Journal of Hydrology (NZ) Vol. 31 No. 2, 1993

NEW METHODS OF UNITGRAPH AND LOSS RATE ESTIMATION APPLIED TO URBAN CATCHMENTS

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ABSTRACT

A method has been developed for estimating a common unitgraph for a set of runoff events, without using rainfall data. For a given event, the calculated input to this unitgraph generally lags behind the actual rainfall hyetograph. The unitgraph derived in this way also has a higher peak and shorter time to peak than unitgraphs derived by conventional methods. Both these outcomes indicate that a nonlinear storage should be included in the rainfall loss model to be used in conjunction with the common unitgraph.

These results are demonstrated using streamflow and rainfall data for two urbanised catchments in the Auckland region. Runoff events have been identified by a baseflow separation algorithm applied to the record of total stream flow. For each catchment, a common unitgraph has been determined for a set of large runoff events, and a best-fit loss model has been estimated. The performance of the overall model is described in terms of prediction of peak flow and time to peak.

INTRODUCTION

The unit hydrograph (or unitgraph) for a catchment, for a time period T, is defined as the discharge which would result from 1 mm of rainfall excess occurring on the catchment at a uniform rate in a time T. For vanishingly small T, the unitgraph is called the instantaneous unit hydrograph (IUH). The rainfall excess is that part of the rainfall which reaches the catchment outlet as surface runoff or quick flow.

Unitgraphs are widely used by practising hydrologists to determine the design flood hydrograph for a catchment; they are also used for flood forecasting and for the estimation of missing flow records. The method is generally considered inapplicable to very large catchments, where there will be variations in the distribution of rainfall excess from one event to another, and to very small and urban catchments, where it has been suggested that the linearity requirement may not be met (Institution of Engineers Australia, 1987, p.153).

The unitgraph is only part of the overall rainfall-runoff model. A loss model first estimates the rainfall excess from the catchment rainfall hyetograph; the unitgraph is applied to this rainfall excess to determine the surface runoff hydrograph; and a baseflow model provides the additional low flow which is combined with the surface runoff to obtain the total streamflow hydrograph. The recognition that the unitgraph is a measure of the response of the catchment to a lumped input has led to the concept of a geomorphological IUH (Rodriguez-Iturbe and Valdes, 1979; Gupta et al., 1980), defined as the probability density function of travel times along all possible paths to the catchment outlet.

The basis of unitgraph theory is the assumption that the catchment runoff process is a linear system, so that if the rainfall excess is doubled, each ordinate of the outflow surface runoff hydrograph will be doubled. It is usually considered (e.g. Institution of Engineers Australia, 1987, p.154) that this assumption is only valid at higher stages (typically with overbank flow) when the stream velocity is almost constant, and that unitgraphs should not be used for smaller events, for which nonlinear routing methods are recommended.

A second underlying assumption of unitgraph theory is that the spatial distribution of rainfall excess on the catchment is always the same. The spatial distribution does not have to be uniform (e.g. there may be consistently higher rainfall in elevated parts of the catchment), but it must be approximately the same from event to event. In some cases it may be possible to derive different unitgraphs for situations with a different spatial distribution, such as a partial-area storm on a large catchment.

The conventional method of deriving unitgraphs uses stream flow and rainfall data, and involves the following steps for each storm:

- Separation of base flow to obtain a surface runoff hydrograph of finite duration;
- Calculation of the hyetograph of catchment rainfall;
- Application of a loss model to the catchment rainfall to obtain a hyetograph of rainfall excess with the same volume as the total surface runoff;
- Operating on the surface runoff hydrograph and the rainfall excess hyetograph to obtain the unitgraph; a least-squares technique (Institution of Engineers Australia, 1987, p.158) is frequently adopted, though many other approaches have been advocated (e.g. O'Donnell, 1960; Dooge, 1965; Eagleson et al., 1966; Neuman and de Marsily, 1976).

