070	0.012	0.085	3	1
50	0.0080	0.012	0.085	3	1
1000	0.0020	0.007	0.085	2	6
1000	0.0030	0.008	0.085	4	5
1000	0.0045	0.008	0.085	7	4
1000	0.0060	0.008	0.085	9	4
1000	0.0075	0.009	0.085	11	3
50	0.0060	0.012	0.1	2	2
50	0.0070	0.012	0.1	2	1
50	0.0080	0.013	0.1	3	1
50	0.0090	0.013	0.1	3	1
50	0.0095	0.013	0.1	3	1
1000	0.0020	0.008	0.1	2	6
1000	0.0030	0.008	0.1	3	5
1000	0.0045	0.008	0.1	6	5
1000	0.0060	0.009	0.1	8	4
1000	0.0075	0.009	0.1	9	4

All earlier work was based on the use of a single representative water discharge for channel flow. While this may be valid for comparison with laboratory channels subjected to steady flow, comparison with natural channels, in which water discharge is highly variable over time, becomes difficult. Research reported herein therefore extends our analysis to unsteady flow, in which the maximum transport capacity parameters are determined from the full flow record, rather than for a single flow. Comparison is then made between actual channel widths of gravel-bed rivers in Westland and Canterbury, New Zealand, and theoretical maximum transport capacity widths based on the channel slope, bed material and flow records at these sites.

Theoretical determination of the maximum transport capacity

Determination of an expression for the maximum transport capacity channel parameters has three requirements: a specified channel shape, a sediment transport relationship and a flow resistance relationship. The analysis by Carson and Griffiths (1987a) was undertaken assuming a rectangular channel, uniform roughness along bed and banks, a Manning roughness co-efficient that is constant irrespective of flow depth (given by the Strickler formula—Meyer-Peter and Muller, 1948), and a bedload capacity given by the Meyer-Peter and Muller (1948) transport formula.

The weakest of these assumptions probably relates to channel roughness. Data provided by Griffiths (1981) for gravel-bed rivers in New Zealand indicated that actual Manning n consistently exceeded Strickler's estimate, the ratio ranging from 1.0 to 2.1, with a mean of 1.3. His data set for gravel channels with *mobile* beds indicated an empirical relationship of the form

$$n = 0.055 \frac{\left[\dfrac{(G_s - 1)d}{g^{1.94} S}\right]^{0.258}}{R^{0.091}} \tag{1}$$

where d is median gravel size, R is hydraulic radius, S is channel slope (assumed equal to energy slope), g is gravitational acceleration and G_s is gravel specific gravity. It is this empirical relationship, rather than the Strickler formula, that is used in the present analysis. In the above equation, roughness is only weakly related to depth. The use of this formula, if Manning n is to be no less than the Strickler particle roughness $n_p = 0.048d^{0.167}$, requires that

$$S < 1.70\frac{(G_s - 1)}{g^{1.94}}\left(\frac{d}{R}\right)^{0.353} \quad (2)$$

This condition is satisfied at the river sites analysed here (Table 1). Based on data taken from Mosley (1981), this slope condition seems to be generally true for gravel channels in New Zealand that have gradients less than 0.01, which is the norm for alluvial channels.

Development of the solution for the maximum transport capacity parameters is similar to that in Appendix IV of Carson and Griffiths (1987a), but is more complex. The analysis (given in the Appendix) reduces to the following implicit expression for the width/depth ratio, A, of the maximum transport capacity channel:

$$A* = \frac{(A+2)^2}{A}\left[\frac{2+1.76A}{5.42+0.05A}\right]^{2.42} \quad (3)$$

Where A* is given by

$$A* = \frac{0.035QS^{2.60}g^{1.31}}{E_f^{\,2.42}d^{2.50}(G_s - 1)^{3.10}} \quad (4)$$

and where Q is water discharge and E_f is the Shields entrainment function, the critical dimensionless shear stress for bed movement (Henderson, 1966, p. 413). This simplifies to

$$A* = \frac{240QS^{2.60}}{d^{2.50}} \quad (5)$$

for the case of $G_s = 2.65$, $g = 9.81$, and $E_f = 0.047$ as used in the Meyer-Peter and Muller transport formula. (Note that their formula uses mean, rather than median, bed material diameter. Our analysis assumes that the two are the same. Strict application of the present approach to any given site will require reconciliation of any difference). The relationship between A* and A is given in Figure 1. Thus A can be determined from any combination of Q, S and d. Determination of the maximum transport capacity width and depth, assuming a rectangular channel, is given by B = R(A+2) and D = R (A+2) / A, where B is channel width and D is flow and channel depth.

