

REGIONAL FLOOD-FREQUENCY ANALYSIS OF SMALL CATCHMENTS IN NORTH AUCKLAND AND COROMANDEL

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ABSTRACT

A method of flood-frequency analysis is developed for estimating design floods at ungauged sites, using data from small catchments in the North-Auckland-Coromandel region to develop a regional frequency curve (Q_T/\bar{Q} vs T), and a regional regression equation which estimates \bar{Q} from catchment and rainfall parameters. The regional curve and equation are compared to those developed by Beable and McKerchar (1982) for this region and found to give comparable results.

INTRODUCTION

Frequently, flood estimates are required for a site which is ungauged or where records are inadequate. The single site analysis can be extended to produce a regional flood-frequency analysis for such sites. Flood peak data for a wide region are combined and analysed to produce a regional frequency distribution.

The regional approach is valid only if the data are collected from catchments which are within a region, homogenous with respect to climate, topography, land use, soil characteristics and hydrology. The annual maxima of the recording stations within a region are collected and analysed to produce

- (i) a regional frequency curve Q_T/\bar{Q} vs t , where Q_T is the T year flood and \bar{Q} is the mean annual flood, and
- (ii) a regional regression equation to estimate the mean annual flood \bar{Q} using catchment and climatic parameters.

The regional equation can be applied to an ungauged site within that region to estimate \bar{Q} . Knowing \bar{Q} , the regional frequency curve can then be used to estimate the design flood Q_T for a specified return period T .

A number of regional methods have been developed, most notably the work of the Natural Environment Research Council of the United Kingdom (NERC, 1975). Following this lead, Beable and McKerchar (1982) carried out a comprehensive study of flood peak data for rural catchments in New Zealand. They divided the country into eight regions for deriving regional frequency curves (Q_T/\bar{Q} vs T), and nine regions for regional regression equations for \bar{Q} . A subsequent report by the authors (1983) outlined a design procedure for their method.

In developing the regional frequency curves and regression equations, they made no distinction between small and large catchments within the regions. Catchments included in their study ranged in area from approximately 20 km² to 7000 km². Only the North Island West Coast region included a catchment as small as 2.5 km². In discussing their model, the authors stressed that, where possible, checks should be made with other methods.

A programme was carried out to determine whether or not a Regional Analysis based on Small Catchments (RANSAC) would produce significantly different results from the Regional Flood Estimation (RFE) method of Beable and McKerchar based on small to large catchments. This paper compares results for the regional frequency curves (Q_T/\bar{Q} vs T) and regional regression equations for \bar{Q} of the two models.

REGIONAL ANALYSIS FOR SMALL CATCHMENTS

Small catchments in the Northland, Auckland and Coromandel areas were selected for study. This region corresponds to the upper part of the North Island West Coast frequency region, and the Northland-Coromandel-East

TABLE I Catchments selected from the Northland-Auckland-Coromandel region.

No.	Site No.	Flow Station	Catchment Area (km)	Period of Record	No. of Annual (and Historical) Flood Peaks
1	3506	Maungaparerua R at Tyrees Fords	11.1	1968-79	12
2	3819	Waiharekeke R at Willow Bank	229.0	1968-77	10
3	4901	Ngunguru R at Dugmore's Rock	12.5	1970-79	10
4	5809	Waiarohia R at Russell Rd	16.2	1958-67	10 + 1 hist
5	8501	Wairoa R at Weir	12.7	1960-74	15
6	9301	Kauaeranga R at Smith's	122.0	1959-79	21 + 1 hist
7*	43602	Waitangi R at SH Bridge	17.6	1966-79	14
8	43803	Papakura R at SH Bridge	52.6	1970-79	10
9	46611	Kaihu R at Gorge	116.0	1970-79	10 + 1 hist
10	46618	Mangakahia R at Gorge	246.0	1961-79	19 + 1 hist
11#	46625	Hikurangi R at Kamo Hikurangi	189.0	1961-68	8
12	46632	Whakapara R at SH Bridge	162.0	1960-76	17
13	46660	Puketurua R at Puketitoti	2.5	1965-76	12
14	47527	Opahi R at Pond	10.6	1967-79	13

Note: Waitangi and Hikurangi were not included in the derivation of the regional curve.

