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### FIELD PROCEDURES AND EVALUATION OF A SLUG DILUTION GAUGING METHOD IN MOUNTAIN STREAMS

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#### ABSTRACT

The application of a method (NaCl) of dilution gauging in turbulent mountain streams is evaluated with discussions about equipment, field procedures, mixing lengths, tracer losses, and the precision of derived hydraulic parameters. Mixing lengths are shown to be equal to 25 mean channel widths, and that this length is independent of the cross-sectional position of injection. Tracer losses are described as linear features, although no explanation can be offered. Precision, expressed as percentage probable error about the means of the four pertinent hydraulic variables, is discussed, with individual errors ranging from 1 to 21 percent, median errors ranging from 4.7 to 7.3 per cent and modal errors ranging from 3.6 to 6.0 percent. Comments on sampling procedures are offered, and several simplified procedures for estimating the mean velocity and discharge are presented.

#### INTRODUCTION

Measurement of flow characteristics in turbulent mountain streams is most satisfactorily accomplished with the use of dilution gauging techniques. The accuracy of more conventional methods, such as current meters, is reduced by irregular cross sections and flow discontinuities. Chemical gauging techniques utilizing introduced or artificial tracers (those either initially absent or in low and steady concentrations) employ two injection methods: a constant rate method in which tracer material is steadily introduced over a finite time period, and a slug or pulse method where a known volume of tracer material is injected instantaneously. These introduced tracers must be passive, in that they must be miscible and not alter the density or the velocity of the fluid medium.

Regardless of the injection method employed, tracer characteristics represent streamflow properties only after an initial, transient period, during which the tracer motion and its dilution is dependent upon the conditions of injection and the velocity distribution. The duration of this period is commonly referred to as the mixing time, and the length of channel required for its attainment is termed the mixing length. For dilution gauging, the mixing length is that channel length required before the tracer has sufficiently sampled the flow so that either the cross-sectional distribution of concentration is nearly uniform for constant

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injection, or the amount of dilution is constant for slug injection. Once the mixing length has been reached, the relevant properties of the tracer cloud are independent of injection.

This paper deals with the application of a slug injection method known as the 'relative salt dilution' method, where a salt (NaCl) solution is used as a tracer. In particular, this paper presents information on mixing length, tracer losses, precision of hydraulic measurements, sampling procedures and equipment. Data are abstracted from a field study of longitudinal dispersion of fluid particles in mountain streams (Day, 1974).

## THEORY

The modern form of the 'relative salt dilution' method was developed by Barbageleta (1928) and by Aastad and Sognen (1954). Several recent critiques include those of Church and Kellerhals (1970), Day (1974) and Church (1975). For the study on which this paper is based, this method was chosen for three reasons: (1) the tracer material does not constitute a health hazard; (2) the complete experiment, including analysis, can be carried out in the field; (3) the equipment and tracer material is readily available and inexpensive.

A measured volume,  $V$ , of tracer material is instantaneously injected into a flowing stream of unknown discharge,  $Q$ . The marked fluid elements pass a downstream location as a 'cloud' or 'wave', with the conductivity rising from some initial value to some peak value and then falling back to the original. Conductivity readings are converted to relative or absolute concentration via a linear calibration graph.

The basic tracer integral,  $I$ , transcribed by a time-concentration curve is

$$I = \int_{t_i}^{t_f} C(t) dt \quad (1)$$

where  $t$  is time,  $t_i$  and  $t_f$  represents the initial and final times of passage, and  $C$  is concentration. If the mixing length,  $x_m$ , has been reached, and if the flow has been steady during the interval  $t = 0$  and  $t = t_f$ , then the discharge is

$$Q = VC_i \div \int_{t_i}^{t_f} C(t) dt = VC_i/I \quad (2)$$

where  $C_i$  is the initial concentration of the slug. When relative concentrations are used, as in the method discussed here, the initial concentration is arbitrarily set equal to one, and equation (2) reduces to

$$Q = V/I \quad (3)$$

This simplification is one of the major advantages of the relative salt dilution method.

The mean travel time,  $t_m$ , of the dispersing tracer mass, between the injection and sampling locations is (Thackston *et al.*, 1967).

$$t_m = \int_{t_i}^{t_f} C(t) dt \div \int_{t_i}^{t_f} C(t) \cdot C dt \quad (4)$$

and is identical to the mean travel time of the stream only if instantaneous vertical and lateral mixing occurs at the injection point. A mean velocity,  $v$ , for the tracer cloud is defined as

$$v = x/t_m \quad (5)$$

where  $x$  is the downstream distance. As instantaneous mixing rarely occurs, the velocity of the wave is not equal to the mean stream velocity,  $u$ , which should be estimated using the 'salt velocity method' (Allen and Taylor, 1923) where

$$u = (x_2 - x_1) / (t_{m2} - t_{m1}) \quad (6)$$

where  $x_2 > x_1 > x_m$  and  $t_{m2} > t_{m1}$ . Both velocities, equations (5) and (6), represent mean particle velocities through the stream reach. A comparison of tracer and stream velocities for two tests is shown in Fig. 1. The higher tracer velocities, particularly within the mixing length, are evident. If the motion of the tracer wave is measured relative to its initiation, then the effect of these initially higher velocities is to produce a tracer velocity greater than the stream velocity for distances downstream of the mixing length. A further hydraulic parameter, the flow area,  $A$ , per unit length of channel is

$$A = Q/u \quad (7)$$

Determination of the flow area is particularly useful when channel cross sections are highly irregular.

## EXPERIMENTAL DATA, EQUIPMENT AND FIELD PROCEDURES

### *Experimental Data*

The data discussed herein are abstracted from an extensive series of dilution experiments in five test reaches located along the courses of five mountain streams which drain from the eastern flanks of the Southern Alps, in approximately the centre of the South Island of New Zealand. These streams are characterized by steep slopes, rough, irregular beds, channel braiding associated with general channel instability and bar formation, and highly variable runoff regimes.