All of the above is based on the assumption that water discharge is steady, or that the variable flow can be represented by a 'dominant discharge' (Carson, 1997). The task remains to extend this analysis to the full flow record of a river rather than basing it upon a single 'representative' water discharge. This was done, for each site (slope and bed material), by deriving a sediment rating curve for bedload transport and river discharge for at least eight assumed values of river width. Rating curves were obtained as follows: for at least five given water discharges, bedload transport rate was computed by the Meyer-Peter and Muller formula and a curve empirically fitted by non-linear polynominal regression using TIDEDA software as described by Rodgers and Thompson (1992). An example is shown in Figure 2.

Each rating curve was then applied to the flow record for that site, thus producing values of mean annual bedload capacity, Ymf, for at least eight river widths at that site. The width corresponding to the maximum transport capacity was then interpolated from these points by fitting a polynomial using non-linear regression (Fig. 3). Values for the Hurunui site are shown in bar chart form in Figure 4, showing the amount of bedload capacity available in the different flow ranges.

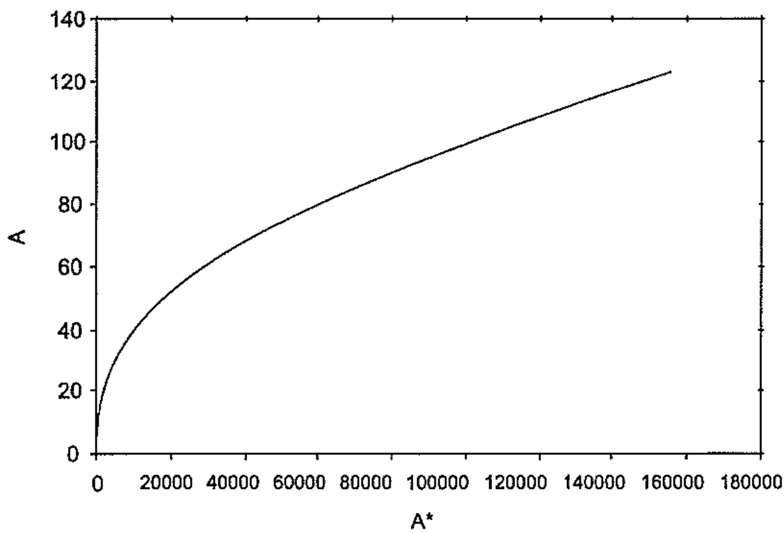

Figure 1 – Relationship between A and A*.

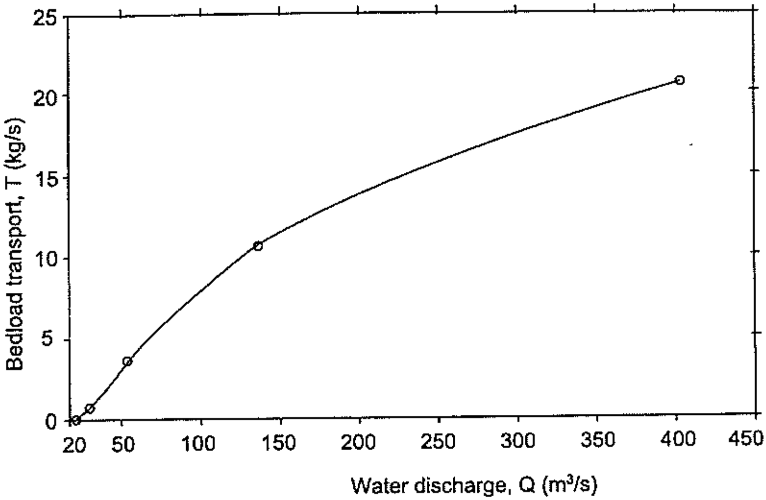

Figure 2 – Example of bedload rating curve used in the analysis: Mangles River at Gorge for B = 6 m.

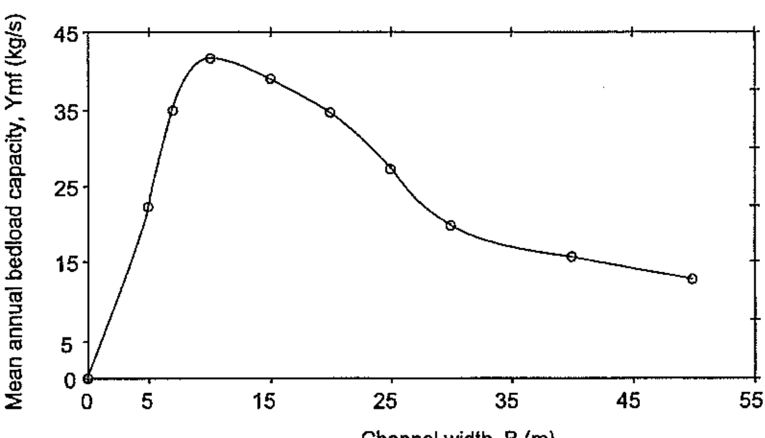

Figure 3 – Example of relationship between annual bedload yield and width: Hurunui at Mandamus.

