HYDRAULIC GEOMETRY RELATIONSHIPS OF SOME NEW ZEALAND GRAVEL BED RIVERS

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ABSTRACT

A statistical analysis is presented of some hydraulic geometry relationships of six large, coarse gravel bed rivers in New Zealand. Simple power-laws involving width, depth, velocity, bedslope, suspended sediment concentration and hydraulic roughness are evaluated at mean annual water discharge, as it varies downstream, the latter being the independent variable in each expression.

Power-law exponents for width, depth, velocity and bedslope are generally consistent (to within one standard error) with traditionally accepted values, and with values obtained from coarse gravel bed rivers of North America and elsewhere. A similiar set of hydraulic relations is deduced theoretically using theoretical-empirical equations of hydraulics and sediment transport.

Further support is provided for current river engineering practice in New Zealand, in that field hydraulic relations satisfy a threshold stable channel design equation, expressing width as a function of bankfull discharge, energy gradient and median size of surface bed material.

Relations for width, depth and velocity only are recommended as a guide in design or research, provided that the constants of the power-laws are calibrated for a particular river or watershed.

INTRODUCTION

Among a catchment's responses to storms are runoff and erosion: most of the water and sediment is eventually delivered to stream channels and transported from the region. For a stream at grade (Mackin, 1948), water discharge and sediment load can be treated as independent variables although, as Kennedy (1975) points out, these are frequently related statistically. Rivers may, in conveying the imposed load, change their transport capacity within wide limits through alterations in planform, slope, cross-section shape and in the ratio of suspended load to bedload (Kuprianov and Kopaliani, 1979). Nevertheless, alluvial rivers demonstrate certain consistencies in the physical solutions they adopt to cater for any given stream flow and sediment discharge, and conditions limiting free channel development, such as unerodible banks. This consistency was expressed statistically by Leopold and Maddock (1953), who, for rivers in the midwestern United States, presented simple power-law relations beween mean annual water discharge, Q_m, as it varies downstream, and the related hydraulic parameters, water surface width, W_m, mean depth, d_m, mean velocity, V_m, water surface slope, S_m, suspended sediment load, G_m, (total load was not available) and hydraulic roughness as reflected by Mannings n. These *downstream hydraulic geometry relations* express the integral of all hydrologic variables operating within a catchment. Specifically the relations are (Leopold, 1953; Leopold and Maddock, 1953; Leopold *et al.*, 1964):

$$W_m \propto Q_m^{0.5} \quad d_m \propto Q_m^{0.4} \quad V_m \propto Q_m^{0.1} \quad S_m \propto Q_m^{-0.5}$$
$$G_m \propto Q_m^{0.8} \quad n_m \propto Q_m^{-0.3} \qquad (1)$$

While the coefficients of proportionality of the equations (1) vary from locality to locality, the exponents are relatively constant, and appear to be independent of location and only weakly dependent on channel type (Parker, 1979).

The purpose of this analysis is to determine whether there is a local variation in the above generally accepted values of the exponents, for New Zealand gravel bed rivers. This statistical work is paralleled by, and used to test, a theoretical analysis employing some empirical-theoretical equations of river engineering. The aim is to provide scientists and engineers with locally applicable downstream hydraulic geometry relations for hydrotechnical design and research.

HYDRAULIC GEOMETRY

Basic Data

The basic data for this study (Table 1) were obtained at 25 flow gauging stations operated by the Ministry of Works and Development on 6 large gravel bed rivers in New Zealand (Ministry of Works and Development, 1979). Choice was limited to rivers having three or more gauging stations with more than 10 years of record and where all the requisite data had been collected. Mean annual water discharge was determined from station flow records using procedures outlined by Thompson and Wrigley (1974). Current-meter flow gauging records with water discharge nearly equal to or equal to Q_m were then selected, and average values of the parameters W_m, d_m and V_m determined from these records. Details of the methods of measurement of these four variables are given by Waugh and Fenwick (1979). As the water surface slope or bedslope is rarely recorded, and most of the gauging station reaches are steep, it was assumed that energy gradient, water surface slope and bedslope are parallel. Average bedslope of the channel reaches was obtained from NZMS 1 topographic maps which have a scale of 1:63360. Hydraulic roughness was estimated in terms of the Darcy-Weisbach friction factor, f_m, defined by

$$f_m = 8gR_mS_m/V_m^2 \qquad (2)$$

in which g is gravitational acceleration, and R_m hydraulic radius, assumed equal to d_m. This assumption is valid because the aspect ratio (W_m/d_m) varied from 19 to 124 with a mean value of 44. Average suspended sediment concentration at mean flow, C_m, was determined from suspended sediment concentration versus water discharge ratings.