* Waitangi was excluded because of the unreliable probability plot of its annual series.

Hikurangi was excluded because of the short length of record.

TABLE 2 — Cross reference between the data used in the RANSAC and RFE models

	Site No.	Annual flood data from B & M*	Historic floods from B & M* (m ³ /s)	Floods added in this study		
				77	78 (m ³ /s)	79
1	3506	1968-77			33.8	55.1
2	3819	1968-77				
3	4901	1970-77			26.6	43.7
4	5809	1958-67	210#			
5	8501	1960-74				
6	9301	1959-76	980#	425.9	907.3	746.9
7	43602					
8	42803	1970-77			7.2	56.7
9	46611	1970-77	303#		100.9	135.0
10	46618	1961-77	961 ⁺		235.5	400.9
11	46625	1960-68				
12	46632	1960-76				
13	46660	1965-76				
14	47527	1966-77			17.4	16.0

* same data as Beale and McKerchar (1982)

historic flood occurred outside the annual series

+ historic flood occurred within the annual series

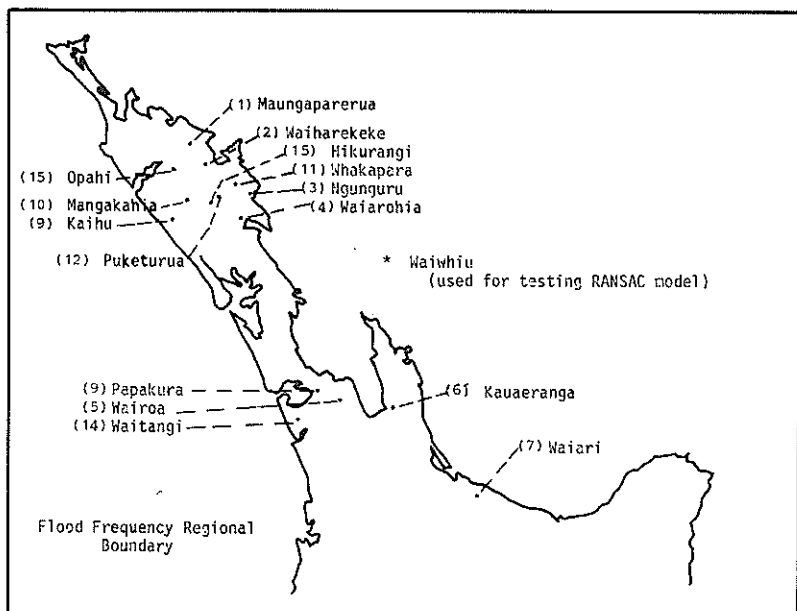


FIG 1—Location of gauging stations within the Northland, Auckland and Coromandel region

Cape mean annual flood region of Beale and McKerchar's study. Catchments with areas less than 250 km² were classified as small.

Only catchments with natural unregulated streams were included, urban areas and catchments with large pondage areas were omitted. The catchments

varied from rolling to hilly, and in some cases rugged, upland country. Table 1 lists catchment area and length of record of the gauging stations used in developing the regional model, and Figure 1 shows the location of gauging stations within the study region. Some of the catchments used were common to both studies, and additional records, where available, were included in the development of the RANSAC model (Table 2).

THE REGIONAL FREQUENCY CURVE

In constructing the probability plot (Q_T/\bar{Q} vs T), the return period T was calculated from the formulae recommended by the Cunnane (1978) to give the unbiased plotting positions. For the extreme value distributions, viz., the EVI and GEV, the Gringorten (1963) formula was used:

$$T = \frac{(N + 0.12)}{(i - 0.44)}$$

The GEV distribution for the random variate x is

$$x = Q/\bar{Q} = u + (\alpha/k)(1 - e^{-ky})$$

where u = a location parameter
 α = a scale parameter
 k = a shape parameter
 $y = (x-u)/\alpha$ a reduced variate
 $x = Q/\bar{Q}$

A special case of the GEV distribution is the EVI curve obtained by setting the parameter $k=0$ to give

$$x = u + \alpha y$$

The normalized data points of all the sites are plotted in Figure 2. The averaged values were taken as the regional data points. As few data points were available for higher return periods, additional information was obtained by extending the observed data set and by including historical floods.