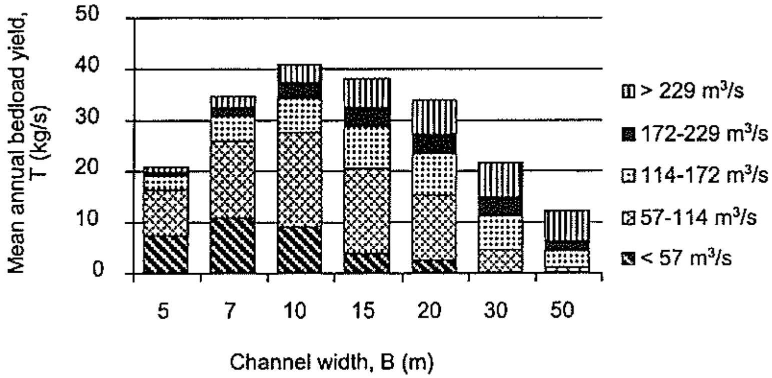

Figure 4 – Annual bedload transport at Hurunui at Mandamus at different widths, for the full flow record, broken down by flow classes.

Comparison of maximum transport capacity width and actual width for South Island rivers

Data were obtained for 12 gravel-bed rivers in Westland and Canterbury from reaches in rock-walled gorges (Table 2). In particular, flow statistics were calculated from stage and flow rating records of the hydrological recording stations listed in Table 2. Values of instantaneous water discharge at 15 minute intervals were generated (see Walter, 1994). The average slopes of the gorges were determined from 1:50,000 scale topographic maps. The median size of the surface bed material was estimated by surface sampling at a number of locations in each gorge. Channel width is the average value for a gorge as determined by field measurement. Values of other channel attributes computed in the maximum transport capacity analysis are also provided in Table 2 for comparison.

At these sites, river width was appreciably less than in alluvial reaches downstream and upstream and appears to be adjusted to present channel conditions (R. McPherson, pers. comm). At low flows gravel bars are evident in the gorges. During large floods the alluvial channel may locally scour to bedrock, but the gorges do not qualify as bedrock channels in the sense of Tinkler and Wohl (1998). The decision to use gorge sites was prompted in part by a simple academic question: do these constricted river reaches act as bottlenecks in the transport of bedload from upstream to downstream? The main reason for the choice of these sites, however, was the availability of long-term flow records and reliable data for slope, bed material size and

Table 2 – Data for South Island river gorge sites

River gorge	Flow recording site and number (Walter, 1994)		Discharge Qd	Qs (m^3/s)	Slope S (m/km)	Gravel d (mm)	Ba	Width Bmf (m)	Bmd	Ya	Bedload Capacity Ymf (kg/s)	Gmd	Ymf/Ya
Hurunui	Mandamus	65104	536	48	5.0	40	45	10	38	14	42	1248	3.00
Waiau	Marble Pt	64602	1026	102	3.0	40	50	15	47	10	22	1005	2.20
Rakaia	Fighting Hill	68526	2573	205	3.2	40	90	22	83	41	118	3501	2.88
Waimakariri	Old Highway Bdg	66401	1487	129	3.2	45	55	18	57	19	39	1734	2.05
Waipara	Ohuriawa Gorge	65901	130	4.9	10.0	30	23	8	23	4	7	948	1.89
Mangles	Gorge	93212	171	41	6.5	65	19	9	17	10	19	339	1.82
Buller	Longford	93202	687	78	4.8	60	50	13	36	8	27	1230	3.26
Rangitata	Kondyke	69302	1019	43	10.0	80	55	17	49	14	39	6801	2.79
N.Ashburton	Old Weir	68810	185	5.8	25.0	100	20	6	22	12	18	4572	1.57
Ahuriri	Sth Diadem	71116	244	24	6.7	50	20	7	24	10	19	759	1.98
Ashley	Gorge	66204	305	46	4.5	40	30	10	26	15	25	503	1.67
Orari	Gorge	69505	202	37	8.0	80	30	8	18	13	25	582	1.92

Qd is dominant discharge (mean annual flood); Qs is the steady discharge that gives the same maximum transport capacity yield as that computed using the full flow record; B is channel width; a is actual; mf is maximum transport capacity value associated with full flow record; md is maximum transport capacity value associated with dominant discharge; Y denotes mean annual value; G denotes instantaneous value.

channel width. Also, the gorge cross-sections, at least over the range of relevant flow depths, were approximately rectangular, as assumed in the theoretical analysis.