TABLE 1: Hydraulic geometry parameters evaluated at mean annual water discharge.

Watershed	Site	Site number	Length of flow record (yrs)	Median size of surface bedmaterial (Wentworth Scale)	Mean annual water discharge, Q_m (m^3/s)	Water surface width, W_m (m)	Mean depth, d_m (m)	Mean velocity, V_m (m/s)	Channel bedslope, S_m	Suspended sediment concentration C_m (p.p.m.)	Darcy-Weisbach friction factor
Buller	Longford	93202	17	pebble	67	41	1.24	1.32	0.0048	10	0.27
	Te Kuha	93203	17	pebble	416	117	2.70	1.38	0.0008	45	0.09
	Landing	93206	17	cobble	72	59	2.00	0.61	0.0033	15	1.39
	Blacks Pt.	93207	15	cobble	15	38	0.57	0.71	0.0054	10	0.48
	Woolfs	93208	17	pebble	240	117	2.30	0.91	0.0017	40	0.36
	Maruia Falls	93209	17	pebble	62	65	1.01	0.93	0.0026	40	0.24
	Mudlake	93211	17	pebble	52	54	1.00	0.96	0.0035	60	0.30
	Mangles Gorge	93212	22	pebble	9	18	0.63	0.74	0.0028	10	0.15
	Lake Rotoiti	93216	29	pebble	13	30	0.76	0.61	0.0061	8	0.98
Ngaruroro	Fernhill	23102	28	pebble	44	78	0.63	0.90	0.003	110	0.18
	Whana Whana	23103	20	pebble	37	36	0.70	1.44	0.0044	40	0.12
	Kuripapango	23104	17	pebble	17	29	0.56	1.04	0.0081	15	0.33
Rangitaiki	Murupara	15408	28	very coarse sand	23	22	0.99	1.00	0.0037	38	0.29
	Galatea	15410	28	pebble	15	28	0.31	1.12	0.0035	16	0.07
	Te Teko	15412	28	gravel	74	58	1.30	1.00	0.0009	36	0.09
	Kopuriki	15432	14	very-coarse sand	54	38	1.35	1.00	0.0028	54	0.30
Wairau	Tuamarina	60108	20	pebble	121	65	2.84	0.66	0.0012	25	0.61
	Dip Flat	60114	30	pebble	25	27	0.81	1.15	0.010	10	0.48
	Hells Gate	60116	12	cobble	8	18	0.45	1.01	0.013	10	0.45
Wanganui	Paetawa	33301	23	pebble	225	93	2.44	1.10	0.0006	35	0.10
	Te Maire	33302	18	pebble	100	56	1.51	1.00	0.0016	30	0.19
	Headwaters	33307	20	cobble	4	14	0.73	0.39	0.014	15	5.27
Whakatane	Waimana	15511	30	pebble	20	28	0.80	0.88	0.006	40	0.49
	Whakatane	15514	24	pebble	58	49	1.59	0.72	0.0009	70	0.22
	Ogilvies Br.	15536	12	pebble	8	17	0.60	0.81	0.0081	30	0.58

Statistical Analysis

Logarithmically transformed values of W_m, d_m, V_m, C_m, S_m, f_m and the independent variable, Q_m, were used to determine best fit simple power-law relations (Table 2, Fig. 1). The use of more complex functions might have resulted in less variance, but this is a first analysis and a

TABLE 2—Summary of downstream hydraulic geometry regression equations and their statistics.