In the NERC study, the data was extended by grouping the records and selecting the four highest values from each group. These were considered to be the four highest in a sample of size M , where M is the total number of station years of that group. In the present study, the RANSAC model using Carrigan's (1971) technique for extending regional record maxima to provide additional data (Fig. 2).

Historical floods, where available, were included in the analysis. The associated plotting positions of the normalized floods were calculated according to the Gringorten formula, but modified according to whether the historical flood occurred within or outside the annual series. In either case the largest historical flood was assigned a return period of

$$T = \frac{(N + J) + 0.12}{(i - 0.44)} = \frac{(N + J) + 0.12}{0.56}$$

where J was assigned a value according to the following:

(a) If the flood occurred outside the annual series of N years, J is the additional

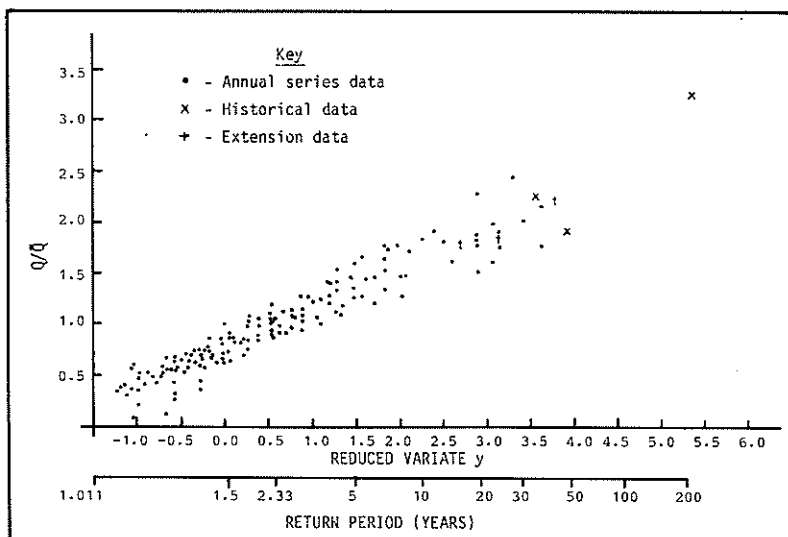


FIG 2—Regional data plot

number of years during which the historical flood is known to have occurred.

(b) If the flood occurred within the annual series of N years, and is known to the largest to have occurred over a period of $N + J$ years.

In either case the plotting positions of the normal annual series were based on the record length of N years.

Figure 3 shows the fit of the GEV and the EVI distributions to the averaged regional data points, and Table 3 compares the coordinates of the two curves. The GEV distribution fits the lower points better than the EVI curve (Fig. 3). However, the EVI distribution gave a better fit at higher return periods. The goodness of fit statistic,

$$\sigma^2 = \sum (Q/\bar{Q}_{\text{obs}} - Q/\bar{Q}_{\text{est}})^2$$

computed for the two curves (Table 3) indicated that the EVI distribution was superior. Thus, the EVI distribution given by the equation

$$Q/\bar{Q} = 0.644 + 0.435y$$

was adopted as the regional frequency curve for the RANSAC model.

TABLE 3 Comparison between the EVI and GEV frequency curves for the RANSAC model.

