Journal of Hydrology (NZ) 38(1): 121-144, 1999

A Conceptual Systems Model of Rainfall-Runoff on the Haast River

Paul W Jowitt

Professor of Civil Engineering Systems Department of Civil & Offshore Engineering Heriot-Watt University Edinburgh Scotland, UK

Abstract

A conceptual systems model of runoff generation is described and applied to hourly rainfall-runoff data from the Haast River. The so-called MaxH model treats the catchment as a statistical population of different-sized water storage elements. The distribution of water within this population is also described statistically. The uncertainty over the choice of probability distributions for the catchment storages and the actual catchment storage within them is dealt with using the principle of maximum statistical entropy. The characteristics of the derived nonlinear model are described and compared to the more familiar linear reservoir. Both model types are then applied to a set of rainfall-runoff data from the Haast River in the western Southern Alps on the South Island of New Zealand. Within the limitations of the study, the MaxH model displays some useful characteristics such as the ability to reproduce a more rapidly rising hydrograph limb with increasing catchment wetness. In other respects, some of the long-standing problems with simple catchment models still remain, such as finding objective ways of determining the percentage runoff, baseflow separation etc.

Introduction

The hydrological literature contains a diverse and rich set of models for simulating rainfall-runoff processes, ranging from black-box time series models to those that are physically-based and with a more iconic structure. In between are conceptual models, some with many system states and model parameters, others much simpler and often better suited for operational uses such as flood forecasting. Of these models, there is the genre based simply on catchment mass (water) balance together with an assumed relationship between catchment storage and runoff. The simplest of these Input-Storage-Output models is the well-known "linear reservoir". A number of nonlinear models can be developed along similar lines. This paper describes an altogether different nonlinear model which derives in part from a logical extension of the distributed function approach to rainfall-runoff modelling proposed by Moore and Clarke (1981) and which eliminates the need for some pragmatic but arbitrary assumptions about storage redistribution. It was also hoped that the model might replicate the tendency for some catchments to exhibit a flashier response as they become more saturated. The resulting MaxH model uses an information theory criterion to establish least-biased estimates of the distribution of catchment capacity and storage. This is coupled with a simple mechanism for generating runoff to produce a nonlinear catchment model whose rate of change of runoff increases with catchment wetness. The dynamic response of this model is demonstrated and then the model applied to hourly rainfall-runoff data from the Haast River.

Linear and nonlinear reservoir models

The linear reservoir assumes that runoff q is directly proportional to catchment storage V through a constant k.

$$q = kV \tag{1}$$

This is coupled with the differential equation describing the catchment mass balance:

$$\frac{dV}{dt} = p - q \tag{2}$$

The response of the linear reservoir model is characterised by an "impulse response" which is exponential in form. And of course, the "principle of superposition" applies so that the additional response of the model to a rainfall event is the same irrespective of the antecedent condition on the catchment. By extension, a range of nonlinear models can be developed along similar lines. For example, the Ibbitt model (Ibbitt, 1976) assumes the form:

$$q = k_I V^n \; ; \; n > 1 \tag{3}$$

The Lambert model (Lambert, 1972) developed for forecasting floods on the River Dee in Wales assumes a logarithmic relationship between runoff and storage:

$$q = \exp\left(\frac{V}{k_{La}}\right) \quad \text{or more familiarly} \quad V = k_{La} \log_e q \tag{4}$$

In both the Ibbitt and Lambert models the rate of change of runoff increases with catchment storage, so the models have some capacity to simulate a flashier response when the catchment is more saturated [1].

The responses of both the Ibbitt and Lambert models are nonlinear and superposition does not apply, so that the "impulse response" depends through the value of V on the antecedent catchment wetness. From a physical point of view, all three models place no upper bound on catchment storage. All three models can also be regarded as building blocks of more complex catchment models. The most well-known of these is the Nash Cascade (also sometimes referred to as the Kalinin-Milyukov model when used in flow routing), comprising a number of identical linear reservoirs in series (Dooge, 1973; Kalinin & Milyukov, 1957). The corresponding impulse response is described by the incomplete gamma function:

$$h(t) = \frac{k^n . t^{n-1} . e^{-kt}}{\Gamma(n)}$$

$$\Gamma(n) = \text{gamma function} = (n-1)! \qquad (5)$$

k = scale parameter

n = shape parameter

Probability-distributed models

Instead of regarding the catchment of a single storage element (or even a series of identical storages elements as with the Nash Cascade), Moore and Clarke (1981) proposed that a catchment could be represented by a statistical population of storage elements of different sizes, characterised by a probability distribution. Water is stored in each of the hypothetical storage elements and the same rainfall-runoff model is applied to each; the results are aggregated to produce the overall runoff. It was assumed that all storages of the same size behave similarly.