Computations of maximum transport capacity values of channel width and annual bedload capacity using dominant discharge were based on the mean annual flood. Actual river widths, Ba, ranged from 19 to 90 metres. The width that maximises bedload transport capacity at these sites, when based on the dominant discharge, Bmd, was virtually the same: these maximum transport capacity widths ranged from 17 to 83 metres. Note that the Bmd values obtained in this new maximum transport capacity analysis (using channel roughness given by equation 1) were not appreciably different from those using the earlier maximum transport capacity analysis by Carson and Griffiths (1987a), which had assumed roughness determined solely by particle size (Table 3). The choice of critical shear stress, E_r, had a greater effect on the maximum transport capacity width, but even then the difference was small.

Table 3 – Variability in width at maximum transport capacity state according to assumptions used in analysis

	River width, Bmd, (m)		
River Gorge	**New MTC Ef=0.047**	**New MTC Ef=0.056**	**C&G1987 Ef=0.056**
Hurunui	38	34	40
Waiau	47	43	49
Rakaia	83	76	90
Waimakariri	57	52	62
Waipara	23	21	25
Mangles	17	15	16
Buller	36	32	38
Rangitata	49	44	50
N.Ashburton	22	20	22
Ahuriri	24	21	25
Ashley	26	24	29
Orari	18	16	17
MEAN	36.7	33.3	38.6
STD DEV (SD)	18.9	17.2	20.7
SD/MEAN	0.51	0.52	0.54

New MTC refers to maximum transport capacity derived for this analysis, based on Equation 1. C&G1987 refers to Carson and Griffiths (1987a).

At first sight, then, it might seem that these rivers have adjusted their gorge widths to correspond, approximately, to the maximum transport capacity. Yet, not only is it difficult to envisage how this might take place, but the correspondence is largely fortuitous. What matters is this analysis is not simply the magnitude of mean annual flood or any other measure of dominant discharge, but the full flow record. When the maximum transport capacity widths are recalculated based on the full flow record, the new values, Bmf, are appreciably smaller: 6 to 22 metres, averaging less than 30% of actual widths (Table 2). The implication is that considerable gravel transport can occur at flows well below the mean annual flood, provided that the channel is narrow enough, as evident in Figure 4. This does not mean that most of the *actual* bedload movement occurs at these lower flows. On the contrary, because, *even in these gorges*, river width is appreciably greater than the maximum transport capacity width, higher water discharges are required to raise bed shear stress sufficiently to move most of the gravel. Our analysis suggests that gravel transport capacity in the South Island gorges, on average, is less than half of that possible at the narrow maximum transport capacity width (Table 2).

Implications of the results

The 'bottleneck effect' of rock gorges

This analysis suggests that rock-walled gorges should not act as bottlenecks for gravel transport because the narrowness of the channels at these sites allows river depths, and hence bed shear stresses, to be sufficiently large to induce gravel movement for much longer periods of time than at wider sites upstream. While the possibility of short-term bottlenecks needs to be appraised (during those periods of high flow when gravel is being supplied from upstream), in the long term these narrow gorges have higher bedload capacities than upriver reaches, assuming similar slope and bed material.

The same conclusion is reached when considering the effects of artificial constrictions of wide channels ('river training') on bedload transport. The results of our analysis confirm the view that river narrowing (up to the limit maximum transport capacity width) increases bedload capacity in a reach. Of course, whether or not such narrowing would actually prevent aggradation in that reach would depend upon whether the narrowing increased bedload transport through the reach *sufficiently* to exceed bedload input from upstream.

In reality, the situation is likely to be more complex because the slope and bed material upstream may differ from that in the narrowed reach, and thus the question of bottlenecks may be complex and case-specific. (Griffiths, 1993).

Inappropriateness of the concept of dominant discharge

For many years the complexity of trying to model river channel behaviour in terms of an unsteady water discharge, as is typical of most natural rivers, has been circumvented by assuming that the variable flows of a river can be represented by a single discharge or 'dominant' discharge that is assumed to be the equivalent, for channel-forming purposes, of the changing river discharge (Lacey, 1929; Blench, 1969; Huang and Nanson, 2000). Typically, the dominant discharge is taken to be either 'bankfull' discharge, mean annual flood, or the 2-year flood peak. Hydraulic geometry relationships based on dominant discharge have generally proved useful for engineering purposes, but this does not necessarily mean that the concept of dominant discharge is valid.