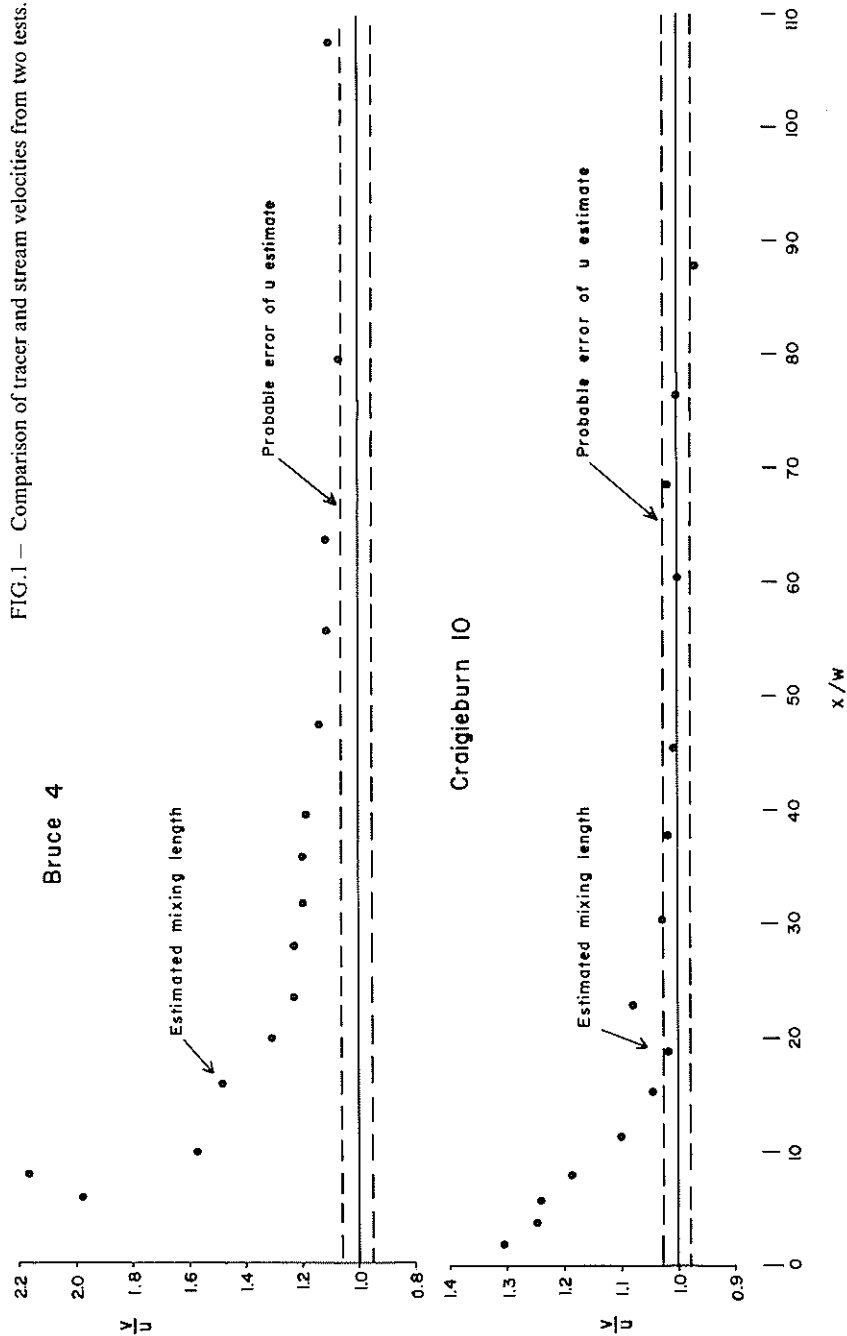
A summary of channel and hydraulic data is presented in Table 1. These data represent 702 time-concentration curves in 49 individual tests. Each test consisted of a series of time-concentration curves which followed the tracer cloud from initiation to as far downstream as possible, always beyond the mixing length.

### *Equipment*

The main disadvantage of the 'relative salt dilution' method is the bulky field equipment. As the recording system had to be operated by a single person, some care was extended in the design of the field equipment. A typical list of equipment was:

- 1 48-litre injection tank
- 1 20-litre calibration/injection tank
- 2 needle gauges
- 1 pail
- 1 500-ml flask

FIG.1 — Comparison of tracer and stream velocities from two tests.



- 4 pipettes
- 2 electrodes and stabilizing weights
- 1 stop watch
- 1 conductivity meter
- 1 thermometer
- 1 1000-ml graduated cylinder

These items are shown in Fig. 2.

The conductivity meter was designed specifically for this study and has excellent operational (linearity of response) and portability (weight and dimensions) characteristics\*. The conductivity meter was interfaced with a Rustrak recorder, first with a two-channel model and finally with a single-channel model (0-1 milliamps range, striking rate of two per second and a chart speed which could be varied from 8.3 to 33.3 mm/s depending upon the gear train used).

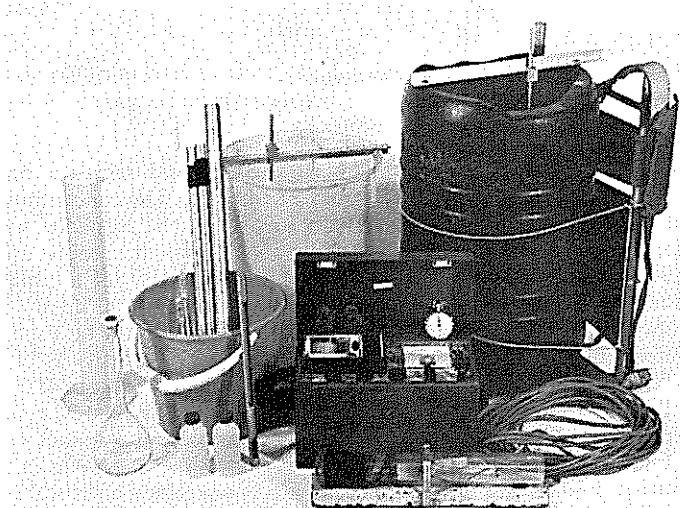


FIG.2 – Field equipment used for dilution gauging. Itemized list is given in the text.

The conductivity probes were constructed from perspex and stainless steel. They consisted of two stainless steel plates (sizes ranged from  $2 \times 3.5$  to  $2 \times 6$  cm) attached to slats of perspex ( $2.5 \times 35$  cm) mounted inside a perspex tube ( $35 \times 5$  cm). When placed horizontally in the stream and parallel to the flow, this tube maintained a constant flow geometry around the plates thereby preventing any flow variation effects. The upstream end of the probe was covered by fine screening to prevent debris from entering the tube and fouling the plates. An alternating current was utilized to prevent polarization on the plate surfaces and permitting the use of stainless steel in the construction of the electrode plates. The

\* The circuit diagram can be obtained either in Day (1974) or from Mr Robin Duff, Technical Enterprises, Christchurch.