Regression Equation	Correlation Coefficient	Factorial Standard Error	F Ratio*	Standard Error of Exponent	Factorial Standard Error of Constant
$W_m = 7.09Q_m^{0.48}$	0.94	1.24	167	0.04	0.39
$d_m = 0.21Q_m^{0.43}$	0.85	1.37	61	0.06	0.39
$V_m = 0.61Q_m^{0.11}$	0.43	1.30	5	0.05	0.79
$C_m = 8.18Q_m^{0.31}$	0.50	1.91	8	0.11	0.34
$S_m = 0.02Q_m^{-0.49}$	0.71	1.79	23	0.10	0.34
$f_m = 1.21Q_m^{-0.38}$	0.46	2.38	6	0.15	0.23

* $F(1,25, 0.95) = 4.24$

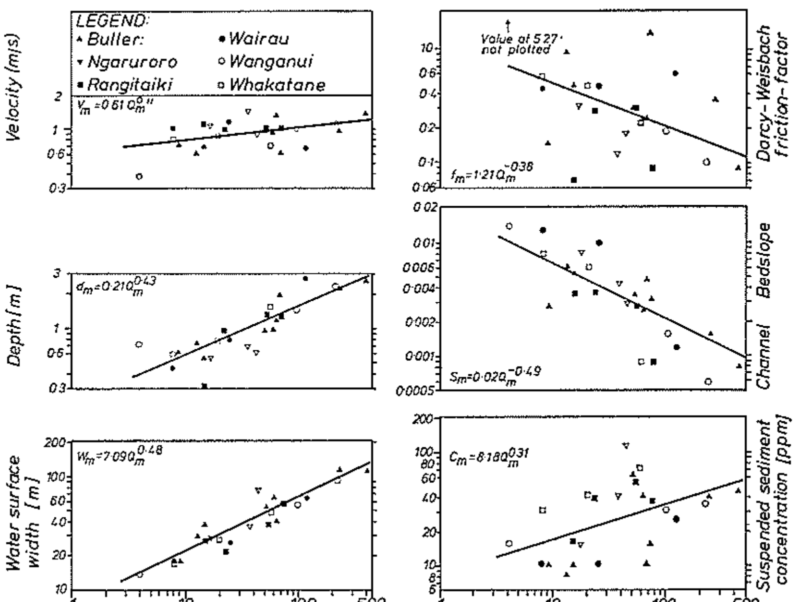

FIG. 1—Width, depth, velocity, bedslope, suspended sediment concentration and hydraulic roughness in relation to mean annual discharge as discharge increases downstream for some New Zealand gravel bed rivers.

comparison with earlier work is sought. Choice of Q_m as the independent variable, apart from the reason of comparison previously mentioned, may be justified because: empirical rules are sought, and, in application, water discharge is normally the parameter whose value is known or assumed; and for the short term time scale defined by the periods of record of Table 1, water discharge and bed sediment size (characterised by d_{50}, the median size of the surface bed material) may be regarded as independent variables.

This choice very probably ensures that single valued functional relations hold with the dependent variables, W_m, d_m, V_m, S_m, G_m and f_m (Kennedy, 1975). Further, the selection of mean annual discharge is based on the need to consider a water discharge, or index thereof, which can shape channels. Finally, the question of whether the short-term average channel slope, S_m, is a dependent variable is a difficult one, owing to a lack of critical data on the subject (Church, 1980). It is probable however, that the adjustment time for S_m is in the order of a decade, which satisfies the record periods of Table 1.

The regression analysis leading to the results of Table 2 does not consider error in the variates. In the sense that we require empirical relations for hydraulic geometry parameters in terms of given values of Q_m, the analysis is rigorous (Mark and Church, 1977). But where a comparison wih theory or other empirical results is required, as is done later, the analysis is valid only if the independent variate is not in error (Lindley, 1947). This assumption for Q_m is made herein.

Q_m was computed by combining more or less continuous records of water stage (recorded every 15 minutes) with stage-discharge relations for varying intervals during the period of record. These relations were defined by numerous flow gaugings (Waugh and Fenwick, 1979). Therefore, given the enormous number of stage observations and the effect of compensating errors in the establishment of flow ratings, the percentage error in Q_m is infinitesimal. Moreover, since the values of W_m, d_m and V_m are averages from the few actual measurements at or about Q_m, these variables are effectively independently determined and by definition so then is f_m. This question does not arise with S_m and C_m. To sum up: spurious correlation effects between the independent and dependent variables may be ignored.