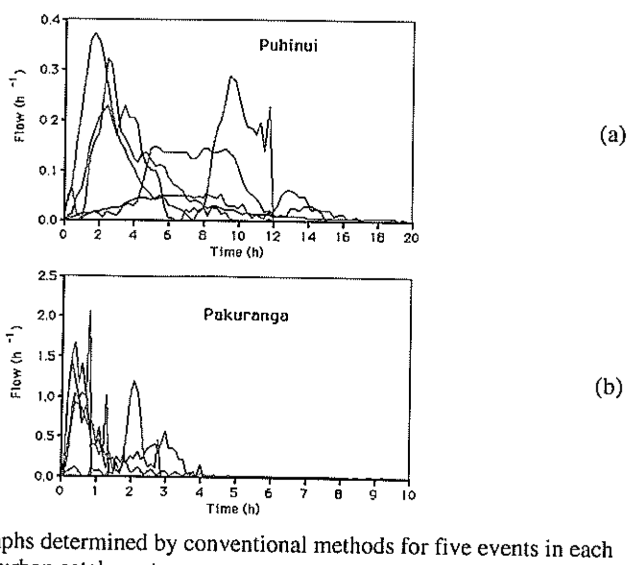


FIG. 1 — Unitgraphs determined by conventional methods for five events in each of two urban catchments..

As illustrated in Figure 1, it is typically found that unitgraphs derived in this way vary greatly from one event to another, and there is therefore considerable doubt about selection of a design unitgraph. These variations in unitgraphs may be partly due to variations in the spatial characteristics of the rainfall from one event to another, but a large factor is the loss model. Malone and Cordery (1989), in a comparison of unitgraphs with nonlinear network models, concluded that the loss model is more critical than the runoff model. There are therefore substantial advantages to be gained from a method of deriving unitgraphs which uses only the streamflow data, and therefore does not require the prior application of an assumed loss model. In fact such an approach can, at least in theory, lead to the estimation of the rainfall excess hyetograph, which can then be compared with the rainfall hyetograph to obtain an objective measure of the loss rate. This approach, applied to two urban catchments in the Auckland region, is the subject of this paper.

The proposition that unitgraphs could be derived from streamflow data only was first put forward by De Laine (1970). This method involved calculation and matching of the roots of polynomial expressions, and while it allowed for the use of more than two outputs, it was discarded by De Laine (1975) in favour of a simple process of successive operations of scaling, superposition, subtraction and shifting, applied to two runoff hydrographs. Childs (1982) developed De Laine's later method to enable determination of the unitgraph from a linear system subject to measurement errors, and his technique was demonstrated for a number of catchments by Chapman (1987). As each pair of events results in an estimated unitgraph, this method also suffers from the problem of estimating an average unitgraph, though the variations between estimates are generally less than those for unitgraphs calculated by the conventional method.

In the next section a method is described which overcomes these difficulties by simultaneously handling any number of events.

OUTLINE OF ANALYSIS AND NUMERICAL METHOD

The only assumption made is that all the runoff hydrographs for different storms on a catchment are the outcome of a linear process which has a common kernel, or unitgraph. It can then be shown (Chapman, 1992) that there will be more equations than unknown variables when at least two hydrographs are available; the solution becomes more constrained as the number of hydrographs increases.

The unitgraph is defined as a vector **u** with ordinates $u_1, u_2, ...u_k$, while the output hydrograph **q** has ordinates $q_1, q_2, ...q_n$ and the rainfall excess **r** has ordinates $r_1, r_2, ...r_j$. The numbers of ordinates are related by

$$j + k = n + 1 \qquad (1)$$

and the unitgraph equations can be expressed by

$$q = [R]\, u \qquad (2)$$

where [R] is a n x k matrix. The first column of this matrix is formed by the elements of the vector **r**, with the remaining entries set to zero. Subsequent columns are obtained by a cyclic permutation in which each element of the previous column is shifted down by one row, and the first entry is replaced by the last.

The conventional least-squares method (e.g. Institution of Engineers Australia, 1987, p.158) determines the unitgraph from given vectors **q** and **r** by

$$u = \{[R]^T[R]\}^{-1} [R]^T q \qquad (3)$$

If we now have a set of output hydrographs $\mathbf{q}_1,...\mathbf{q}_i,....\mathbf{q}_h$ corresponding to (unknown) inputs $\mathbf{r}_1,...\mathbf{r}_i,....\mathbf{r}_h$, each input can be determined from an equation similar to (3) as

$$r_i = \{[U]^T[U]\}^{-1} [U]^T q_i \qquad (4)$$

where the n x j matrix [U] is formed from the elements of **u** in the same way that [R] was formed from the elements of **r**.