Our findings above, in fact, seriously question the premise underlying the use of dominant discharge, Qd, in channel studies. Estimates of maximum transport capacity width based on dominant discharge, Bmd, were 2.2 to 3.8 times those based on the full flow, Bmf, averaging three times higher (Table 4). In these analyses, the *steady* water discharge, Qs, that would produce a maximum transport capacity width equal in magnitude to that determined from the full flow record was also calculated. These discharges, shown in Table 2, are significantly lower than the mean annual flood, and in fact approach the mean annual flow. Doubtless the relationship between Qd and Qs will vary depending on the nature of the flow duration curve.

In order to examine directly the effect of variable flow conditions on both the maximum transport capacity width and the bedload yield occurring at that width, the flow records for the 12 river sites were standardised to a common mean annual flood (that of the Hurunui River). The analysis of bedload yields in relation to channel width was then repeated for the S and d values of the Hurunui River site. Discharge was standardised by dividing each river's water discharge values by that river's mean annual flood value, Qmaf, and then multiplying by the Qmaf value for the Hurunui. The results of this analysis are summarised in Table 5 and reveal two important observations.

The first point is that there is significant variation in bedload yield under maximum transport capacity conditions, for fixed Qmaf and site conditions, depending on the flow record, ranging from a minimum of 10.2 kg/s (322 Mt/yr) based on the Ashley flow record to 42.2 kg/s (1332 Mt/yr) using the Rangitata flow record. The variation is primarily a reflection of how much of each flow record is above the threshold water discharge for bedload transport. Figure 4 (which gives a breakdown of Hurunui bedload transport by arbitrary flow classes) shows that, at the Hurunui site, this threshold is about 57 m^3/s, though it does, of course, vary with width. Figure 5 shows that the maximum transport capacity bedload yield at the 12 sites, predicted

Table 4 – Comparison of maximum transport capacity data using full flow records with maximum transport capacity data based on dominant discharge

	Bmd/Ba	Bmf/Ba	Bmf/Bmd	Bmd/Bmf	Gmd/Ymf
Hurunui	0.84	0.22	0.26	3.79	30
Waiau	0.95	0.30	0.32	3.16	46
Rakaia	0.92	0.24	0.27	3.77	30
Waimakariri	1.04	0.33	0.31	3.19	44
Waipara	1.02	0.33	0.33	3.04	144
Mangles	0.88	0.45	0.51	1.96	18
Buller	0.72	0.26	0.36	2.77	46
Rangitata	0.88	0.31	0.35	2.86	174
N.Ashburton	1.08	0.29	0.27	3.73	254
Ahuriri	1.19	0.35	0.29	3.39	40
Ashley	0.88	0.33	0.38	2.64	20
Orari	0.59	0.27	0.45	2.23	23
MEAN	0.92	0.31	0.34	3.04	72.4
STD DEV (SD)	0.15	0.06	0.07	0.57	72.7
SD/MEAN	0.17	0.18	0.21	0.19	1.0

B is channel width; Y is mean annual bedload capacity; G is instantaneous bedload capacity; a is actual; md is maximum transport capacity condition at dominant discharge; mf is maximum transport capacity condition based on full flow record.

Table 5 – Bedload yields computed using standardized flow record applied to Hurunui site conditions.

	% flow > 58 m³/s	Annual bedload yields (kg/s) at the given channel widths (m)						
		5	7	10	15	20	30	50
Hurunui	28	21	34	41	38	34	22	12
Waiau	27	20	34	40	38	33	22	13
Rakaia	20	16	27	31	29	25	16	9
Waimakariri	19	16	26	31	29	26	17	11
Waipara	4	3	5	7	7	7	6	4
Mangles	11	9	15	18	18	17	12	8
Buller	36	25	42	51	50	45	30	17
Rangitata	29	22	36	39	42	35	22	13
N.Ashburton	9	7	13	17	21	22	21	19
Ahuriri	28	21	35	41	37	33	21	12
Ashley	6	5	9	10	10	9	7	5
Orari	8	7	11	12	11	10	7	5

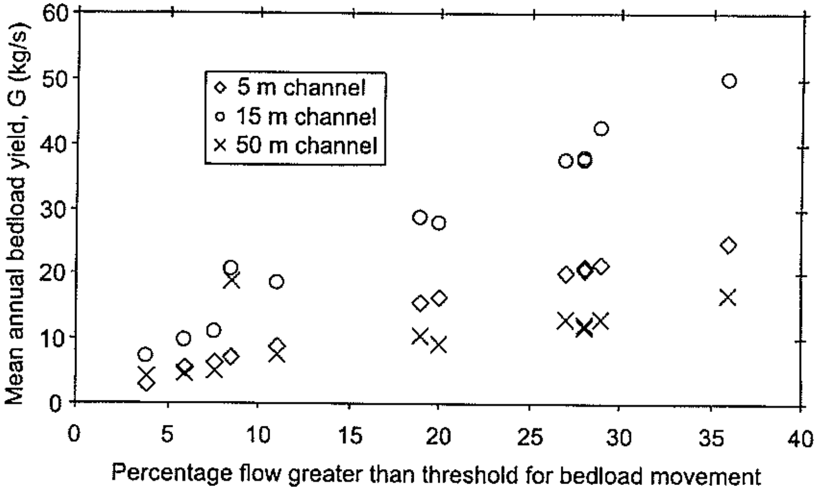

Figure 5 – Annual bedload yield at 12 gorge sites as a function of percentage of flow above threshold discharge: (a) 5 m channel; (b) 15 m channel; (c) 50 m channel.