In their original development, Moore and Clarke assume that runoff develops only from stores which become full and overflow, which fits nicely with the idea that the contributing area increases with catchment wetness. Evaporation takes place from any store which is not empty, leading to a storage deficit d and a storage v (= s - d). This is simple and straightforward, but it results in a rather cumbersome description of the system states, which tends to grow in complexity as the system evolves. Figure 1 illus-

---

[1] To maintain dimensional consistency, and to eliminate the absurd possibility of negative storage, the Lambert model would be better expressed in a form such as:

$$V = k_{LJ} . \log_e (1 - q/q_0)$$

where q_0 is some dimensionally consistent positive constant. To this author's knowledge, this form has not been explored.

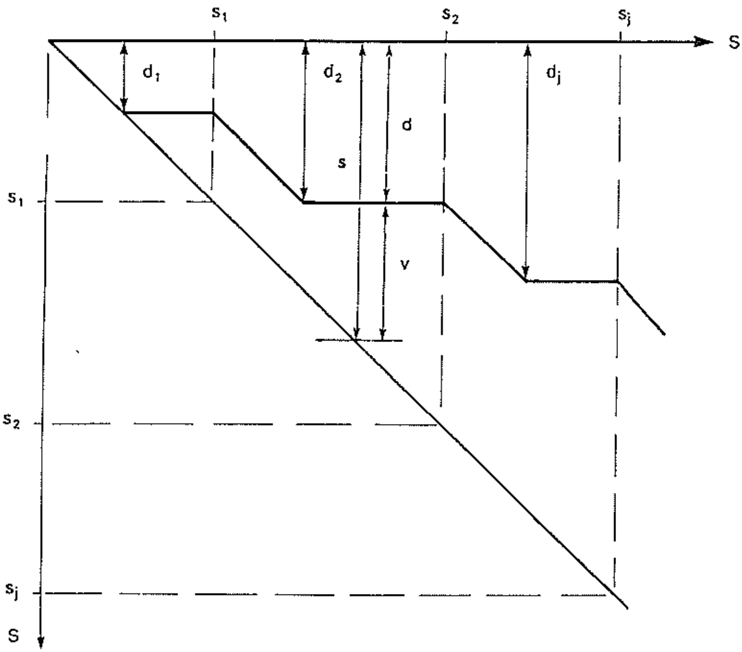

Figure 1. – Profile of contents v within storage elements of capacity s.

trates the variation in deficits d across the range of storage elements of capacity s (Moore and Clarke, 1981).

The state of the system comes to be characterised by a terraced pattern of actual storage within the range of catchment storage elements. The diagram (Figure 1) can be thought of in terms of a set of tubes, one for each value of s, arranged vertically in increasing magnitude from left to right, with their tops level along the horizontal axis. The area between the terraced line and the sloping line represents stored water within the storage elements, and the area between the terraced line and the horizontal axis represents the storage deficits.

The level segments of the terraced pattern are characterised by the same value of *deficit* d_j between storage values d_i and s_j. The sloping segments are defined between storage values s_j and $(s_j+d_{j+1}-d_j)$. These storage elements all contain the same *storage* (s_j-d_j). The number of such segments (and the values of s_j, d_j, s_{j+1}, d_{j+1} etc) is indeterminate unless the initial condition is completely specified, which is practically impossible.

To avoid this and the related problem of system observability, Moore (1985) simplified the model by re-distributing the catchment storage at the end of each rainfall episode, filling all the small stores to capacity and the remainder to equal depth. Although this preserves the notion that contributing area increases with catchment wetness, it remains rather arbitrary.

Jowitt (1989) proposed an alternative model — the *MaxH* — model preserving the probability distributed approach but making as few assumptions as possible about the distribution of total catchment storage amongst the population of storage elements. The model is fully described elsewhere (Jowitt, 1991) and this presentation will be restricted to essential points. Although the MaxH model appeared in the literature some years ago, its dynamic characteristics were never explored; far less applied to actual data. Indeed, at the time the model appeared too unwieldy for numerical solutions. As it turns out, the difficulties are far less than they first appeared.

The *MaxH* catchment model

In Moore and Clarke's model, the population of catchment storage elements are described by a chosen probability distribution. Actual storage within these elements is described deterministically. In contrast, in the *MaxH* model, *both* the catchment storage elements and the catchment water stored within them are described in probabilistic terms.

A marginal probability density function f(s) describes the distribution of catchment storage elements. A set of conditional density functions f(v/s) describes the distribution of water within storage elements of size s. In contrast to Moore and Clarke's model, in the *MaxH* model not all storage elements of size s will contain the same amount of stored water and therefore not all elements of size s will behave similarly.