It then follows readily that the least-squares solution for the common unitgraph **u**, given the h output hydrographs $\mathbf{q}_i$ and the inputs $\mathbf{r}_i$ calculated from (4), is

$$u = \left[\sum_{i=1}^{h} \{[R_i]^T[R_i]\} \right]^{-1} \sum_{i=1}^{h} [R_i]^T q_i \qquad (5)$$

Equations (4) and (5) form the basis of an iterative procedure, which starts with an assumed common unitgraph **u**, calculates a set of values $\mathbf{r}_i$ for the inputs from (4), and then calculates a new estimate of **u** from (5). These equations however do not take account of the necessary constraints that the ordinates of **u** and each $\mathbf{r}_i$ must be non-negative, the total input for each event must be equal to the total output, and the ordinates of the unitgraph must sum to unity. Rather than including these constraints in the solution, the approach used in work to date has been to apply them after each iteration through equations (4) and (5).

To implement the above analysis, a computer program has been written to perform the joint task of identifying the common unitgraph for a set of surface runoff events and calculating the corresponding input hyetographs for each individual event. Details of particular aspects are given in Chapman (1992); the computation sequence is listed here:

1. The ordinates of each runoff hydrograph are scaled so that their sum is unity; the scaling factors are retained.
2. An initial unitgraph **u** is specified.
3. Rainfall excess vectors $\mathbf{r}_i$ are calculated for each event, using Equation 4.
4. In each of these vectors, any negative ordinates are set to zero, and the remaining ordinates are scaled so that their sum is unity.
5. A new unitgraph is calculated from the surface runoff and rainfall excess vectors, using Equation 5.
6. Any negative ordinates in the unitgraph are set to zero, and the remainder are scaled so that their sum is unity.
7. Each rainfall excess vector is convolved with the unitgraph to obtain a set of predicted runoff hydrographs.
8. The sum of squares of differences between ordinates of each predicted and observed runoff hydrograph is calculated, and the total sum of squares for all events is converted to a root-mean-square (rms) value.

9. If the change in the rms value in the last two iterations is less than 10^{-6} or a preset maximum number of iterations has been reached, the ordinates of the common unitgraph, the rainfall excess hyetographs and the predicted runoff hydrographs (both multiplied by the previously calculated scaling factor) are output to a file, and the program ends.
10. Otherwise, a new unitgraph is calculated from the values obtained in the last two iterations.
11. The program returns to step 3.

An additional statistic which is output in step 8 is the percentage contribution of each event to the total sum of squares. If one event stands out as responsible for most of the error, the operator can omit that event from the data set and/or investigate whether there is an error in the calculation of the observed hydrograph.

The initial unitgraph used has been a truncated gamma distribution scaled to unit area and with its time to peak equal to one-third of the base length; but tests have shown that the method is quite insensitive to the shape of the initial unitgraph. Of more consequence is the need to experiment with different base lengths for the unitgraph, since this is unknown a priori. The method adopted is to start with the maximum possible length, defined by the number of ordinates in the shortest runoff hydrograph (see Equation 1 with j=1), and to reduce this length in successive trials. The rms error decreases as the number of ordinates gets less, but at some point the predicted unitgraph becomes discontinuous or oscillatory. The unitgraph selected is that which has the lowest rms error and a plausible shape.

When Equation 5 alone is used to obtain the new unitgraph, convergence becomes very slow after the first few events. The program therefore provides an acceleration algorithm (Chapman, 1992) which calculates the new unitgraph from the values obtained in the last two steps.

TABLE 1 — Catchments and data sets - Pakuranga and Puhinui catchments, Auckland.

Stream name	Pakuranga	Puhinui
Gauging site	Moony's bridge	Drop structure
Catchment no.	8207	43807
Rain gauge no.	649927	740815
Rain gauge site	Substation	Puhinui gardens
Catchment Area (km^2)	3.08	11.6
Urbanised area (km^2)	3.08	4.0
Elevation range (m)	15-76	20-157
Data period	1/4/85- 28/11/85	1/3/90- 28/2/91
Sample interval (h)	0.1	0.25
No. events > 100 Ls^{-1}	102	65

CATCHMENTS AND DATA SETS

The above analysis and numerical procedure has been implemented with data from two catchments in the Auckland region. Details of the catchments and data are shown in Table 1.