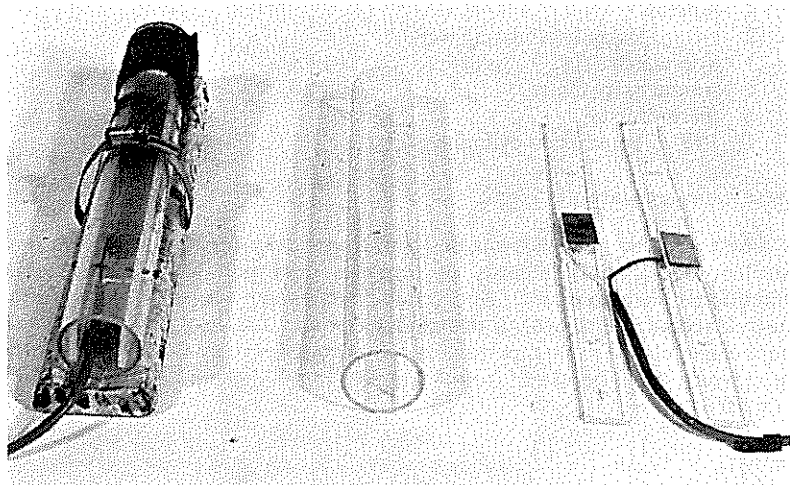


FIG.3 — Probe components showing perspex tube, electrodes and an assembled probe with weight.

#### *Field Procedures*

Common salt, NaCl, was used as the tracer. The dissolved tracer was hand injected as a slug whose volumes ranged from 3 to 48 litres. The concentration and volume of the slug, although constant for any single test sequence, were varied according to the flow rate, background conductivity and the probes used. Ostrem (1964, p. 26) stated that the amount of salt necessary for a dilution gauging should be about  $0.5 \text{ kg m}^{-3} \text{ s}^{-1}$ . However, quantities of  $0.3 \text{ kg m}^{-3} \text{ s}^{-1}$  proved to be quite adequate, and even smaller amounts could be used in conjunction with more sensitive probes. The maximum amount of salt was 5.5 kg in 45 litres (concentration of 121 g per litre) and the minimum, 1.1 kg in 48 litres (concentration of 23.6 g per litre). For smaller injection slugs (i.e. 3 litres) a stock solution was mixed in a large tank from which the appropriate volumes were taken. All injections were made as instantaneous as possible, at a point in the cross section (cf. Fig. 4), and through inertial effects resembled a vertical line source.

During low flows the tracer slug was injected into the main body of the flow, which is not necessarily the centre of the channel. During higher flows when access to the main flow was restricted, the slug was injected as far out into the flow as safely possible. The probes were placed in the channel with the same criteria as those of the injection design (cf. Fig. 5). The probes were always placed within 2.5 cm of the bed and away from any obvious dead zones or back eddies.

plates were approximately 2.5 cm apart. The sensitivity of the probes can be varied by altering either the spacing or the plate area or both. A 5-kg weight was used to stabilize the probe when in the channel. Probe assembly is shown in Fig. 3.

### MIXING LENGTHS

#### *Definition*

Upon injection into a flowing stream, the initially concentrated slug is rapidly advected downstream. Dispersal of the marked fluid elements throughout the



FIG.4 — Porter test reach showing slug injection at a point in the cross section.

flow is accomplished by two opposing mechanisms, transverse velocity variations and turbulent velocity fluctuations. The latter mechanism is important in transporting tracer material across streamlines into zones of differing longitudinal velocity, resulting in a rapidly dispersing tracer cloud. Eventually, the tracer is dispersed throughout the flow cross section, and its mean motion adopts the mean longitudinal properties of the streamflow. At this time the dilution of the tracer becomes constant and can be determined from a single time-concentration measurement at any point in the flow cross section.

#### *Conventional Formulae*

Many attempts have been made to relate the mixing length,  $x_m$ , to various hydraulic, dispersive and geometric properties of the stream. Some common mixing length formulae are listed in Table 2. Application of these formulae to mountain streams is not straightforward. On theoretical grounds the direct application of conventional diffusion theory (e.g. the formulae of Fischer, and of Yotsukura and Cobb) to these highly turbulent natural flows is inappropriate, as pointed out by Day (1974). Dispersion of fluid particles in these channels has been shown to be distinctly different from that predicted by conventional theory, and probably caused by the entrapment and subsequent slow release of fluid particles from dead or slowly moving zones along the flow boundary. Also, the lateral dispersal of the tracer is enhanced by the mechanical mixing effects associated with large protruding boundary elements and macro forms such as riffles. Another problem arises in their applicability, as these formulae require some prior knowledge of flow hydraulics and geometry. Field determination of these parameters is time consuming, particularly in mountain streams where numerous changes in flow behaviour occur. Recourse to a simpler method would be most beneficial.

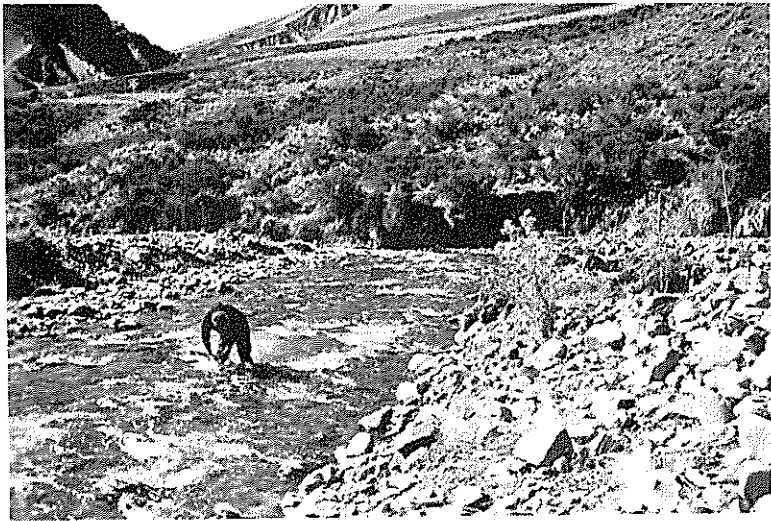


FIG.5 – Porter test showing placement of probe in the cross section.