Objectivity in the choice of f(s) and f(v|s) is maintained by using Jaynes' principle of maximum statistical entropy. Detailed explanations of the maximum entropy formalism can be found in Jaynes (1983) and Jessop (1995). An application to flood frequency is given by Jowitt (1979). The underlying idea behind the maximum entropy formalism is to select that probability density function which satisfies the known information but which otherwise seeks to avoid favouring any one outcome over another without good reason. In simple terms, it attempts to make the probability density function as flat as possible.

Application of the principle of maximum entropy to the *MaxH* model

For a discrete probability distribution, the expression for entropy (Shannon, 1948) is simply:

$$H = -\sum_{i=1}^{N} p_i . \log_e p_i \qquad (6)$$

H measures "uncertainty" or "missing information" (i.e. the extent to which there is uncertainty about what will happen). When one of the N probabilities is unity, there is no uncertainty and H = 0. When all the probabilities are equal (=1/N), then there is maximum uncertainty and H takes the maximum possible value of $\log_e N$. The larger the value of H, then the more the uncertainty. The expression for entropy in the continuous case is:

$$H = -\int_0^\infty f(x).\log_e \frac{f(x)}{m(x)}.\, dx \tag{7}$$

where f(x) is the probability density function of random variable x and m(x) is an "invariant measure function" which ensures that the entropy measure remains invariant under allowable changes of variable (such as a change of scale or location). (see Jaynes, 1983)

The two probability distributions of interest herein are f(s), describing the population of storage elements and f(v|s), describing the water content v within storage elements of size s. Both are generated using the entropy formalism.

The distribution of storage elements: f(s)

For f(s), all that is assumed to be known is that the mean storage element volume is proportional to the total catchment storage capacity V_{max}. When the model is applied to real data, V_{max} will become a model parameter, but for now it assumed known. The lower bound on s is taken as zero. For computational convenience, the upper bound is taken as infinity, which produces approximately the same result as taking an upper bound of several multiples of the mean storage. This means that all that is assumed known about f(s) is:

$$\int_0^\infty f(s)\,.\,ds = 1 \tag{8a}$$

$$\int_0^\infty s.\, f(s).ds = V_{max} \tag{8b}$$

Applying the principle of maximum entropy to determine f(s) therefore requires the solution to the following mathematical program:

Maximise: $H_s = -\int_0^\infty f(s).\log_e \frac{f(s)}{m(s)}.\, ds$ with respect to f(s) where m(s) is the appropriate invariant measure function and subject to constraints (8a) and (8b)	(9)

The solution to this mathematical program is found using Lagrange's method of undetermined multipliers (Strang, 1986). The required solution takes the form:

$$f(s) = m(s).\exp[-\lambda_0 - \lambda] \qquad (10)$$

where λ_0 and λ are the two Lagrangian multipliers whose values are such that Equations 8a and 8b are satisfied.

For the distribution of storage elements s, the only allowable change of variable is restricted to a change of scale; i.e. the same result should be obtained irrespective of the units used to specify V_{max}. This requirement for scale invariance requires that m(s) = constant and results simply in the negative exponential distribution:

$$f(s) = \lambda.\exp[-\lambda s] \quad \text{where } \lambda = 1/V_{max} \qquad (11)$$

The distribution of water within the storage elements: f(v|s)

The form of the density function f(v|s) is required for all values of s. All that is assumed to be known is the total amount of water stored in the catchment, V. Nothing is known about the mean storage contained in storage elements of a particular size s, except that, over all values of s, the overall mean storage in the catchment is V. Thus f(v|s) must satisfy:

$$\int_0^{\infty} f(s) \int_0^{s} f(v|s).\,dv.\,ds = 1 \qquad (12a)$$

$$\int_0^{\infty} f(s) \int_0^{s} v.f(v|s).\,dv.\,ds = 1 \qquad (12b)$$

The form of f(v|s) is thus generated from the following mathematical program:

$$\text{Maximise: } H_v = -\int_0^{\infty} f(s) \int_0^{s} f(v|s).\log_e \frac{f(v|s)}{m(v|s)}.\,ds.dv \text{ with respect to } f(v|s) \qquad (13)$$

where m(s) is the appropriate invariant measure function

and subject to constraints (8a) and (8b)

Again, using Lagrange's method of undetermined multipliers, this leads to solutions of the form:

$$f(v|s) = m(v|s).\exp[-\phi_0-\phi v] \qquad (14)$$

where ϕ_0 and ϕ are the Lagrangian multipliers which ensure constraints 12a and 12b are satisfied.

Scale invariance requires values of f(v|s) to depend on the ratio of v to s. This means that m(v|s) is independent of v and so can be absorbed into the normalisation constant. Thus f(v|s) becomes a truncated exponential distribution; a negative exponential when $V < V_{max}$, a positive exponential when $V > V_{max}$ and uniform when $V = V_{max}$. Thus:

$$f(v|s) = \frac{\phi.\exp[-\phi v]}{1-\exp[-\phi s]} \qquad (15)$$

and where ϕ is such that equation (12b) is satisfied.