BASEFLOW SEPARATION

Since the unitgraph is a model of surface runoff, base or slow flow must first be subtracted from the streamflow hydrographs in order to obtain discrete surface (or quick flow) hydrographs for individual storms. For consistency in this study, the same separation algorithm (Chapman, 1991) has been used throughout. This algorithm is based on the assumption that the base flow is a weighted average of the previous base flow and the average of the current and previous direct runoff. When expressed as a relation between the base flow q_{bi} and the total flow q_{ti} at time interval i, it takes the form

$$q_{bi} = \frac{3k-1}{3-k} q_{b,i-1} + \frac{1-k}{3-k} (q_{ti} + q_{t,i-1}) \qquad (6)$$

subject to $q_{bi} \leq q_{ti}$.

This algorithm requires specification of one parameter 'k', which can be identified with the baseflow recession constant. This algorithm works well in catchments with an exponential recession characteristic, but was only partially successful on the Pakuranga and Puhinui catchments, where the base flow after a storm may decrease and then slowly increase again for some time. While this obviously requires a more complex separation algorithm, for the current study plausible separations were achieved by using Equation 6 with the further constraint that initiation of surface runoff required an increase in stream flow of more than 10% in one time interval. The adopted values for k were 0.995 for the Pakuranga and 0.997 for the Puhinui. Examples of the separation are shown in Figure 2.

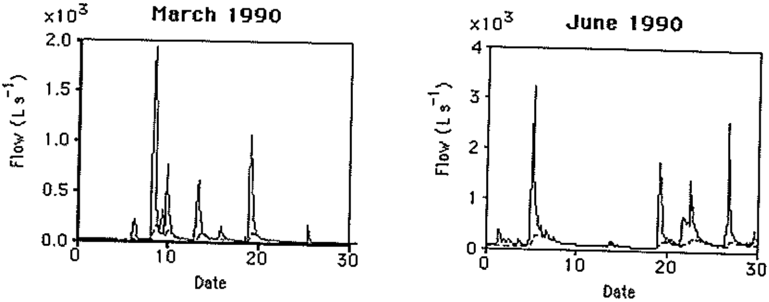


FIG. 2—Base flow separation for the Puhinui at drop structure.

EXAMINATION OF SAMPLING INTERVAL

Practical applications of unitgraph theory require that the observed hydrographs be sampled at equal time intervals. In order to avoid unnecessary computation, these intervals should be as long as possible, subject to not losing any information in the sampling process. Childs (1982) used an approximate estimate for the Nyquist sampling interval developed by Goodspeed (1968) to show that the hydrograph can be reconstructed in continuous time if samples are taken at time intervals no greater than Δt_m, defined by

$$\Delta t_m = \pi M / 6s \tag{7}$$

where M is the maximum flow ordinate, and s is the maximum slope (positive or negative) of the hydrograph.

Values of Δt_m were calculated for each surface hydrograph, and the range of ratios of adequate to actual sample interval Δt are shown in Table 2. The results suggest that the 0.25 h interval for the Puhinui is adequate, but the 0.1 h interval for the Pakuranga is too long in about 10% of the events, illustrating the very flashy nature of this stream.

TABLE 2—Actual and adequate sampling intervals — Pakuranga and Puhinui catchments

Catchment	Pakuranga	Puhinui
Sample interval Δt	0.1 h	0.25 h
$\Delta t_m/\Delta t$	0.76 - 5.34	1.10 - 6.07
No. events	111	95
Events with $\Delta t_m/\Delta t<1$	11	0

DERIVATION OF COMMON UNITGRAPHS

For each catchment, the set of events with peak flows greater than 1000 L s^{-1} was used to estimate the common unitgraph and the corresponding inputs. The fit obtained was excellent, with rms errors of 0.0004 and 0.0007 for the Pakuranga and Puhinui catchments respectively. Figure 3 shows comparisons of observed flows with the predicted flows obtained by convolving the calculated inputs with the common unitgraph, for the largest and smallest event in each set. While these events were among those used to calculate the common unitgraph, similar results were obtained for the events with peak flows less than 1000 L s^{-1}, which were not used in the unitgraph calculation.