A common dimensionless form for mixing length estimates is

$$x_m/w = a_0 (w/d) (u/u_*) \quad (8)$$

and shows that the dimensionless mixing length,  $x_m/w$ , is dependent upon two dimensionless parameters: one which represents the channel geometry (width–depth ratio),  $w/d$ , and a second, a velocity ratio,  $u/u_*$  (where  $u_*$  is the shear velocity). All the equations in Table 2 can be manipulated to the form of equation (8), with only Barsby's formula raising the velocity ratio to the second power (also, there is no constant,  $a_0$ ). The value of the constant will vary with the lateral position of the injection and these values are shown in Table 2. Ward's formula implies a further factor referred to as a dimensionless time ( $a_0 = tdu_*/w^2$ ) which is proportional to the degree of mixing. Ward's coefficients also imply that, for 99 percent mixing, the mixing length for either centre or bank injection is approximately half that predicted by the other formulae.

#### *Mixing Length Determination*

The mixing lengths for Day's (1974) data were determined from visual inspections of the longitudinal pattern of the tracer integral,  $I$ . As the mixing length is reached, the value of  $I$  should become constant. A typical downstream pattern is shown in Fig. 6. The accuracy of these estimates is influenced by the spacing of the sampling points in the vicinity of  $x_m$ . The precision of the successive  $I$  measurements is also of importance and will be discussed in a later section.

The mixing-length data indicate that for all but the Bealey data, lateral mixing was completed by 250 m. The shortest observed mixing length was 40 m for a Thomas test ( $u = 0.51$  m/s). The longest mixing length of 360 m ( $x_m/w = 21$ ) occurred on the Bealey reach, which is consistent with its larger width.

Mixing-length data for the Bruce and Porter test reaches are listed in Table



TABLE 1 — Basic channel and hydraulic data.

Terr reach	Maximum channel length (m)	Slope (m/s)	No. of tests	Range of discharges, Q (m <sup>3</sup> /s)	Range of mean velocities, u (m/s)	Range of mean width, w (m)
Bealey	660	0.0094	7	6.03–15.00	0.85–1.27	18.4–21.8
Bruce	775	0.0203	10	0.56–7.29	0.47–1.57	6.0–9.1
Craigieburn	780	0.0234	11	0.23–5.34	0.32–1.32	4.9–10.2
Porter	825	0.0176	13	0.35–9.13	0.60–1.46	4.3–11.4
Thomas	2250	0.0273	8	0.12–1.59	0.51–0.99	2.8–5.0

TABLE 2 — Mixing Length Formulae.

Source	Formula	Comments	Bank injection	Centre injection	Constant $a_0$
Barby (1968)	$x_m = (w^2/d) (u/u_s)^2$	based on Rimmar (1953)	—	—	—
Glover (1968)	$I_s = w/\sqrt{4E_y l}$	$I_s$ an index of lateral solution concentration which approaches unity as $x_m$ is reached	1.0	0.25	0.25
Fischer (1966)	$x_m = 1.8 l^2 u/Ru_s$	$x_m$ is really the length of the convective period	1.8	0.45	0.45
Yotsukura and Cobb (1972)	$x_m = 0.25 uw^2/u_s d$	for midstream injection only	—	0.25	0.25
Ward (1973)	$x_m = (T/C_y) (8/lT) (w^2/d)$	for 99% mixing	0.50	0.14	0.14

Notation:  $w$  is the flow width;  $d$  is the mean cross-sectional depth;  $u$  is the mean cross-sectional velocity which is replaced by the mean particle velocity (equation 5);  $u_s$  is the shear velocity;  $E_y$  is the lateral mixing coefficient;  $l$  is a characteristic length taken as the distance between the farthest bank and the maximum velocity thread;  $R$  is the hydraulic radius;  $T$  is equal to  $(E_y t)/w^2$ ;  $C_y$  is equal to  $(E_y/du_s)$ ;  $ff$  is the Darcy-Weisbach resistance coefficient.

3. Perhaps the most startling conclusion arising from these data (as well as for the remaining data) is that  $x_m$  does not vary with injection position. Side injections for this study were made about one-quarter to one-third into the flow cross section. According to the previously presented formulae, bank injections should increase the mixing length by a factor of four. No increase is evident in the data of Table 3. Non-dimensional mixing lengths,  $x_m/w$ , for all reaches range from 8.2 to 37.5. Separating these values on the basis of injection position, the mean  $x_m/w$  for centre injection is 17.7 with a standard deviation of 6.7, where the mean for side injection is only 13.2 with a standard deviation of 4.6.

Although there is a general trend of increasing  $x_m$  with increasing flow scale, trends for individual streams can be complex. In the case of the Porter test reach the mixing length is constant for a considerable range of flow events. During a flood in October 1972 all channels experienced some readjustments. In the case of the Porter, the channel at the injection point shifted laterally about 30 m, with a large riffle forming about 100 m downstream in the new channel. This riffle controlled mixing for all subsequent experiments regardless of flow scale.

With the exception of two Craigieburn tests (5 and 7,  $x_m/w = 37.5, 35.8$  respectively) all tests exhibited mixing lengths within about  $25 x/w$ . Although the apparent anomalous nature of these two Craigieburn tests cannot be explained, it is probably safe to ignore them in the face of the bulk of the remaining data. Also, Day (1974) has shown that the mean position of the mid point of the leading edge of the time-concentration curves is located at  $x/w = 24$  when the mixing length is reached (judged by the mean longitudinal motion of the tracer cloud). On the basis of these observations,  $x/w = 25$  appears to be the length of channel required for complete lateral mixing in mountain streams, regardless of injection position.