Distribution	Regional Curve ordinates						u	α	k	σ^2
	$\frac{Q_{2.33}}{\bar{Q}}$	$\frac{Q_5}{\bar{Q}}$	$\frac{Q_{10}}{\bar{Q}}$	$\frac{Q_{20}}{\bar{Q}}$	$\frac{Q_{50}}{\bar{Q}}$	$\frac{Q_{100}}{\bar{Q}}$				
EVI*	0.92	1.32	1.64	1.96	2.14	2.67	0.664	0.435	0.00	0.0626
GEV	0.88	1.25	1.57	1.89	2.10	2.74	0.667	0.360	0.09	0.6176

* selected as the RANSAC regional frequency curve

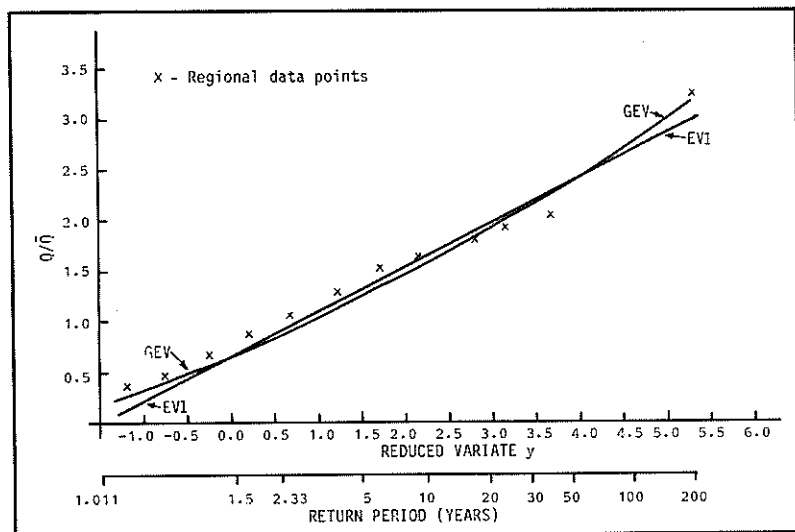


FIG 3—RANSAC model regional frequency curves for the EVI and GEV distributions

COMPARISON OF THE RFE AND RANSAC CURVES

Beable and Mc Kerchar used the EVI curve for the North Island West Coast regional data given by

$$Q/\bar{Q} = 0.804 + 0.330y$$

If the coordinates of the RFE curve and the one derived for the RANSAC model (Table 4) and the regional curves (Fig. 4) are compared, the following similarities and differences can be seen:

- (i) The ordinates of the two curves are approximately equal at lower return periods. As return period increases, the difference between the RFE and RANSAC curves increases.
- (ii) The RANSAC curve fits the data from small catchments better than the RFE curve. For the RANSAC curve $\sigma^2 = 0.06$, whereas $\sigma^2 = 0.42$ for the RFE model applied to the small catchment data.
- (iii) The gradient of the RANSAC curve is steeper than the RFE curve. Hence, the former will predict larger floods at higher return periods.
- (iv) At the 100 year return period, the ordinates for the RANSAC and RFE models are

$$(Q_{100}/\bar{Q})_{\text{RANSAC}} = 2.67$$

$$(Q_{100}/\bar{Q})_{\text{RFE}} = 2.32$$

The difference between these 100 year ordinates is 0.35 which is less than the standard error (se) of the RANSAC ordinate for 100 years ($se(Q_{100}/\bar{Q})_{\text{RANSAC}} = 0.437$). Hence the difference between the 100 year estimates appears not to have statistical significance. The differences at higher return periods was not investigated because the RANSAC regional curve was derived from short records and the estimates may not be reliably extrapolated.