The common unitgraphs are compared with conventional unitgraphs in Figure 4. The conventional unitgraphs were calculated by the least squares method for each event in the data set by assuming an initial loss and constant continuing-loss model. The initial loss was assumed to be the rainfall occurring before the first rise of the hydrograph, and the continuing loss was calculated to preserve the water balance. An 'average' unitgraph was obtained by a procedure similar to that recommended in Institution of Engineers Australia (1987), which involved selecting the median values of peak flow, time to peak, and total duration of the event unitgraphs, and then sketching a curve which conformed to their general shape.

There is a striking difference between the shapes of the unitgraphs, with the common unitgraph having a higher peak, shorter time to peak, and shorter duration than the conventional unitgraph. This difference has been observed on all previous catchments for which common unitgraphs have been determined (Chapman, 1992), and suggests that the loss model to be used with a common unitgraph must have a storage component, unlike the loss models used with the conventional method.

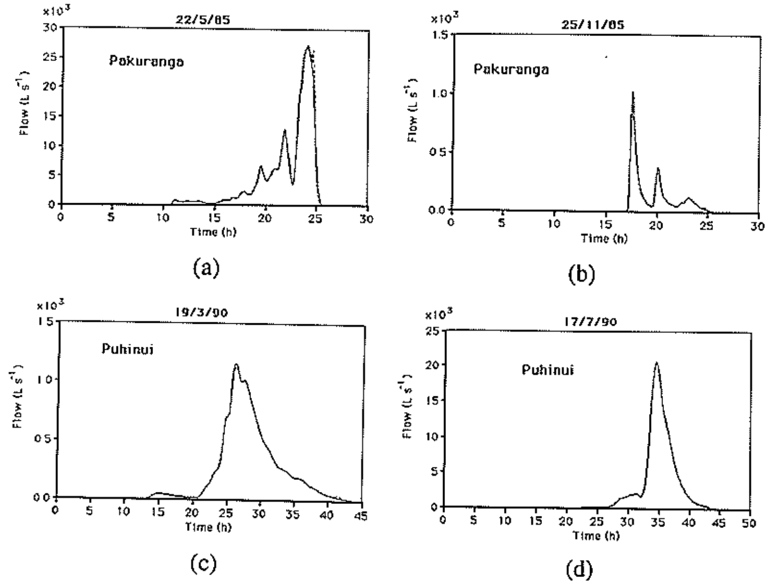


FIG. 3 — Observed (full lines) and predicted (dashed lines) hydrographs obtained with the common unitgraph and calculated input hyetographs

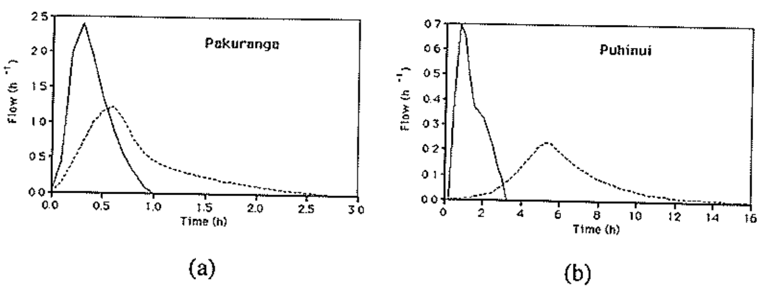
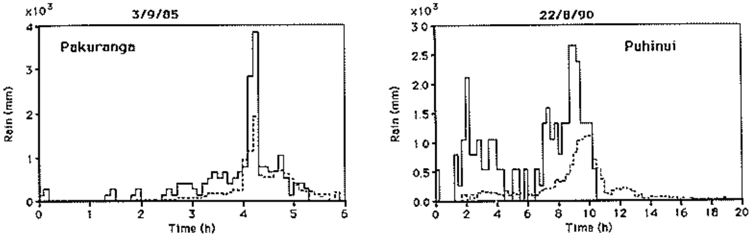


FIG. 4 — Common (full lines) and conventional (dashed lines) unitgraphs.