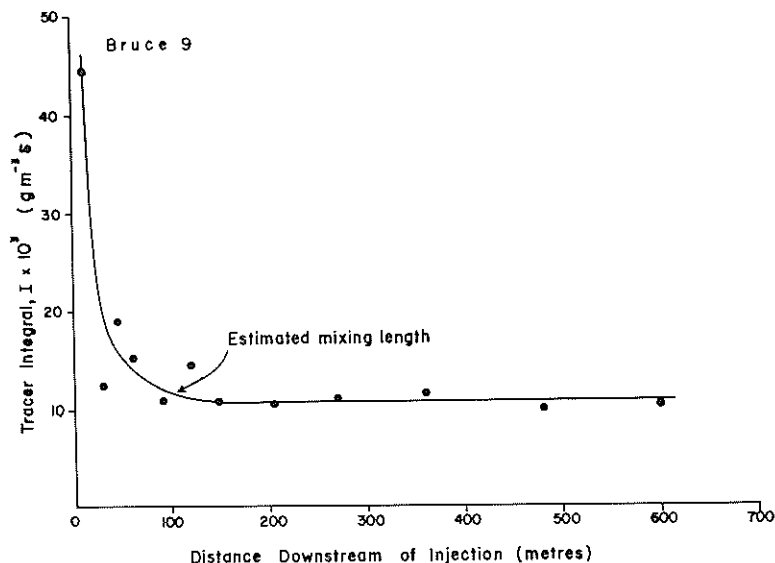


FIG.6— Downstream changes in the dilution of a tracer mass. Within the mixing length dilution is rapid, but becomes constant when the mixing length is reached.

### Comparison of Observed and Predicted Mixing Lengths

A comparison between the observed  $x_m$  and that derived from the various formulae is shown in Table 4 for the Bruce data. For centre injections the estimates of Glover and of Yotsukura and Cobbs are best. All estimates are derived from transforming the equations into the relevant non-dimensional form of equation (8). As expected, Ward's estimated values are considerably lower and about 30 to 40 percent less than the observed values.

The greatest discrepancies occur among the estimates for side injections. Differences between observed and predicted can be as large as six times. Ward's estimates are again the most conservative, but are still about three times the observed values.

Although there is no reason to expect formulae based on assumptions of symmetrical channel geometry, uniform velocity distributions and uniform longitudinal geometry to apply to mountain streams, the paucity of more appropriate ones made this resource necessary. The very irregular geometries and high particle roughness must considerably enhance lateral mixing through increased turbulence and mechanical effects.

### FIELD SAMPLING PROCEDURES

Once the appropriate mixing length, injection volume, etc. have been determined then the remaining problem is the sampling period in terms of the arrival and passage of the tracer cloud. If continuous recording equipment is used, then prediction of the sampling period is not important unless distances are great and operation expense (chart paper, battery power) is of concern. However,

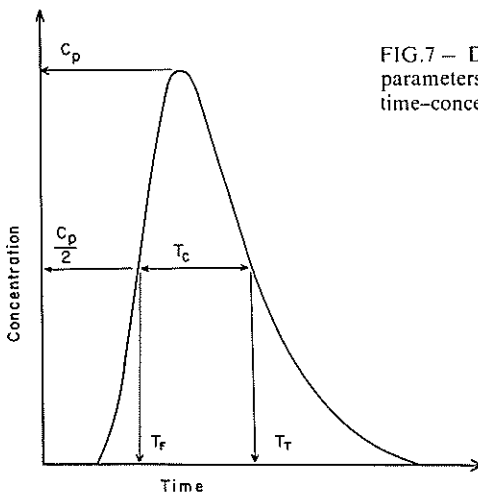


FIG.7 — Definition sketch of scaling parameters for non-dimensionalizing a time-concentration curve.

if discrete samples are to be collected, then prediction of the time parameters of the tracer wave is important.

The sampling procedures to be presented here are based upon the similarity characteristics of the mean motion of the dispersing tracer mass. Day (1974) showed that dispersion in natural channels exhibits the characteristics, constant velocity ratio and geometric form, of a self-similar process. Any time-concentration curve can be non-dimensionalized by: (1) dividing concentrations

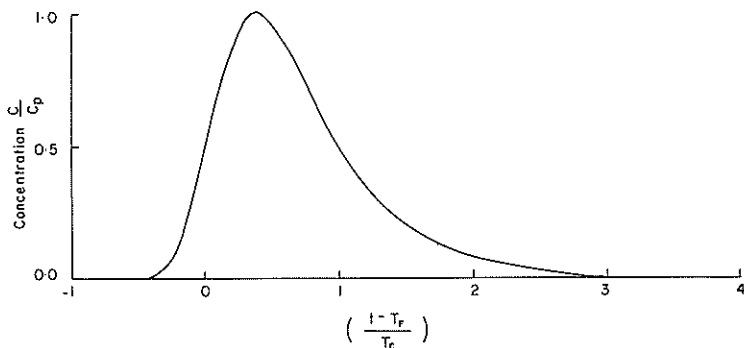


FIG. 8 — A mean dimensionless shape for time-concentration curves measured in rough, irregular channels.

by the peak concentration,  $C_p$ , and (2) scaling the time co-ordinates by a time parameter which represent both the spread of the tracer and the position of the spread relative to initiation; such a parameter is  $(t - T_F)/T_C$ . A definition sketch of these parameters is shown in Fig. 7. A non-dimensional time-concentration is shown in Fig. 8, the co-ordinates of this form are listed in Table 5. Day (1974) showed that the form shown in Fig. 8 is probably common to all rough mountain streams.

#### Sampling Period

The linearity of the spreading and the resultant constant geometric form offer some possibilities for predicting the tracer dimensions, time of passage, etc. If the stream travel-time velocity can be estimated by several current meterings, float velocities or from previous gaugings (i.e. if a stage-discharge relationship is being established) the sampling period can be outlined quickly. For any distance and known mean stream velocity, the mean travel time can be estimated from  $t_m = x/u$ . An analysis of Day's (1974) data indicates that the arrival time of the wave can be approximated as  $t_i = 0.77t_m$  and the final time as  $t_f = 2t_m$ . To ensure that the complete wave is sampled, the value for  $t_i$  should be considered a maximum, and the value for  $t_f$  a minimum. Once the sampling period is chosen then the sampling interval needs to be determined. Although there is no set number of data points required to define a time-concentration curve, it is important that they are fairly well distributed over the wave. Any number from 15 to 30 should do, with the greater number of points when the accuracy of the sampling period is uncertain.