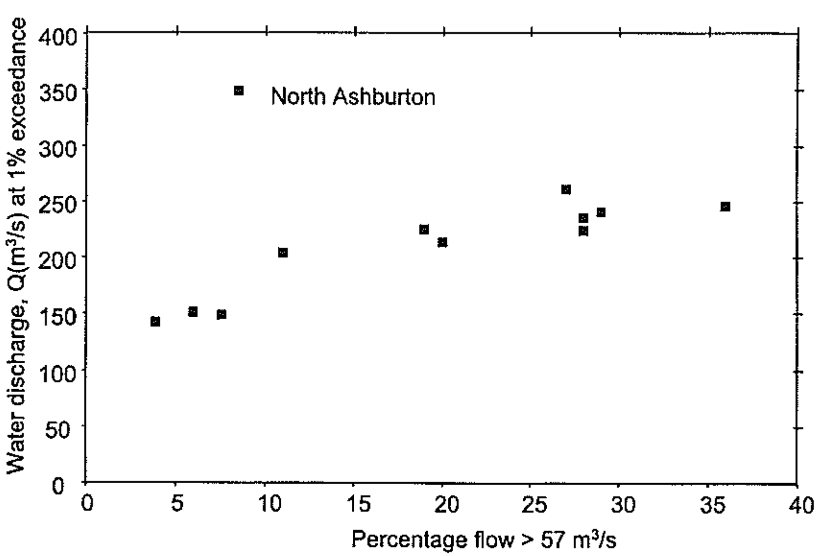

Figure 6 – Hydrologic record characteristics of 12 gorge sites, showing anomalously high Q1% for North Ashburton River at Old Weir.

from the standardised flow record, is strongly correlated with the % flow that is above the threshold discharge, though the anomaly of North Ashburton becomes quite pronounced in the case of wider channels.

The second point is that *the actual width of the maximum transport capacity channel itself* is not uniquely determined by channel slope, bed material, and dominant discharge, but is also affected by the nature of the flow record. While most flow records maximise bedload yield at a width of about 10 metres, in the case of the Waipara and Rangitata sites it is 15m, while the maximum transport capacity width for the North Ashburton is 20 m.

The North Ashburton, as noted previously, is somewhat anomalous, owing to peculiarities in its flow record or flow duration curve at higher flows. As shown in Figure 6, while there is a strong correlation between Q at 1% exceedance and the percentage of flow greater than 57 m^3/s among the other 11 rivers, the North Ashburton has a much higher flow at the 1% exceedance level (even though standardised to the same mean annual flood). It is for this reason that its bedload yield (relative to the duration of bedload-moving flows, Figure 5) and the width of its maximum transport capacity channel are higher than those of the other rivers.

Discordance between maximum transport capacity and regime widths

Our third general finding relates to the view expressed by some authors that 'regime channels', defined as those with a rough equilibrium between input and output of bed sediment, actually correspond to the state of maximum transport capacity. White *et al.* (1982), for example, used as a working hypothesis that 'an alluvial channel adjusts its geometric characteristics and gradient in such a way that the sediment transporting capacity is maximised', and then proceeded to verify this by showing good agreement between maximum transport capacity widths and the widths of both 'regime' canals and 'regime' gravel channels (based largely on the data of Bray (1982) in Alberta, Canada). For further recent work on this topic see Huang and Nanson (2000).

Such a conclusion, that natural channels in equilibrium adjust their widths to a magnitude that, based on prevailing discharge, slope and bed material, maximises bedload transport, raises many questions. Not least of these is the mechanism by which such channels prevent excess bank erosion which would otherwise widen the channels beyond the maximum transport capacity state. Our own findings above indicate that the hypothesis of White *et al.* (1982) is certainly inapplicable in gravel-bed rivers of South Island, New Zealand. Specifically, even where confined by rock-walled gorges, these channels are 2 to 5 times the maximum transport capacity width. In the alluvial reaches of these rivers downstream of these gorges, with essentially the same Q, and roughly similar S and d, the width of the actual gravel bed may be 2 to 20 times that of the width in the gorges, and thus even more discordant with the maximum transport capacity width.