In regard to precision, it has been inferred that the error in Q_m is negligible: the precision errors in the dependent variables are also negligible in comparison with their natural variability. Therefore the standard errors of the regressions (Table 2) give unbiased and efficient estimates of precision error. Probable accuracy, however, cannot be ascertained owing to lack of opportunities to compare results with other determinations whose precision is known.

The correlation coefficients and F ratios (Table 2) show that statistically significant relations hold only for the variables W_m, d_m and perhaps S_m, and also that regression equation standard errors are unacceptably large for design purposes. Nevertheless, the trends of the relationship (Fig. 1) are reasonably well defined, and for wide, coarse gravel bed rivers have not been disputed from either theory or observation. The constraint imposed by continuity of water discharge (Equation 7), that the exponents of the relations for W_m, d_m and V_m sum to 1.00, is satisfied within the limits of the precision of the determinations (Table 2, sum is 1.02). An inspection of the standard errors of the constants and the exponents reveals that the constants have much greater variability. The trends of data from individual rivers, or rivers of a single drainage basin, are more consistent than the relations between data from different rivers (Fig. 1). This suggests that the exponents represent the physical dependence of the variables upon water discharge quantities, and the constants reflect the geological and hydrological variables unique to each river or its basin. This statement supports and expands that of Parker (1979) quoted previously.

Traditionally accepted values of the exponents (Leopold *et al.*, 1964) for W_m, d_m, V_m and S_m generally agree with those for New Zealand gravel bed rivers (Table 3). Discrepancies exist with suspended sediment load or concentration, and with hydraulic roughness expressions. Leopold *et al.* (1964), however, also report load exponents up to 1.3 and their roughness coefficients apply to sand bed rivers, which exhibit quite different hydraulic resistance characteristics from gravel bed rivers (Vanoni, 1975). New Zealand exponents for W_m, d_m and V_m (Table 3) and those obtained from North American gravel bed rivers by Kellerhals (1967), Emmett (1972), Bray (1973) and Drage (1976), as reported in Drage and Carlson (1977), agree generally to within one standard error (Table 2). Parker (1979) has demonstrated a similar agreement for W_m, d_m and S_m for a small sample of gravel bed rivers from Canada, United States and Britain. Although exponent values for hydraulic geometry relations for W_m, V_m, d_m, and perhaps S_m are apparently consistent for coarse gravel bed rivers, much greater variability than one standard error (Table 2) is reported by Park (1977) in an analysis of world-wide variations. Park (1977) demonstrates, however, that exponents vary less if data are analysed according to gross climatic regions. Major problems in comparing exponent values are discussed by Park (1977); they include unassessed differences between quality of data, consistency of measurements, definition of variables and methods of fitting power-laws to the data. Church (1980) also discusses these problems and concludes that hydraulic geometry of alluvial channels presents a relatively consistent picture, once data have been appropriately sorted. These several arguments, together with the results of Table 2, suggest that for steep, wide, coarse gravel bed rivers of New Zealand, hydraulic geometry proportional relations (Table 3) for width, depth and velocity may prove a useful guide in design and research. The relations, however, require calibration for the relevant river or watershed. The conclusions of Leopold and Maddock (1953) apparently hold for New Zealand gravel bed rivers; their most relevant conclusion, for this study, is that the hydraulic geometry exponents are consistent for all flows between mean annual and bankfull discharge. Support for this proposition is provided by Church (1980) using data from an Alberta gravel bed river. Limited data, determined at bankfull discharge at Buller River sites (Table 1), also demonstrates (for bankfull discharge) this consistency (Griffiths, 1981b).

TABLE 3—Values of exponents in the relations for downstream hydraulic geometry of river channels.