#### Mean Travel Time

Once the time-concentration curve has been sampled the mean travel time can be quickly estimated from  $t_m = T_F + 0.63 T_C$ , where 0.63 is the dimensionless position of  $t_m$  relative to  $T_C$ . Besides offering rapid velocity estimates, this method of estimating  $t_m$  is particularly useful if the wave is not well defined. Most problems in wave definition occur for times distant from the peak, particularly along the trailing limb. As calculation of  $t_m$  is very sensitive to the ends of the wave, recourse to another method is beneficial.

#### Discharge

The flow discharge can be determined from knowledge of only  $V, C_p, T_C$ , and the dimensionless form. By definition, the area under a time-concentration curve is

TABLE 3 — Mixing length data for two test reaches.

<i>Test*</i>		<i>Channel length, x (m) †</i>	<i>Flow width, w (m) ‡</i>	<i>Mean velocity, u (m/s)</i>	<i>Observed mixing length, x<sub>m</sub></i>	<i>x<sub>m</sub>/w</i>
Bruce	1c	500	6.2	0.67	120	20.2
	2c	664T	6.5	0.61	100	15.4
	3c	650	6.4	0.57	125	19.5
	4c	675	6.3	0.59	100	15.9
	5c	500	6.0	0.47	100	16.7
	6c	500	5.6	0.51	75	13.4
	7s	775T	7.2	1.11	125	17.4
	8s	600T	9.1	1.57	125	13.7
	9s	600	8.2	1.40	75	9.2
	10s	600	8.7	1.37	150	17.2
Porter	1c	700	4.3	0.65	50	11.6
	2c	825	4.4	0.60	100	22.7
	3c	825	7.0	0.90	175	25.0
	4c	825	6.9	0.88	125	18.1
	5c	825	6.8	0.84	150	22.1
	6c	525	6.2	0.80	100	16.1
	7c	600	8.0	0.98	100	12.5
	8c	825	7.2	1.01	100	13.9
	9c	825	7.6	1.25	100	13.1
	10s	550	11.4	1.46	100	8.8
	11s	550	10.8	1.19	100	9.3
	12s	550	9.7	1.19	100	10.3
	13s	550	10.0	1.17	100	10.0

\* In this column the s represents side injection, and the c represents centre injection.

† In this column the T indicates that tracer losses occurred.

‡ In this column the mean flow width was based on 8 to 13.

TABLE 4 — Comparison of mixing length estimates for Bruce River data.

<i>Test*</i>	<i>Observed (m)</i>	<i>Barby's estimate (m)</i>	<i>Glover's estimate (m)</i>	<i>Fischer's estimate (m) †</i>	<i>Yotsukura's &amp; Cobb's estimate (m)</i>	<i>Ward's estimate (m)</i>
1c	125	299	144	297	144	80
2c	100	255	129	267	129	72
3c	125	195	118	242	118	66
4c	100	221	124	251	124	69
5c	100	153	100	208	100	56
6c	75	199	106	215	106	60
7s	125		611	684		306
8s	125		824	926		498
9s	75		897	1007		449
10s	150		799	1013		397

\* In this column the s represents a side injection and the c represents a centre injection.

† Side injection estimated as  $3/4w$ .

$$I = I' C_p T_C \quad (9)$$

where  $I'$  is the area under the dimensionless curve and equal to 1.12. The discharge is then

$$Q = V/I = V/1.12 C_p T_C \quad (10)$$

Equation 10 is most useful if the tails of the wave are not adequately defined.

#### *Final Decline of the Tracer Wave*

The problem of adequately defining the final decline of the tracer wave for discharge and velocity determinations can be approached from several ways: (1) by accepting the method outlined above, (2) by extending the field data using the co-ordinates of the dimensionless curve listed in Table 8, and (3) by fitting a simple linear least-squares model to those data points on the final decline. This latter method was suggested by Florkowski *et al.* (1969) and an appropriate model is

$$C(t) = a \exp[-b(t-t_i)] \quad (11)$$

where  $a$  and  $b$  are regression constants. Day (1974) found this to be an adequate method, with the regression equations explaining over 90 percent of the variance, exemplifying the representativeness of equation (11) When using this equation the limits of integration can be standardized by truncating the wave at predetermined concentrations. Yotsukura *et al.* (1970) used truncation points of 1 and 3 percent of the peak concentration.

TABLE 5 — Co-ordinates of a dimensionless time-concentration curve.

No.	$C/C_p$	$(t-T_p)/T_C$
1	0.0	-0.406
2	0.05	-0.282
3	0.10	-0.219
4	0.15	-0.188
5	0.20	-0.145
6	0.30	-0.094
7	0.40	-0.056
8	0.50	0.0
9	0.60	0.044
10	0.70	0.088
11	0.80	0.140
12	0.90	0.219
13	0.95	0.282
14	1.00	0.375
15	0.95	0.500
16	0.90	0.575
17	0.80	0.688
18	0.70	0.788
19	0.60	0.875
20	0.50	1.000
21	0.40	1.119
22	0.30	1.281
23	0.20	1.500
24	0.15	1.656
25	0.10	1.856
26	0.05	2.281
27	0.0	3.156

## PRECISION OF MEASUREMENTS

### *Sources of Error*

The precision or repeatability of the method depends upon the accuracy of the calibration procedure and the precision (1) volumetric errors associated with pipette usage and the preparation of the primary, secondary and calibration volume; (2) statistical errors of the calibration curve fitted to the calibration points; (3) errors of the conductivity values as read during the experiment; (4) errors associated with the existence of natural electrolytes in the stream.

Gross errors occur if: (1) measurements are taken before the mixing length is reached; (2) the electrode is placed in a dead zone on the stream bed; (3) significant temperature changes occur during the test period. Temperature changes are most important near 0°C when conductivity sensitivity is greatest. In the experiments discussed here, no temperature changes greater than 0.5°C occurred during the recording of any single wave. By comparison with temperature calibration graphs, these changes in water temperature were found to be insignificant and no corrections were made.

Precision error is present whenever successive measurements of an unchanging quantity yield numerically different values. The statistical nature of precision error precludes an absolutely correct datum from arising in a single measurement. Consequently, precision estimates must be in terms of statistical and probabilistic parameters of the error distributions. In the experiments described here, the true flow conditions in the channel remain unknown, and can only be estimated from mean values while errors are evaluated from the deviations about these means.