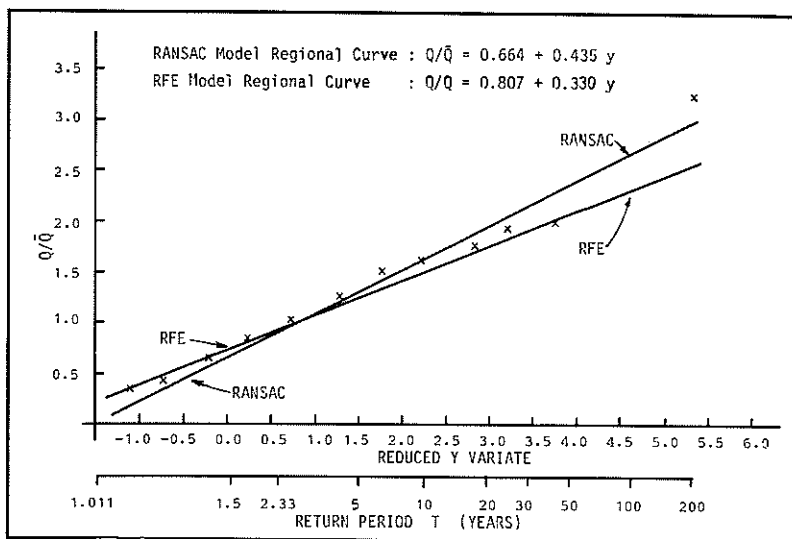


FIG 4—RANSAC and RFE regional frequency distributions

TABLE 4 — Comparison between the RANSAC and RFE regional frequency curves

		Regional Curve ordinates								
Model	Region	$\frac{Q_{2.33}}{\bar{Q}}$	$\frac{Q_5}{\bar{Q}}$	$\frac{Q_{10}}{\bar{Q}}$	$\frac{Q_{20}}{\bar{Q}}$	$\frac{Q_{50}}{\bar{Q}}$	$\frac{Q_{100}}{\bar{Q}}$	u	α	σ^2
RANSAC	Northland, Auckland, Coromandel	0.92	1.32	1.64	1.96	2.14	2.67	0.664	0.435	0.06
RFE	North Island West Coast	1.00	1.30	1.55	1.78	2.09	2.32	0.804	0.330	0.42

REGIONAL REGRESSION EQUATION

The Waitangi and Hikurangi stations were included in developing the RANSAC mean regional flood equation for the study region. This region corresponded to the Northland-Coromandel-East Cape region of Beable and McKerchar's study. Some lengths of record were 10 years or less which was too short to reliably estimate the mean annual flood \bar{Q} . The sampling error of \bar{Q} may be reduced by extending the effective length of a short record (Fiering, 1963). A short record can be extended by correlating it with a longer record of a suitable neighbouring station. This study adopted the simple linear regression equation

$$\log y = B_0 + B_1 \log x$$

where y = the annual flood for a particular year from the shorter record
 x = the annual flood for a particular year from the longer record
 B_0, B_1 = coefficients

TABLE 5 — Data for the Derivation of the RANSAC Regional Equation

Station	Catchment Characteristics									
	AREA (km ²)	MRAIN (mm)	1224 (mm)	LENGTH (km)	S1085 (m/m)	FOREST %	STMFCY ($\mu\text{m}/\text{km}^2$)	ELEV (RLΔ(m))	Q̄ (m ³ /s)	
1 Maungaparerua*	11.1	2250	105	7.0	0.0260	0	0.90	250	59.3	
2 Waiharakeke	229.0	1510	109	40.1	0.0014	29	0.45	112	107.3	
3 Ngunguru*	12.5	1940	124	4.6	0.0094	48	1.84	180	77.7	
4 Waitarohia*	16.2	1700	102	7.1	0.0516	64	1.70	168	54.6	
5 Wairoa	12.7	1790	74	8.5	0.0130	100	5.40	214	31.2	
6 Kauaeranga	122.0	3090	84	23.8	0.0177	82	2.30	330	514.6	
7 Waitangi	17.6	1360	75	5.7	0.0092	0	1.20	50	20.2	
8 Papakura	52.6	1340	69	18.5	0.0050	6	1.30	73	34.4	
9 Kaihu	116.0	1620	117	22.7	0.0190	14	1.53	310	135.4	
10 Mangakahia	246.0	1950	117	26.8	0.0190	31	0.92	352	448.1	
11 Hikurangi*	189.0	1630	101	23.2	0.0040	16	0.80	155	197.1	
12 Whakapara	162.0	1820	147	20.9	0.0070	40	1.50	186	184.4	
13 Puketurua	2.5	1540	100	2.7	0.0210	0	0.00	80	9.8	
14 Opahi*	10.6	1800	114	6.0	0.0102	8	0.47	236	29.8	

* stations with extended data

Where a record was extended the annual mean flood \bar{Q} was estimated from the longer record (Table 6).