RELATION	EXPONENT e		
	Traditionally accepted value (bankfull or mean annual water discharge)	Field value for N.Z. gravel bed rivers (mean annual water discharge)	Derived theoretical value (bankfull water discharge)
$W \propto Q^e$	0.5	0.48	0.44
$d \propto Q^e$	0.4	0.43	0.44
$V \propto Q^e$	0.1	0.11	0.11
$C \propto Q^e$	—0.2*	0.31	—0.33
$G \propto Q^e$	0.8	1.31	0.67*
$S \propto Q^e$	—0.5	—0.49	—0.44
$f \propto Q^e$	—0.3*	—0.38	—0.22
$n \propto Q^e$	—0.3	—0.07*	—0.04*

* denotes exponent derived from definitions, $C_m \propto C_m Q_m$: $f \propto d_m S_m / V^2_m$: $n \propto d_m^{0.66} S_m^{0.5} / V_m$; using values listed.

Theoretical Analysis

Three conceptually different approaches provide a theoretical basis for hydraulic geometry. Langbein (1964) performed a formal statistical exercise, involving minimisation of exponent variance, to define the exponents of Equations 1 (except for G_m) subject to certain boundary conditions or physical constraints concerning the expenditure of stream power. Engelund and Hansen (1967) proceeded from similarity principles. Theoretical-empirical equations of hydraulics and sediment transport were manipulated by Parker (1979) to derive hydraulic geometry relations for W_b, d_b and S_b, evaluated at bankfull discharge (denoted by the subscript b). Similarly, Li *et al.* (1976) extended the work on stable channel design by Lane *et al.* (1959) to develop relations for W_b, d_b, V_b and S_b, in relatively steep, cobble bed streams of small aspect ratio ($W_b/d_b < 12$). This analysis can be extended to wide ($W_b/d_b > 15$) gravel bed rivers of relatively steep slope. First, however, it is convenient to introduce the dimensionless variables:

$$W_b^* = W_b/d_{50},\ d_b^* = d_b/d_{50},\ V_b^* = V_b/\{g(S_s - 1)d_{50}\}^{0.5},$$
$$Q_b^* = Q_b/\{g(S_s - 1)\}^{0.5}\, d_{50}^{2.5}$$

in which S_s is the specific gravity of the sediment.

For stable, self-formed, straight reaches with gravel size or larger bed material, the following empirical-theoretical relations apply:

(a) Uniform flow hydraulic equation:

$$f_b \propto d_b^* S / V_b^{*2} \qquad (3)$$

Hey (1978) points out that it is more rigorous to employ f_b than n_b.

(b) Empirical definition of f_b :

Using data from New Zealand gravel bed rivers and elsewhere Griffiths (1981a) obtained the relation:

$$1/\sqrt{f} = 0.76 + 1.98\log_{10}(R/d_{50})$$

which may be approximated in the relevant region of interest ($5 \leqslant R/d_{50} \leqslant 200$) by

$$f_b \propto (d_{50}/d_b)^{0\cdot52} \qquad (4)$$

where at $f = f_b$
$R = R_b \simeq d_b$

Equation 4 becomes

$$f_b \propto (1/d_b^*)^{0\cdot52} \qquad (5)$$

Bray (1979) has recently developed a relation similar to Equation 4, having an exponent of 0.56.

(c) Sediment transport:
At bankfull discharge, Q_b, it is reasonable to assume (see, for example, Henderson, 1966; Kellerhals, 1967; and in particular Parker (1978) for a detailed discussion supported by field evidence) that bed particles of about size d_{50} are on the point of movement. The bed surface of a stable reach is normally armoured, and limited bed load transport of only fine particles usually occurs. There is often bank erosion with an equilibrium between (bank) erosion and deposition. Thus, as Leopold *et al.* (1964) pointed out, the form, but not necessarily the position, of a cross-section is stable. At these threshold bed conditions Shields' relation (see Henderson, 1966) for incipient bed particle motion is assumed to hold, namely

$$d_{50} \propto d_b S_b$$
$$\text{or } d_b^* \propto 1/S_b \qquad (6)$$

(d) Continuity of water discharge:

$$Q_b^* = V_b^* W_b^* d_b^* \qquad (7)$$

To make further progress, it is assumed that aspect ratio is approximately constant: this assumption is supported, for example, by Li *et al.* (1976), and also *a posteriori* by the New Zealand field results of Table 2. It follows that, for a given river

$$d_b^* \propto W_b^* \qquad (8)$$

Combination of Equations 3, 5, 6, 7 and 8 yields

$$W_b^* \propto Q_b^{*0\cdot44},\ d_b^* \propto Q_b^{*0\cdot44},\ V_b^* \propto Q_b^{*0\cdot11},\ S_b \propto Q_b^{*-0\cdot44},\ f_b \propto Q_b^{*-0\cdot22} \qquad (9)$$