### *Measurement of Precision*

It is convenient to express precision, or the lack of it, by a single precision index. Types of indices include variance, standard deviation, coefficient of variability and probable error. The probable error,  $\phi$ , adopted here is defined as that deviation which encloses exactly one-half of the total sample, with 25 percent on either side of the mean. This deviation envelope is the region for 'one-to-one' odds; that is, the chance that any readings will have a deviation greater than  $\phi$  is the same as that it will have one less than  $\phi$ .

The probable error is restricted, however, to normally distributed errors. The normality of the deviations about their respective mean was investigated using the 'unit normal deviate' form of residuals (Draper and Smith, 1966, p. 88). If approximately 95 per cent of the deviations fall within two standard deviations of the mean, then they are considered to follow a normal distribution. This criterion was met by the data except where unsteady flow or tracer losses were recognized. If the deviations are normally distributed then the probable error is related to the sample standard deviation by a simple proportion,  $\phi = 0.675\sigma$  (Schenck, 1968).

Day (1976) has presented a discussion of the precision of these data. A summary of these results are shown in Table 6. Precision estimates indicated here refer to those data points located at and downstream of the mixing length. This mixing parameter was determined from the longitudinal behaviour of the tracer integral as described before.

The equation for error propagation in any relationship such as equations (2), (5) and (7) is, for example

$$\phi/A = [(\phi/Q)^2 + (\phi/u)^2]^{1/2} \quad (12)$$

the  $\phi$  values are for those parameters shown in the denominator (Schenck, 1968). In the case of the tracer integral,  $\phi$  can only be determined from the standard deviation. In the nine tests where tracer losses were observed, only those measurements immediately downstream of  $x_m$  which were judged to be consistent with the value of  $I$  at  $x_m$  were considered. The mean velocity estimates were calculated by the salt velocity method, equation (6), taking the difference between the mean travel time at  $x_m$ , and the distance at  $x_m$  and each successive downstream measurement. In this manner each estimate extends over an increasing channel length, and the average of these values were taken as the estimate of the mean stream velocity. As the error in distance measurements is unknown the  $\phi$  estimate for the mean velocity also had to be determined from the sample standard deviation. A similar approach was necessary for determining  $\phi$  for the discharge (equation 2) as the errors in  $V$ , the injection volume, are unknown. However, as Church (1975) estimated that the error in  $V$  should only be about 0.1 percent, then the probable error in  $Q$  should be virtually the same as  $I$  [the term  $(\phi/V)^2$  would be equal to  $1 \times 10^{-6}$  and quite negligible].

TABLE 6 — Summary of precision data.

Parameter	Error equation	Probable error*, $\phi$ (%)		
		Median	Mode	Range
Tracer integral, $I$	$0.675\sigma_I$	5.1	3.6	1.0–13.8
Mean velocity, $u$	$0.675\sigma_u$	4.7	4.5	2.1–14.2
Discharge, $Q$	$0.675\sigma_Q$	5.4	3.8	0.9–15.4
Flow area, $A$	$\phi/A = [(\phi/Q)^2 + (\phi/u)^2]^{1/2}$	7.3	6.8	3.2–20.9

\* The  $\pm$  of the probable errors is implied. Distributional characteristics are described by medians and modes rather than means because the error distributions are asymmetrical.

#### Precision Estimates

Day (1976) states that the largest mean velocity errors occur when  $u$  is greater than 1 m/s, although not all high velocities exhibit lower precision. Tracer losses do not appear to affect estimates of  $t_m$  and hence  $u$ . For Thomas test 8, which extended over 2250 m and exhibited larger tracer losses, the  $\phi$  of  $u$  is only 1.9 percent over the entire reach. The apparent effect of losses is to 'shrink' the wave, maintaining similarity (Day, 1974), and altering all time parameters except the mean travel time. Also, any errors in measuring times (e.g. time variations due to cross-sectional position of the sample) should be very small relative to the total time elapsed from initiation. By the nature of error propagation, determination of  $A$  is less precise than either discharge or mean velocity. The largest errors are in fact associated with  $A$ , as are the mean distributional statistics (Table 6). A more complete presentation of error sources and magnitudes is shown in Table 7, for the Craigeburn test reach.

As the deviations about the mean of individual test sequences are normally distributed, measurement errors are apparently random, although there is also the possibility of consistent systematic errors which can result from incorrect



calibration, instrument faults, etc. Although such errors are a possible explanation for variations among the test sequences, there is no evidence that significant systematic errors existed during any individual test sequence.

The magnitude of errors resulting from chemical gaugings is expected, under proper conditions, to be similar to those resulting from current-meter gauging. Bell (1969, p. 29) stated that the precision of both methods should be about  $\pm 5$  percent under reasonable field conditions. The relative precision of the slug injection method discussed here was field tested against a constant injection method (sodium dichromate) on the Porter test reach. Discharge by the slug method was determined to be about 9 percent larger, a result somewhat greater than suggested by Bell, but judging from the ranges of the discharge errors of the test data, a difference which must be expected.

It follows from Bell's statement that if the discharge precision among methods is in the order of  $\pm 5$  percent, then the precision of any single discharge measurement (test sequences in this case) must also be within these limits. In point of fact the modal discharge error for all the test data is only  $\pm 3.8$  percent, with a median of  $\pm 5.4$  percent. However, the range is considerable; from  $\pm 0.9$  to  $\pm 15.4$  percent.

Inspection of the data indicates no relationships between error magnitude and such variables as flow scale, temperature, temperature range or channel length. Suspended sediment concentrations also appear to have no determinable effect, as any flow above 1 m/s contained such material, and no noticeable relationship exists. Similarly, as all channels have similar bed and banks (gravel and no vegetative matter) boundary properties are common and again no variations are evident. Errors encountered in these experiments therefore appear to be dominantly random ones associated with field applications.

## TRACER LOSSES

Tracer losses were first identified by the existence of a decreasing tracer integral with increasing distance (no increases in tracer integrals occurred). The structure and significance of these losses were investigated by regressing tracer integral against distance. Linear equations were fitted to all measurements at and downstream of  $x_m$ . A summary of this analysis is shown in Table 8. The amount of variation in  $I$ , explained by the variation in distance ( $r^2$ ), ranges from 45 to 93 percent. A 't' test (Steel and Torrie, 1969, p. 161) was used to test for the significance of the regression slope. Tracer losses were considered to be present when this slope was significantly different for zero. The intercepts of the regression equations are not listed, as they are functions of the initial concentrations. Further curve fitting in the form of second-order parabolas resulted in only one case of an increased  $r^2$ , from 65 to 69 percent for Craigieburn test 11. This increase was, however, statistically insignificant. Although it is implied in this analysis (by fitting the regression equation through the estimated  $x_m$ ), tracer losses do not necessarily begin at the mixing length. The choice was one of convenience to demonstrate the nature of the losses downstream of this point.