(a) (b)

FIG. 5 — Observed rainfall per time interval (full lines) and calculated input to common unitgraph (dashed lines).

LOSS RATES AND LOSS MODELS

Previous work (Chapman, 1992) has shown that the input hyetograph calculated from the common unitgraph identification process has peaks which lag behind corresponding peaks in the observed rainfall, and continues beyond the end of the rainfall. The examples shown in Figure 5 indicate that this also occurs with the Auckland catchment data.

This supports the conclusion that the loss model to be used in conjunction with a common unitgraph must have a storage component which will achieve a time lag. A possible form for this component is a power function relation of the form

$$S = K\,Q^m \qquad (8)$$

where S is the volume in storage, Q is the output of the lagged loss model, and the exponent m would be expected to be less than 1. This type of relation is widely used in both flood routing and runoff routing (Institution of Engineers Australia, 1987), although alternative forms have been suggested by Bates and Pilgrim (1986).

The storage S is here conceived as the volume of non-channelised surface water in transit in the catchment. The inflow to this storage is the rainfall excess calculated as the difference between the rainfall intensity and the loss rate, while the outflow from the storage is the input to the common unitgraph.

The initial loss in an event can then be regarded as a negative value of S, which must be satisfied by rainfall before any surface accumulation of water occurs. Once S becomes positive, the continuing loss for the sample interval is subtracted, and the value of Q is calculated from Equation 8. The set of values of Q becomes the input vector **r** to the common unitgraph.

For practical use of the procedure in flood forecasting or design it is necessary to have average or design values of the loss and storage parameters that give the best overall results for the calibration events. This has been achieved by the following procedure, using a constant continuing-loss rate:

1. Select trial values of the three parameters: loss rate, K and m.
2. For each event, calculate the initial loss which satisfies the water balance; calculate the lagged rainfall excess and convolve it with the common unitgraph to get the predicted runoff; calculate the sum of squares of differences between the ordinates of the predicted and observed runoff for

this event, and divide this value by the square of the total observed surface runoff.

3. Add the values for each event to get a total weighted sum of squares.
4. Repeat steps 1,2,3 according to the optimisation algorithm, until the weighted sum of squares has been minimised.
5. Perform a final run through steps 1,2,3 with the optimal values of loss rate, K and m.

The weighting system used in this calculation is in conformity with the principle used throughout this study of scaling rms errors to the area under the relevant hydrograph. The optimisation procedure was a modification of the simplex search technique of Nelder and Mead (1965).

Figure 6 shows the predicted and observed hydrographs for the four events used previously in Figure 3. The optimal values of the parameters, fitted to all 17 events in each catchment are given in Table 3.

The performance of the common unitgraph approach can be assessed by the measures used by Malone and Cordery (1989), that is, the ratio of predicted to observed peak flow and the difference between modelled and observed time to peak. The means and standard deviations of these quantities are shown in Table 4.

The results for the ratio of peak flows are similar to the results obtained for an Australian catchment (Chapman, 1991), which compared well with corresponding statistics for the conventional unitgraph method. However, the results for the time to peak show that the model predicts the peak to occur later than has been observed.