The value of ϕ is thus defined through the equation:

$$V= \frac{1}{\phi} - \int_0^{\infty} \frac{1.e^{-\lambda s}.s.e^{-\phi s}}{(1-e^{-\phi s})}.\,ds \qquad (16)$$

The runoff generating mechanism and catchment dynamics

The runoff generating mechanism is assumed to be based in part on the familiar Linear Reservoir model (q = kV) and in part on the Moore and Clarke model. Thus runoff is proportional to catchment storage, *but only* from those storage elements which are full; i.e. $q = k_H v$, weighted by f(s) and f(v|s) with *v set to the value s*, and where k_H is some model parameter. This results in the following expression for the runoff q:

$$q = k_H.\int_0^{\infty} f(s).(v{=}s).f(v{=}s|s).\,ds \quad = k_H.\int_0^{\infty} \frac{\lambda.e^{-\lambda s}.s.\phi.e^{-\phi s}}{(1-e^{-\phi s})}.ds \qquad (17)$$

and where

$$\frac{1}{\lambda}=V_{max}= \text{a catchment property, and} \qquad (18)$$

$$\phi = \text{a function of catchment wetness.} \qquad (19)$$

From the similarity between the integrals in equations (16) and (17), a simple relationship between q and V becomes apparent:

$$q=k_H.(1-\phi V) \qquad (20)$$

The catchment dynamics are thus given by:

$$\frac{dV}{dt} = p - q = p - k_H.(1-\phi V) \qquad (21)$$

where ϕ is a function $\phi(V)$ of V.

Numerical solutions

The integral in equation (16) for V (and which defines the relationship between ϕ and V, and in turn q) has no general closed form. It is in fact related to Riemann's 2-parameter zeta function, for which no comprehensive tables were generally available in the public literature - hence the pessimism expressed in Jowitt (1991) over the possibility of ever testing the model against field data.

Riemann's 2-parameter zeta function is defined as follows (Gradshtein and Ryzhik, 1980):

$$\zeta(z,\mu) = \frac{1}{\Gamma(z)}.\int_0^\infty \frac{x^{z-1}e^{-\mu x}}{(1-e^{-x})}.dx \quad ; \text{ Re } \mu > 0; \text{ Re } z > 1 \qquad (22)$$

and which can also be expressed as an infinite series:

$$\zeta(z,\mu) = \sum_{n=0}^{\infty} \frac{1}{(\mu+n)^z} \quad ; \text{ Re } z > 1 \qquad (23)$$

Note also the recurrence relation $\zeta(z,\mu) = \zeta(z,1+\mu) + \frac{1}{\mu^z}$ (24)

Using relations (22), (23) and (24), together with a change to the dimensionless variable:

$$w = -\phi/\lambda = -\phi.V_{max} \qquad (25)$$

results in the set of model equations:

$$\frac{dV}{dt} = p - q \qquad (26)$$

and from equations (20) and (25)

$$q = k_H.(1+\frac{w.V}{V_{max}}) = \frac{k}{2}.[V_{max}+w.V] \qquad (27)$$

where

$w = w(V,V_{max})$

$= w(\phi(V),V_{max})$ is a dimensionless index of catchment wetness (28) and $KH = \frac{k}{2}.V_{max}$ (29)

and with

$$V = \frac{V_{max}}{w}.[1+\frac{1}{w}.\zeta(2,1-\frac{1}{w})] \quad ; \ w<0\ ; V<\frac{V_{max}}{2} \tag{30a}$$

$$V = \frac{V_{max}}{2} \quad ; \ w=0\ ; \tag{30b}$$

$$V = \frac{V_{max}}{w}.[1+\frac{1}{w}.\zeta(2,\frac{1}{w})] \quad ; \ w<0\ ; V<\frac{V_{max}}{2} \tag{30c}$$

and where

$$\zeta(2,\frac{1}{w}) = \text{Riemann's 2 – parameter zeta function} = \int_0^\infty \frac{x.e^{-\frac{x}{w}}}{(1-e^{-x})}.dx \tag{31}$$