Losses of salt tracer may result from several causes: e.g. adsorption on to bed and suspended particles, inflow-outflow exchange through the beds and banks, and insensitivity of the measurement system to low concentrations. It is also possible to have apparent tracer losses if the flow rate is increasing (more dilution); however, in these data no correlation between declining tracer integrals and increasing velocities exist.

TABLE 7 — Precision data for Craigieburn test reach.

Tev*	Maximum channel length (m)†	Mixing length (m)	Volume of injection (litres)	No. of tracer measurements downstream of $x_m$ ‡	Tracer integral		Hydraulic parameters: mean and % probable error §		Flow area, A			
					Mean (ppm.s)	Error, $\rho$ (%)	Mean (m/s)	Error, $\rho$ (%)	Mean (m <sup>2</sup> /s)	Error, $\rho$ (%)	Mean (m <sup>2</sup> )	Error, $\rho$ (%)
1 T	540 (210)	30	48	13(7)	240848	1.7	0.35	5.0	0.20	1.7	0.57 ± 0.03	5.3
2 T	540 (270)	30	16	15(10)	48219	6.5	0.42	3.5	0.34	6.4	0.80 ± 0.06	7.3
3	780	120	16	13	38058	12.9	0.43	6.0	0.44	12.3	1.02 ± 0.14	13.7
4 T	780 (240)	90	16	14(6)	22780	2.7	0.55	4.8	0.70	2.7	1.28 ± 0.07	5.5
5	480	240	16	5	37405	1.0	0.38	3.2	0.43	0.9	1.13 ± 0.04	3.3
6	660	150	45	8	20337	3.8	1.01	4.6	2.27	5.4	2.25 ± 0.16	7.1
7 T	780 (420)	240	45	5(4)	34235	3.2	0.80	3.7	1.32	3.3	1.65 ± 0.08	5.0
8	476	180	45	5	11176	11.6	1.49	4.8	4.12	11.5	2.77 ± 0.35	12.5
9	693	150	48	9	14423	4.5	1.18	2.1	3.29	4.6	2.79 ± 0.14	5.1
10	693	210	45	7	10568	10.8	1.16	2.3	4.35	10.7	3.75 ± 0.79	20.9
11	693 (360)	180	45	7(3)	30205	2.5	0.83	--	1.49	2.5	1.80 ± --	--

\* T denotes tracer losses as indicated by decreasing tracer integrate.

† Brackets denote length of channel over which tracer losses do not appear significant.

‡ Brackets denote number of measurements used to estimate mean hydraulics when tracer losses are present.

§ Number of measurements for calculation of mean velocity is always one less than in column 7 (— denotes insufficient data).

TABLE 8 — Tracer loss data.

<i>Test</i>	<i>No. of tracer measurements*</i>		<i>Slope of regression equation</i>	<i>Amount of explained variance (%)</i>
Bruce	2	14	- 13.0	45
	7	15	- 4.9	56
	8	6	- 8.6	93
Craigieburn	1	13	-102.9	63
	2	15	- 44.2	47
	4	14	- 11.5	77
	7	5	- 12.8	78
	11	7	- 15.8	65
Thomas	8	10	- 3.7	84

\* The degrees of freedom associated with the regressions are two less than the number of tracer measurements.

There is no consistent information to suggest tracer losses are associated with changes in water temperature and flow scale. Also, for all but the long Thomas test 8 (2250 m), channel length does not consistently affect either the occurrence or magnitude of losses. All remaining tests were measured within 200 mean channel widths downstream. On the other hand, the long Thomas test extended over 800 channel widths. Tracer losses were severe over these distances with the last measured time-concentration curve having an  $I$  of only 29 percent of that determined near  $x_m$ . The mean velocity estimates remained remarkably consistent, with a probable error of only 1.9 percent, indicating that the salt velocity method remains valid over these distances and tracer losses, where the salt dilution gauging does not.

Significant concentrations of suspended sediment were present in most higher flows ( $u > 0.8$  m/s), and five of the nine tests which exhibited losses were recorded under these conditions. However, as tracer losses do not occur for all such flows then suspended sediment cannot be offered as a complete explanation.

### CONCLUSION

The application of slug injection gauging in turbulent mountain streams has been considered. The theory of its application and an outline of a data collection system is offered. Data presented from a large number of gauging experiments show that lateral mixing can be quickly estimated from a single geometric parameter, the mean flow width, using a simple equation,  $x_m = 25w$ . These mixing lengths are shown to be usually shorter than those predicted from conventional formulae. Injection position in the cross section does not have any effect on the location of the mixing length.

Although the slug injection technique discussed here is reasonably well known, little information on its precision under field conditions is available. The errors discussed here, for data on a wide range of flow events and stream scales, and collected under a variety of field conditions, indicate that the maximum probable errors in the order of  $\pm 10$  to  $\pm 20$  percent can be expected, although their occurrence is infrequent. Most errors are in the order of  $\pm 4$  to  $\pm 7$  percent.

Losses of salt tracer are shown to be linear features with a constant rate of loss along the channel. Explanation of both the causes of these losses and their linear nature is unfortunately not as yet possible.

Some procedures for determining sampling periods and intervals are given. Also, based on the similarity in structure of the dispersion process, several rapid methods are presented for the determination of the mean travel definition of inadequately defined time-concentration curves are outlined.

The results presented here also indicate that the salt velocity method remains quite valid through large channel lengths and large tracer losses. On the other hand, as tracer losses appear unpredictable, dilution gauging for discharge and flow area determinations should be restricted to much shorter channel lengths. A better estimate of hydraulic parameters will result from two time/concentration curves measured at two distances downstream of the mixing length. Should only one tracing of  $C(t)$  be possible, then the mean wave-velocity estimate will vary from the stream velocity depending upon the downstream location of the sampling site.

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