The relationship between \bar{Q} and measurable catchment and climatic characteristics can be expressed in the form of

$$\bar{Q} = a X_1^{b_1} X_2^{b_2} \dots X_p^{b_p}$$

where $X_1, X_2, X_3 \dots$ are catchment and climatic parameters
 $a, b_1, b_2 \dots$ are regression constants

The regression can be linearized by taking logarithms to give

$$\log \bar{Q} = a + b_1 \log X_1 + b_2 \log X_2 + \dots + b_p \log X_p$$

A major difficulty is determining which parameters should be included in the equation. To develop a useful design procedure parameters should be limited to those readily available from published information. For example, the variables initially chosen by Beable and McKerchar (1982) are listed below. (Refer to Beable and McKerchar, p 53-54 (1982) for a fuller definition of these variables.)

Variable	Symbol	Units
1 Catchment area	AREA	km ²
2 Main channel length	LENGTH	km
3 Main channel slope	S1085	m/m
4 Mean catchment elevation	ELEV	RL(m)
5 Stream frequency	STMFCY	jtn/km ²
6 Forest cover	FOREST	% (area)
7 Mean annual rainfall over catchment	MARAIN	mm
8 Rainfall intensity (2 year return period, 24 hour duration)	1224	mm

The values of these variables for the catchments are listed in Table 5. Even with this small set of variables a large number of combinations are possible. To select a subset of variables to produce a set of preferred regression equations, the prior analysis of Daniel and Wood (1971) was adopted.

The prior analysis indicated that catchment area, mean annual rainfall, main channel length and mean catchment elevation should be included in the regional equation. Three equations were produced using combinations of these variables. The least squares fitting procedure gave the final preferred equations as

$$(1) \bar{Q} = 1.66 \times 10^{-8} \text{ AREA}^{0.61} \text{ MARAIN}^{2.68} \quad (R = 0.974, \text{ se} = 0.122)$$

$$(2) \bar{Q} = 5.01 \times 10^{-9} \text{ AREA}^{1.05} \text{ MARAIN}^{2.68} \text{ LENGTH}^{-0.82} \quad (R = 0.985, \text{ se} = 0.09)$$

$$(3) \bar{Q} = 1.17 \times 10^{-6} \text{ AREA}^{1.04} \text{ MARAIN}^{1.93} \text{ LENGTH}^{-0.88} \text{ ELEV}^{0.38} \quad (R = 0.995, \text{ se} = 0.07)$$

Equation 3 was adopted as the preferred RANSAC regional equation for the study region.

The residuals ($\log \bar{Q}_{\text{obs}} - \log \bar{Q}_{\text{est}}$) based on this equation were calculated and plotted onto a map (Fig. 5). The residuals are low and uniformly distributed within the redefined region. Furthermore, there are an equal number of stations with positive and negative residuals approximating a normal distribution with zero mean.