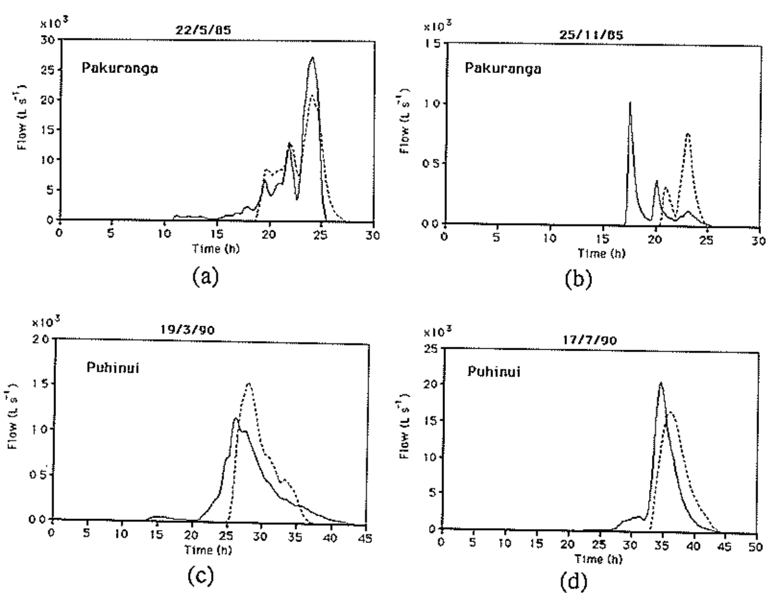


FIG. 6 — Observed (full lines) and predicted (dashed lines) hydrographs using an initial and constant continuing-loss model combined with a nonlinear storage. Joint calibration for 17 events in each catchment.

TABLE 3—Fitted values of parameters for an initial and constant continuing loss model, combined with a nonlinear storage.

Catchment	Pakuranga	Puhinui
Loss rate (mm h^{-1})	0.0	0.44
K	6.3	14.3
m	1.00	0.72

TABLE 4—Ratio of observed to predicted peak flow, and difference between predicted and observed time to peak (expressed in units of sampling time interval.

	Pakuranga		Puhinui	
	$\frac{Q_{pmod}}{Q_{pobs}}$	$T_{pmod}-T_{pobs}$	$\frac{Q_{pmod}}{Q_{pobs}}$	$T_{pmod} - T_{pobs}$
Mean	0.97	7.0	1.11	6.2
SD	0.33	15.1	0.40	6.7

DISCUSSION

The results of this study may be interpreted in terms of either hydrological processes or system functions. From the process viewpoint, it can be postulated that the rainfall loss and nonlinear store algorithm models the processes of infiltration and overland flow, while the common unitgraph routs the stream flow from source areas to the catchment outlet. From the functional viewpoint, the unitgraph identification procedure can be said to remove all common linearities from the set of surface runoff hydrographs, leaving the loss process and the nonlinearities.

The process interpretation, while probably only roughly correct, is at least preferable to that underlying the conventional loss and unitgraph model, in which it is assumed that a linear system is valid for the whole routing process from the points of impact of rainfall to the catchment outlet. While it can be argued that stream flow behaves approximately linearly, particularly at higher overbank flows, the process of runoff generation and routing to the nearest channel has no such justification. Modern runoff routing models such as RORB (Laurenson, 1964) and WBNM (Boyd et al., 1979) assume the whole process at subcatchment level is nonlinear, with a value of the exponent m typically around 0.8. What has been done here is to separate this process into a more strongly nonlinear storage routing, followed by a linear transformation.

There is now scope for experimentation with different combinations of loss models and nonlinear storage functions, in order to obtain better predictions with the common unitgraph. While reasonable results have been obtained here with the initial and constant continuing-loss model and the power function form of nonlinear storage, different algorithms may well be more successful in these or other catchments. In particular, a runoff coefficient form of loss model may be more appropriate in areas with lower rainfall intensities, such as the Auckland region, than the constant continuing-loss model popular in Australia, which has been used in this study.

CONCLUSIONS

It has been shown that common unitgraphs can be obtained from streamflow data alone for a set of runoff events, even in urban catchments which are generally considered to be unsuitable for a linear systems approach. These common unitgraphs have higher peaks, shorter times to peak and generally shorter total durations than average unitgraphs obtained by conventional techniques from rainfall and streamflow data for single events.

The input which drives the common unitgraph lags behind corresponding peaks in the rainfall, and continues after rainfall has ceased. It can best be interpreted as the output from a process which follows a conventional loss model with a strongly nonlinear storage. Thus the overall transformation from rainfall to catchment stream flow takes the form of three processes in series: a loss process, a nonlinear storage, and a linear-routing process.

There is great scope for study of the most appropriate forms for the rainfall loss algorithm and the nonlinear storage, with the reasonable expectation that improvements will result in an overall model which will give a better fit over a wider range of flows than predictions from conventional unitgraph methods.

ACKOWLEDGEMENTS

The author wishes to thank Royd Cumming of the Auckland Regional Council, Environment and Planning Division, for providing the data used in this study.

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Manuscript received 23 October 1992; accepted for publication 14 September 1993