The remaining hydraulic geometry relation is for suspended sediment concentration, C. A large number of theoretical formulae have been derived for the estimation of bed material load. (Bed material load = bed load + suspended load.) In a gravel bed river at bankfull discharge the proportion of bed load is likely to be less than 10% of the total sediment load (total load = washload + bed material load), and may be ignored herein; but the proportion of washload may be around 60% (see, for example, Emmett *et al.* 1978; Seitz, 1976; Griffiths, 1979). However, it is reasonable to assume that the washload concentration versus discharge rating has a similar gradient to that of the suspended component of the bed material load, for mean annual to bankfull discharges (Vanoni, 1975). This gradient assumption is supported in New Zealand gravel bed rivers by, for example, Griffiths (1979) and Jowett (1979). These authors present ratings of suspended sediment load versus water discharge for Waimakariri River and tributaries of Clutha River respectively, which have well-defined gradients and for which the greater proportion of the suspended material is washload.

With these assumptions, bed material load formulae derived by Engelund and Hansen and by Ackers and White—as reported by White *et al.* (1973) are suitably simple for analysis. The Englund and Hansen formula may be written as

$$C_b \text{ (bed material load)} \propto V_b^* (d_b^* S)^{1 \cdot 5}/d_b^* \qquad (10)$$

for sediment of constant specific gravity. From Equation set 9, Equation 10 becomes

$$C_b \propto Q^{*-0 \cdot 33} \qquad (11)$$

An identical result follows from the Ackers and White formula. Equation set 9 and Equation 11 are the theoretically-derived downstream hydraulic geometry relations for wide gravel bed rivers (Table 3). In comparisons beween theoretical and field relations (Table 3) it must be noted that the dimensionless theoretical relations (Equation set 9) contain the variable d_{50} implicitly. Church (1980) has shown, however, in a regression analysis using gravel bed river data and employing expressions of the form of Equation set 9, that d_{50} is not statistically significant in each case. Anyway d_{50} usually changes only slowly downstream in New Zealand gravel bed rivers (see, for example, Griffiths (1979)).

There is generally a remarkable similarity between power-law exponents of the theoretical and field relations (Table 3). A similar degree of correspondence in the exponents is also achieved by Langbein (1964), Engelund and Hansen (1967), Li *et al.* (1959) and Parker (1979) in their respective theoretical-empirical analyses. This is surprising considering the doubts concerning both their and the writer's assumptions. The agreement between the theoretical analyses herein, and the field results suggests that the assumptions made in the theoretical treatment are reasonable working hypotheses.

Exceptions to similarity of theoretical and field exponents for New Zealand gravel bed rivers (Table 3) are evident for suspended sediment and hydraulic roughness equations (C, G and f, n). For sediment, this disagreement may reflect the inapplicability of theoretical bed material load formulae: for roughness the variance of Equation 4 (see Griffiths (1981a) and Bray (1979)) is probably largely responsible. Finally, it is important to remember the inference above that the field exponents (Table 2) hold for mean annual to bankfull discharges.

Stable Channel Design

A frequent problem in hydraulics and sedimentation is the design of a stable channel within banks confining flood flow, where Q_b is determined from upstream, S_b is controlled by the country slope, and

d_{50} by existing surface gravels. Within usual economic constraints the only real degree of freedom open to the designer is channel width, given from Equations 3, 5, 6 and 7 by

$$W_b^* \propto Q_b^* S_b^{1\cdot 26} \qquad (12)$$

Expressions with slightly different exponents have been put forward or may be derived from studies by, for instance, Henderson (1966), Kellerhals (1967) and Parker (1979). In particular, Henderson's result

$$W_b^* \propto Q_b^* S_b^{1\cdot 17} \qquad (13)$$

is employed widely in New Zealand.