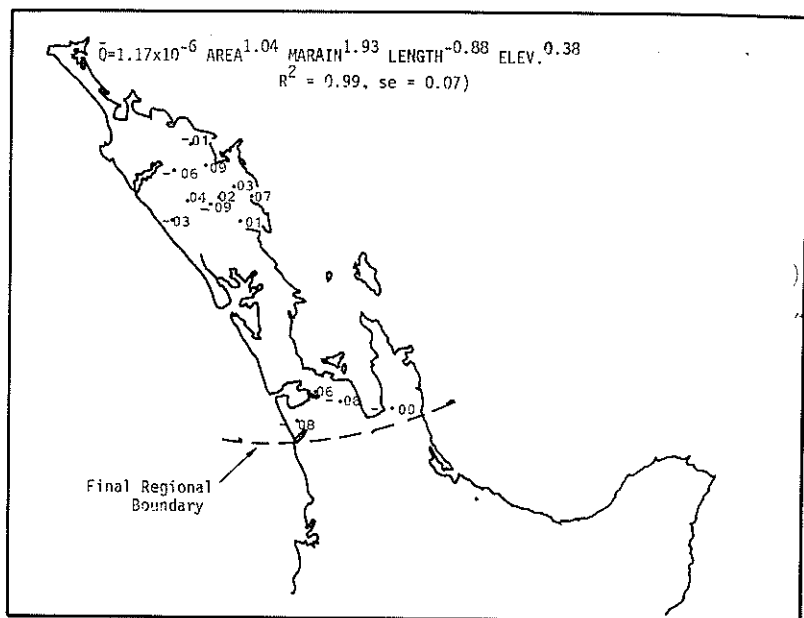


FIG 5—Distribution of logarithmic residuals for the final regional equation

The variables mean catchment area, mean annual rainfall and mean catchment elevation have a positive exponent which agrees with the catchment response to rainfall. Large catchment areas, higher rainfalls and the steeper slopes at higher elevations all contribute towards greater runoff. On the other hand, the negative exponent for main channel length shows that catchments with longer channel lengths extend the time of concentration, thereby reducing the runoff rate.

Beable and McKerchar used 21 catchments in the Northland-Auckland-Coromandel-East Cape to develop their regional \bar{Q} equation. They used the stepwise linear regression analysis to produce the following equations:

- (1) $\bar{Q} = 6.42 \text{ AREA}^{0.70}$ ($R = 0.927$, $se = .237$)
- (2) $\bar{Q} = 2.18 \times 10^{-7} \text{ AREA}^{0.64} \text{ MARAIN}^{2.33}$ ($R = 0.983$, $se = 0.119$)
- (3) $\bar{Q} = 8.24 \times 10^{-7} \text{ AREA}^{0.62} \text{ MARAIN}^{2.01} \text{ ELEV}^{0.22}$ ($R = 0.986$, $se = 0.110$)
- (4) $\bar{Q} = 8.71 \times 10^{-7} \text{ AREA}^{0.84} \text{ MARAIN}^{2.03} \text{ ELEV}^{0.21} \text{ LENGTH}^{-0.39}$ ($R = 0.988$, $se = 0.107$)

Equation 2 was selected as the regional mean annual flood equation for the RFE model. In their recent publication (1983) this equation was expressed with a different multiplier as

$$\bar{Q} = 2.13 \text{ AREA}^{0.64} \text{ MARAIN}^{2.33} \quad (R = 0.983, se = 0.119)$$

where the mean annual rainfall MARAIN is expressed in m/year.

COMPARISON BETWEEN THE RFE AND RANSAC MODELS

The RANSAC model was used to estimate \bar{Q} and Q_{100} values for a small

catchment in the study region. For the purpose of comparison, the RFE model and a single site frequency analysis were also used to estimate \bar{Q} and Q_{100} . The site chosen was Station No 43702, Waiwhiu River at Dome Shadow, Northland which had 10 years of historical records. This site was not included in the development of the RANSAC model because of uncertainty over land use since 1973. The catchment characteristics for the Waiwhiu River are: AREA = 8.0 km², MARAIN = 1570 mm, LENGTH = 5.7 km, ELEV = 270 m. The annual floods are listed in the Appendix.

For comparison, five methods were used to estimate the 100 year return flood Q_{100} as follows:

- (1) assuming no flood records, estimate \bar{Q} using the RANSAC regional equation and then obtain Q_{100}/\bar{Q} from the RANSAC regional frequency curve
- (2) as for (1) but use the RFE model
- (3) obtain \bar{Q} from the 10 years of observed record, and then calculate Q_{100}/\bar{Q} using the RANSAC regional frequency curve
- (4) as for (3) but use the RFE model
- (5) using the 10 years of observed record estimate Q_{100} using the simple site frequency analysis.

Table 6 summarizes the estimates of \bar{Q} and Q_{100} using the five methods.

TABLE 6 — Summary of \bar{Q} and Q_{100} estimated and their associated standard errors obtained by the RANSAC and RFE models and single site frequency analysis.