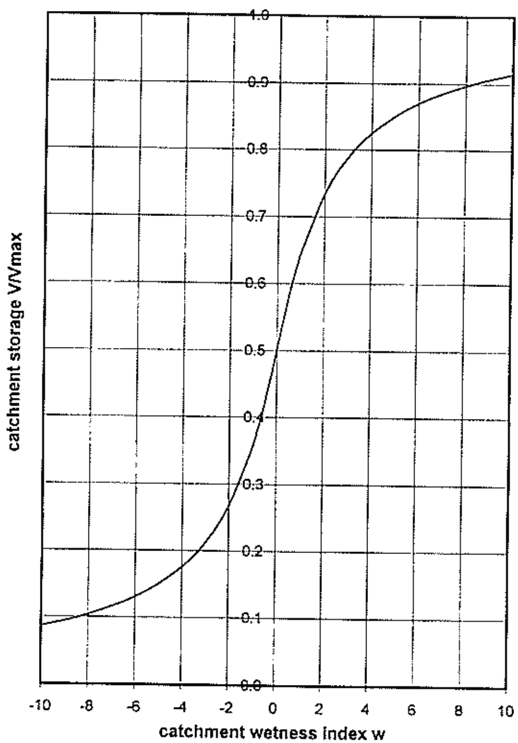

Figure 2. – Variation of $V/_{max}$ with catchment wetness index w.

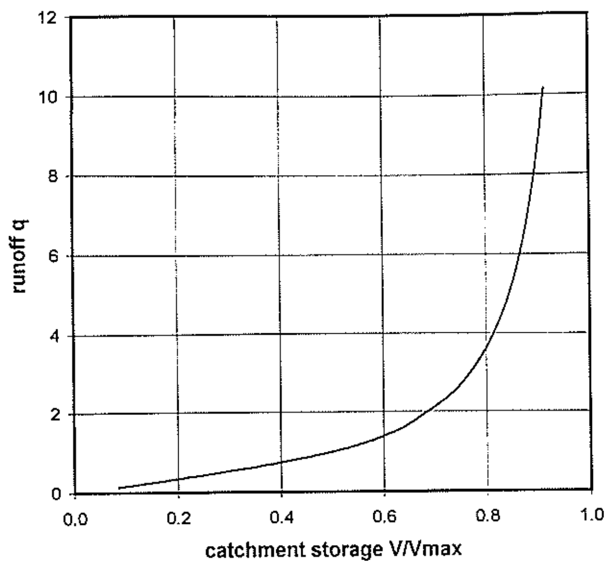

Figure 3. – Variation of runoff q with catchment storage V/V_{max}.

For values of $|w|$ not close to zero ($|w| > 0.1$), values of the zeta function can be found by numerical integration and organised as a table. Close to zero the integral becomes ill-conditioned but values can be found to sufficient accuracy by expansion of the terms in the original integral (Hunt ,1997) to complete the table.

The variation of V/V_{max} with the dimensionless catchment wetness index w, and the variation of dimensionless flow q (corresponding to $k_H = 1$) with V/V_{max} are shown in Figures 2 and 3. Tabulated values are available in Jowitt (1997).

Dynamic properties of the *MaxH* model

The tabulated values of $\zeta(2, \frac{1}{w})$ for $w > 0$ and $\zeta(2, 1 - \frac{1}{w})$ for $w > 0$ were organised as a table within a suitable numerical simulation package. The *ModelMaker* package (Cherwell Scientific, 1995) proved a suitable vehicle not only for the dynamic simulation of the *MaxH* model itself but also as an integrator to establish values of the zeta function away from w~0. The dynamic behaviour of the *MaxH* model is now briefly presented before applying the model to data from the Haast River. At this point *MaxH* model is compared to the with the usual linear reservoir model (L) in which $q_L = kV_L$ with runoff q_L and storage V_L.

MAX ENTROPY RESERVOIR MODEL	LINEAR RESERVOIR MODEL

Mass balance

$$\frac{dV}{dt} = p(t) - q(t) \qquad\qquad \frac{dV_L}{dt} = p(t) - q_L(t) \tag{32a,b}$$

Runoff generation

$$q = \frac{k}{2}.[V_{max} + w.V] \qquad\qquad q_L = k.V_L \tag{33a,b}$$

so that when $V = \dfrac{V_{max}}{2}$

$$q = \frac{k.V_{max}}{2} = q_L \tag{34}$$

The nonlinear nature of the *MaxH* model is clear and comes through the catchment wetness index w. The *MaxH* and linear reservoir models have the same relationship between runoff and catchment storage when the *MaxH* storage is exactly half of its maximum value. But at values of $V > V_{max}/2$, the rate of runoff in the *MaxH* model is greater than that in the linear reservoir model, and vice versa for $V < V_{max}/2$.

These properties are reassuring in the following sense: the essence of the entropy formalism is to infer as little as possible beyond that which is known. In this case, this prior knowledge comprised the following minimal set of model assumptions:

1. The total actual storage in the catchment is bounded between zero and some maximum value V_{max} (in contrast to the linear reservoir which places no limit on catchment storage),
2. Runoff is generated only from those individual storage elements which are at capacity.