The question thus arises as to why the White hypothesis is repudiated in our sample, while being seemingly verified in their earlier work. There are at least two reasons. One is that White *et al.* (1982) based their determination of the maximum transport capacity width on dominant discharge (bankfull discharge), and not the full flow record. As noted above, use of dominant discharge leads to much higher estimates of the maximum transport capacity width, although in the New Zealand study even these inflated maximum transport capacity widths (while comparable to the gorge widths) were much less than those of the corresponding river in nearby alluvial reaches. The second reason reflects the contrast in the channels studied. The New Zealand rivers are highly active and transport considerable quantities of bedload. In contrast, the Alberta rivers, with comparable bed material size and discharge, have gradients very close to the threshold for bed mobility (Carson and Griffiths, 1987b, Tables 5.2 and 5.3). Indeed, recognising this, there is a strange inconsistency in attempting to 'explain' the width of *low-transport* channels in terms of a model based on the condition of *maximum transport* capacity. It seems more likely that the reasonable results obtained for the Alberta gravel channels were largely fortuitous.

Our analysis above indicates that the maximum transport capacity condition is an inherently unstable channel condition, with severe shear stresses on channel banks as well as on channel bed material. Even rock-walled channels, lateral erosion is so intense that channels are widened well beyond the maximum transport capacity width. It is unrealistic to expect channels formed in alluvial material to be able to withstand such high bank shear stresses. For this reason it should be expected that all natural bedload channels will exist at widths much greater than given by maximum transport capacity analysis.

Conclusion

The analysis described above is, to our knowledge, the first attempt to predict maximum transport capacity widths and bedload yields of gravel bed channels utilising actual flow records. The results demonstrate that predictions of maximum transport capacity conditions based on dominant discharge are subject to large errors when compared with calculations based on the full flow record for the channel. Comparison with actual channel widths in rock gorges in South Island, New Zealand, show that natural channels will invariably exceed the width that maximises bedload transport capacity because of the huge shear stresses on channel sides. This discrepancy is even more pronounced in alluvial reaches downstream of these gorges. It is thus clear that the notion of 'equilibrium' channels in nature attaining the condition of maximum transport, as sometimes entertained in the past, is invalid.

References

Blench, T. 1969: *Mobile-Bed Fluviology*. University of Alberta Press, Canada, 168p.

Bray, D.I. 1982: Regime equations for gravel-bed rivers. In R.D. Hey; J.C. Bathhurst; C.R. Thorne (*eds.*), *Gravel Bed Rivers*. John Wiley and Sons: 517-542

Carson, M.A. 1997: Optimum channel width for gravel bedload transport in channels of given discharge, slope and bed material size. Report U99/70, Canterbury Regional Council, Christchurch, New Zealand, 12p.

Carson, M.A.; Griffiths, G.A. 1987a: Influence of channel width on bedload transport capacity. *Journal of Hydraulic Engineering 113(12):* 1489-1509.

Carson, M.A.; Griffiths, G.A. 1987b: Bedload transport in gravel channels. *Journal of Hydrology (New Zealand) 26 (1):* 1-151.

Davies, T.R.H.; Lee, A.L. 1988: Physical hydraulic modelling of width reduction and bed level change in braided rivers. *Journal of Hydrology (New Zealand) 27 (2):* 113-127.

Griffiths, G.A. 1981: Flow resistance in coarse gravel bed rivers. *Journal of the Hydraulics Division, ASCE, 107 (HY7):* 899-918.

Griffiths, G.A. 1984: Extremal hypotheses for river regime: an illusion of progress. *Water Resources Research 20(1):* 113-118.

Griffiths, G.A. 1993: Sediment translation waves in braided gravel bed rivers. *Journal of Hydraulic Engineering 119(8):* 924-937.

Henderson, F.M. 1966: *Open Channel Flow.* Macmillan, New York, USA, 522p.

Huang, H.Q.; Nanson, G.C. 2000: Hydraulic geometry and maximum flow efficiency as products of the principle of least action. *Earth Surface Processes and Landforms, 25(1):*1-16.

Lacey, G. 1929: Stable channels in alluvium. *Proceedings of the Institution of Civil Engineers 229:* 259-292.

Mosley, M.P. 1981: Semi-determinate hydraulic geometry of river channels, South Island, New Zealand. *Earth Surface Processes and Landforms 6:* 127-137.

Meyer-Peter, E; Muller. R. 1948: Formulas for bedload transport. Proceedings of the International Association for Hydraulic Research, 3rd Annual Conference, Stockholm: 39-64.

Nevins, T.H.F. 1969: River-training: the single-thread channel. *New Zealand Engineering 24:* 367-373.

Pickup, G. 1976: Adjustment of stream-channel shape to hydrologic regime. *Journal of Hydrology 30:* 365-373.