Description of Method	\bar{Q} m ³ s ⁻¹	se(\bar{Q})* (%)	Q_{100} m ³ s ⁻¹	se(Q_{100})* (%)	one std error intervals of Q_{100}
1 \bar{Q} and Q_{100}/\bar{Q} estimated from RANSAC model regional equation and curve	27.2	16	73	22	57 to 89
2 \bar{Q} and Q_{100}/\bar{Q} estimated from RFE model regional equation and curve	23.0	28	53	34	35 to 71
3 \bar{Q}_{obs} estimated from 10 years record, Q_{100}/\bar{Q} estimated from RANSAC model regional curve	32.7	15	87	22	67 to 107
4 \bar{Q}_{obs} estimated from 10 years record, Q_{100}/\bar{Q} estimated from RFE model regional curve	32.7	17	76	25	57 to 95
5 Q_{100} estimated from single site frequency analysis			80	27	59 to 102

* the appropriate equations used in calculating the respective standard errors (se) are listed in the Appendix

DISCUSSION

(a) \bar{Q} estimate for Waiwhiu

The mean annual flood estimated from the 10 years of historical record was $\bar{Q}_{obs} = 32.7$ m³/s. The RANSAC model returned a value of 27.2 m³/s. which was just outside the one standard error level of \bar{Q}_{obs} . The RFE value of 23.0 m³/s. was even lower.

It was concluded that the RANSAC regional equation performed better than RFE model because the \bar{Q} estimate was closer to \bar{Q}_{obs} , and the standard error of \bar{Q} for the RANSAC model was smaller.

(b) Q_{100} estimate for Waiwahi

The single site frequency analysis (method 5) yielded a value of $Q_{100} = 80 \text{ m}^3/\text{s}$. The RANSAC model of methods 1 and 3 gave values which were close to the above value. The RFE model using method 4 based on the observed \bar{Q}_{obs} returned a value of $76 \text{ m}^3/\text{s}$ which was also close to the observed value.

Using the both the RFE regional equation for \bar{Q} and its frequency curve for Q_{100}/\bar{Q} for the estimation (method 2) resulted in a value of $Q_{100} = 53 \text{ m}^3/\text{s}$ which was outside the one standard error level confidence limit of the observed value. This method gave the lowest result.

CONCLUSIONS

There does not appear to any significant difference between RANSAC and RFE frequency curves for predicting Q_T/\bar{Q} for the 100 year return period flood. Both models predicted flood values within the one standard error level of confidence.

APPENDIX

Reliability Analysis

The equations used for estimating standard errors are as follows:

Standard Error of \bar{Q} [se (\bar{Q})]

$$se(\bar{Q}) = [\text{var}(\bar{Q})]^{1/2}$$

(a) \bar{Q} estimated from the regional equation

$$\text{var}(\bar{Q}) = C_R \cdot \bar{Q}^2$$

where	$C_R = 0.026$	(RANSAC)
	$C_R = 0.078$	(RFE)

(b) \bar{Q} estimated from N years of record

$$\text{var}(\bar{Q}) = (C_v \cdot \bar{Q})^2 / N$$

where	$C_v = 0.47$	(RANSAC)
	$C_v = 0.54$	(RFE)

Standard Error of Q_{100} [se(Q_{100})]

$$se(Q_{100}) = [\text{var}(Q_{100})]^{1/2}$$

$$\text{var}(Q_{100}) = [E(\bar{Q})]^2 \cdot \text{var}(Q_{100}/\bar{Q}) + [E(Q_{100}/\bar{Q})]^2 \cdot \text{var}(\bar{Q})$$

where $E(\bar{Q})$ = mean annual flood \bar{Q}
 $\text{var}(\bar{Q})$ = variance of \bar{Q} as calculated above
 $E(Q_{100}/\bar{Q})$ = expected value of Q_{100}/\bar{Q} as given by the regional curve
 $\text{var}(Q_{100}/\bar{Q}) = [C_1 \cdot (Q_{100}/\bar{Q})]^2$

where $C_i = (2.43 + 3.05 \ln 100)/100$ (RANSAC)
 $C_i = (0.94 + 3.93 \ln 100)/100$ (RFE)

Annual floods for Waiwhiu River. Station No 43702 (m³/s)

1968	21.7	1973	24.2
1969	29.4	1974	8.8
1970	27.3	1975	37.9
1971	41.4	1976	67.1
1972	42.6	1977	26.6

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