Figures 4 and 5 show the variations of runoff q and catchment storage V in response to a set of synthetic rainfall input rainfall sequences, an example of which is shown in Figure 6. Also shown is the response of the corresponding linear reservoir. In both cases, k = 1. For the *MaxH* model, V_{max} is set to 20. The set of rainfall sequences consist of an impulse, and later a step function, superimposed on a set of 5 background steady state "rainfall" values (b_f) which are intended to imitate the effect of baseflow.

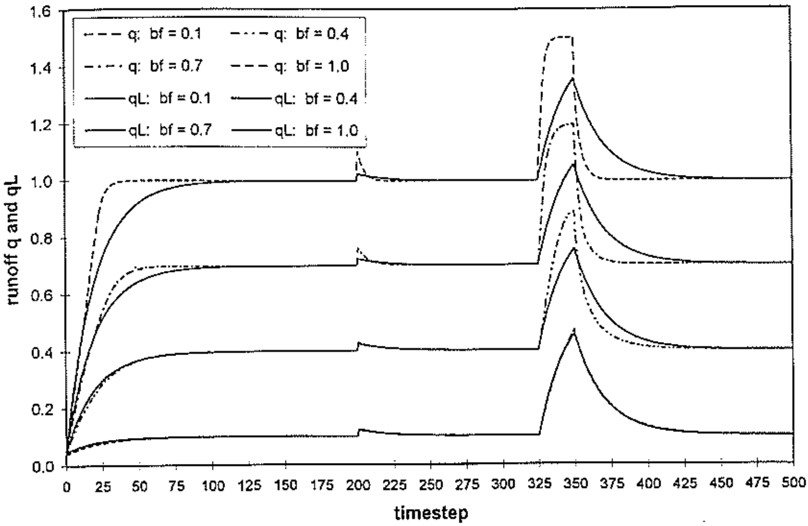

Figure 4. – *Max H* runoff q and linear reservoir runoff q_L for various values of baseflow b_F.

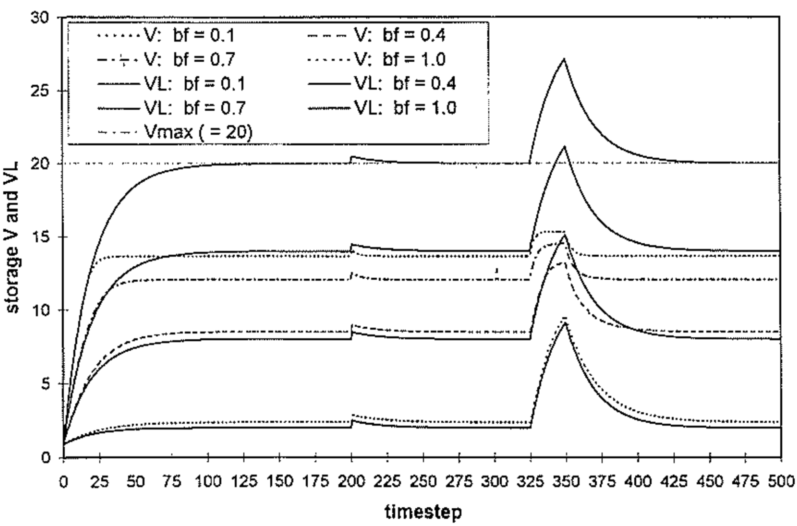

Figure 5. – Variation of catchment storage V and V_L for various values of baseflow b_F.

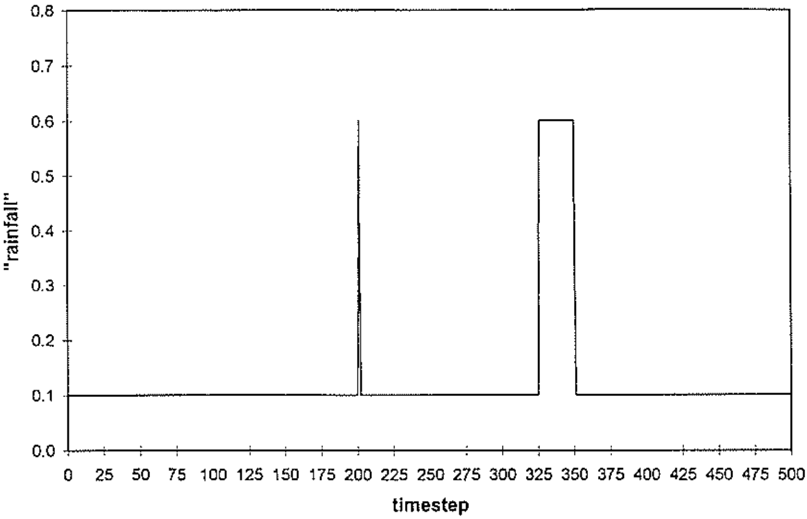

Figure 6. – Example of synthetic "rainfall" input: (b_f (= 0.1) + impulse + step)

Of course, for the linear reservoir model, once steady-state runoff is attained, the transient response to the impulse and step function is identical. This is not so for the MaxH model; because of the limit of V_{max}, the runoff shows a much flashier response as the catchment becomes saturated. [2]

This highly non-linear response can cause some occasional instability with particular values of model parameters. Such behaviour might be expected given that the model relies on a tabulated function within a numerical integration routine. No major problems however were encountered during the model calibration exercise, which is described below.