Rodgers, M.W.; Thompson, S.M. 1992: Tideda Reference Manual (2nd Edition). Freshwater Division, National Institute of Water and Atmospheric Research, Christchurch, New Zealand, 222p.

Tinkler, K.J.; Wohl, E.E. 1998: *Rivers over Rock: Fluvial Processes in Bedrock Channels.* American Geophysical Union, Washington, DC, USA, 323p.

Walter, K.M. 1994: Index to Hydrological Recording Stations in New Zealand. NIWA Science and Technology Series No. 17, National Institute of Water and Atmospheric Research, Christchurch, 152p.

Warburton, J.; Davies, T.R.H. 1994: Variability of bedload transport and channel morphology in a braided river hydraulic model. *Earth Surface Processes and Landforms 19:* 403-421.

White, W.R.; Bettess, R.; Paris, E. 1982: Analytical approach to river regime. *Journal of the Hydraulics Division, ASCE, 108 (HY10):* 1179-1193.

APPENDIX

Development of a solution for the maximum transport capacity condition in a rectangular channel with uniform roughness along bed and banks is similar to that in Appendix IV of Carson and Griffiths (1987a), but is more complex.

The flow resistance equation for mobile gravel beds adopted from Griffiths (1981) is:

$$\frac{1}{\sqrt{f}} = 2.41\left[\frac{V}{\sqrt{g(G_s-1)d}}\right]^{0.340} \qquad \text{A1}$$

where f is the Darcy-Weisbach friction factor and V is the mean velocity of flow. By definition (Henderson, 1966)

$$f = \frac{8gRS}{V^2} \qquad \text{A2}$$

and

$$n = \frac{BDR^{0.667}S^{0.5}}{Q} \qquad \text{A3}$$

Combination of Equations A1, A2 and A3 yields, with Q=VBD,

$$Q = 18.3\frac{BD(RS)^{0.758}g^{0.5}}{\left[(G_s-1)d\right]^{0.258}} \qquad \text{A4}$$

Equation A4 is comparable to Equation 43 in Carson and Griffiths (1987a).

This equation now needs to be combined with the Meyer-Peter and Muller (1948) bedload transport formula to determine an extreme value in the transport function, subject to the roughness and shape conditions embodied in A4.

The Meyer-Peter and Muller formula may be expressed as

$$G = 8B\gamma G_s[g(G_s - 1)d^3]^{0.5}\left[\left(\frac{n_p}{n}\right)^{1.5}\frac{RS}{(G_s-1)d} - E_f\right]^{0.5} \qquad \text{A5}$$

where G is the bedload transport rate by weight, γ is the specific weight of water, n_p is the Manning-Strickler co-efficient for particle roughness, and d is the mean size (taken here to be approximately equal to the median gravel size, as noted previously).

Now the Langrangian for G from A4 and A5 is

$$G = 8B\gamma G_s[g(G_s - 1)d^3]^{0.5}\left[\left(\frac{n_p}{n}\right)^{1.5}\frac{RS}{(G_s-1)d} - E_f\right]^{0.5}$$

$$+\lambda\left\{BDR^{0.758} - 0.055\left[\frac{(G_s-1)d}{S_g^{1.94}}\right]^{0.258}\frac{Q}{S^{0.5}}\right\} \qquad \text{A6}$$

Where λ is the Lagrangian multiplier. Further detail on the deviation of Equation A6 is given in Griffiths (1984, p. 114).

To obtain an extreme value from A6 we eliminate λ between $\partial G/\partial B = O$ and $\partial G/\partial D = O$ to arrive at, after lengthy manipulation:

$$R - 0.022\left\{\frac{E_f}{n_p^{1.5}g^{0.75}}\left[\frac{(G_s-1)d}{S}\right]^{1.39}\left(\frac{2+1.76A}{5.42+0.05A}\right)\right\}^{0.879} \qquad \text{A7}$$

It may be readily shown that this value of R produces a maximum value of G. Equation A7 is comparable to Equation 46 of Carson and Griffiths (1987a).

Elimination of R between Equations A4 and A7 gives, using the Strickler formula, $n_p = 0.048\ d^{0.167}$ (Meyer-Peter and Muller, 1948)

$$A^* = \frac{0.035QS^{2.60}g^{1.31}}{E_f^{2.42}d^{2.50}(G_s-1)^{3.10}} \qquad \text{A8}$$

where

$$A^* = \frac{(A+2)^2}{A}\left[\frac{2+1.76A}{5.42+0.05A}\right]^{2.42} \qquad \text{A9}$$

Equations A8 and A9 are comparable to Equations 14a and 14b respectively in Carson and Griffiths (1987a).

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