Combination of Equations 6 and 12 yields.

$$W_b^* \propto Q_b^*/d_b^{*1\cdot 26} \qquad (14)$$

and if the field hydraulic geometry relation for d_m, given by (Table 3)

$$d_m \propto d_b \propto Q^{*0\cdot 43}$$

is substituted into Equation 14, we deduce

$$W_b^* \propto Q_b^{*0\cdot 46} \qquad (15)$$

Henderson's (1966) equation (Equation 13) gives

$$W_b^* \propto Q_b^{*0\cdot 50} \qquad (16)$$

Both equations are indistinguishable from the field result

$$W_m \propto W_b \propto Q^{0\cdot 48} \qquad (17)$$

given the precision limits of its determination (Tables 2 and 3).

Such conformity is of course guaranteed by the previously established agreement between field and theoretical results (Table 3), and it provides support for current engineering practice for New Zealand gravel bed rivers.

In design, numerous writers advocate use of the values of width a river adopts in its natural channel to calibrate the constant of the $W_b \propto Q_b$ relation and sometimes also the exponent. For example, Parker (1979) recommends that a rule of the above form should be found empirically for each watershed. Once this rule is defined, the other downstream hydraulic geometry relations are easily deduced theoretically.

This philosophy has been extended by Griffiths (1981b). He rewrites Equation 12 as

$$W_b\, d_{50}^{1\cdot 5}\, g^{0\cdot 5}/Q_b\, S_b^{1\cdot 26} = \text{constant} \qquad (18)$$

and suggests that the dimensionless constant be determined for a particular river by computing an average value from the values of the variables at several cross-sections either known, or strongly suspected, to be stable in the long term (100 years). Then, in the design reach, Equation 18 yields values of W_b, when, as discussed above, the other variables are more or less predetermined. The precision or range of applicability may be further increased by employing an empirical relation (Davies, 1974) for d_{50} of the design reach bed material, in terms of the grain size statistics of the country or existing river gravels (Griffiths, 1981b).

CONCLUSIONS

Exponents of power-law downstream hydraulic geometry relations for width, depth, velocity and bedslope, derived for steep, wide, coarse gravel bed rivers of New Zealand, are generally consistent, to within one standard error, with traditionally accepted exponents. The exponents also agree with values obtained from gravel bed rivers of North America and Britain.

Power-law exponents represent the physical dependence of hydraulic geometry variables upon mean annual water discharge. The constants, which have large standard errors, probably reflect the geological and hydrological variables unique to each river or its drainage basin.

Theoretical power-law relations developed from equations of open channel flow and sediment transport have similar exponents to those of field relations, except for suspended sediment concentration or load and hydraulic roughness. The theoretical analysis in these instances is, however, poorly defined.

A threshold stable channel design equation in current use satisfies downstream hydraulic geometry field relations. This gives further support for its continuing use in New Zealand. Design performance should improve, however, if the constant of the equation is calibrated for the particular river where it is to be applied.

Hydraulic geometry relations for width, depth and velocity only are recommended as a guide in design or research and these power-laws should also be calibrated for a particular river or watershed.

ACKNOWLEDGEMENTS

The author recognises and appreciates the invaluable work of a large number of Ministry of Works and Development and Catchment Authority field and office staff who collect and analyse sediment and flow data. For assistance with various aspects I thank Kathy Walter, Maurice McSaveney, Horace Freestone, Ross Christian, Murray Hicks and Derek Goring. The Commissioner of Works has given permission for this publication.

NOTATION

General

C — suspended sediment concentration
d — mean flow depth
d_{50} — median size of surface bed material
f — Darcy-Weisbach friction factor
g — gravitational acceleration
G — suspended sediment load
n — Manning hydraulic roughness coefficient
Q — water discharge
R — hydraulic radius
S — energy gradient assumed equal to water surface slope and bedslope
S_s — specific gravity of sediment
V — mean flow velocity
W — water surface width

Superscripts

e — generalised hydraulic geometry power-law exponent
* — denotes non-dimensional form of a particular variable

Subscripts

b — denotes evaluation of parameter at bankfull discharge conditions
m — denotes evaluation of parameter at mean annual discharge conditions

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