The Haast River

The Haast River in the western Southern Alps on South Island is one of New Zealand's largest in terms of discharge, having the tenth highest mean annual discharge (Carson *et al*, 1997). The characteristics of the Haast catchment are summarised in Table 1. For this study, the runoff data is taken from Roaring Billy (NIWA flow gauge number 86802) and the rainfall record comes from NIWA station number 399410. Rainfall and runoff are recorded hourly. For modelling, both sets of data were re-scaled to give

[2] In the words of an old friend and colleague, Victor Appleby - now aged 99 and still working on his PhD on nonlinear rainfall runoff models (eg Appleby, 1974) - it shows the characteristics of those "Walls of Water", a term often used by survivors of real floods and which he thought encapsulated the behaviour of catastrophic floodwaters.

Table 1 – The Haast River at Roaring Billy New Zealand western Southern Alps Region of South Island

Catchment Area	1026 km2
Altitudinal Range	53-2650 m
Tributaries	Landsborough, Wills, Clarke Rivers
Topography	Steep/very steep mountain slopes under perennial ice & snow; Icefields/glaciers along NW perimeter; small areas of flood plains; Steep-sided valleys running NE - SW
Mean annual discharge (1971-96)	191.9 m3/s
Mean Annual Rainfall	~6500 mm/yr
Mean Annual Evaporation	~570 mm/yr
Time of Concentration	~7 hours
	Source: Information based on Carson et al (1997).

an hourly mean of unity. For model calibration, the first 1600 hours of the 1992 record were used to estimate the parameters, and for model validation, the fitted models were run against the first 4000 hours of 1995 data. Examples of the rainfall and runoff records are shown in Figures 7 and 8.

Modelling preliminaries

The river flows are sustained by snowmelt from the upper catchment. The rainfall pattern with its alternating episodes of rainfall and no rainfall gives rise to the transient runoff events. The flow record therefore contains runoff phenomena at two different timescales and from different runoff mechanisms - slow and fast. The modelling objective was to model the

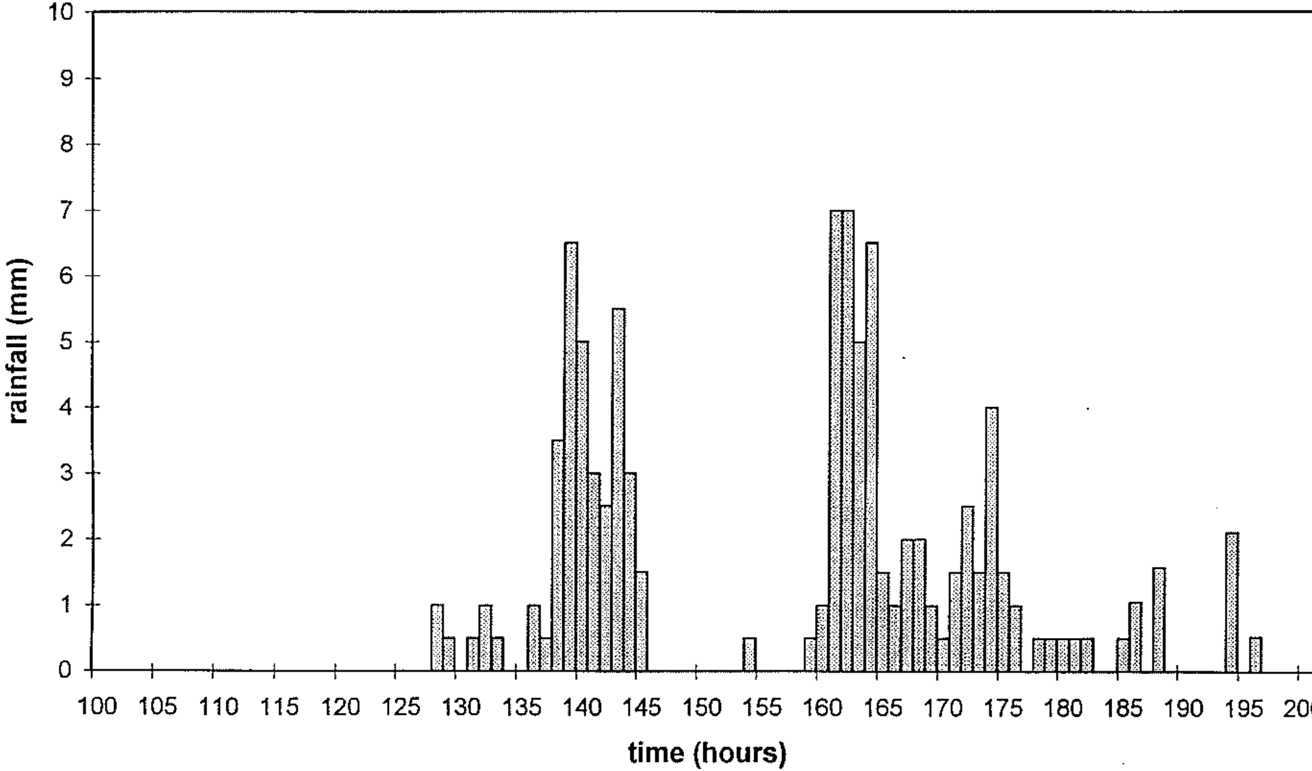

Figure 7. – Haast 1992: rainfall (mm) at Guage 399410.

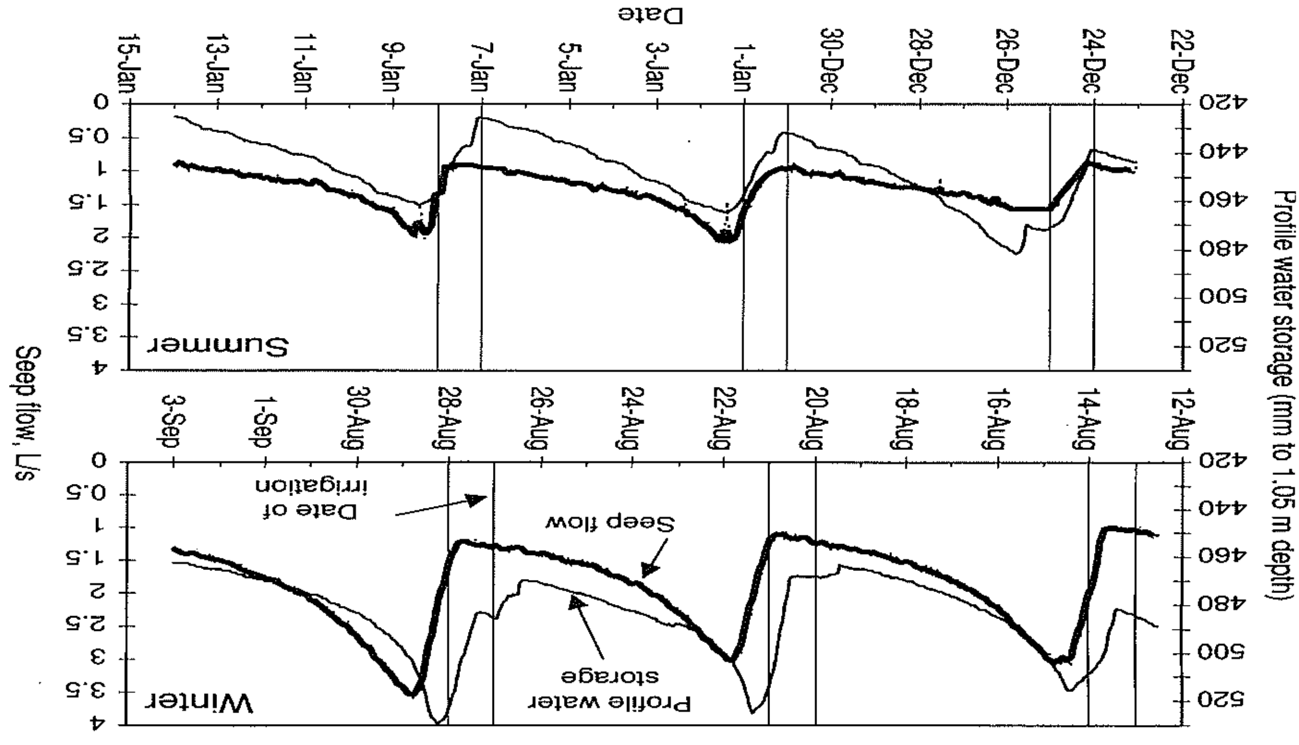

Figure 8. – Haast 1992: runoff (cumecs) at Roaring Billy – Guage 86802.

whole flow record, rather than just storm runoff. Neither the linear reservoir nor *MaxH* models can model both fast and slow runoff responses simultaneously. To overcome this difficulty, the simple expedient was used of including a background inflow—essentially a baseflow component—into each of the models. [3] To compensate, the rainfall component was factored by a runoff coefficient, a. The catchment mass balance equation used in the modelling studies then took the form

$$\frac{dV}{dt} = B + a.p - q \qquad (35)$$

where a = runoff coefficient, and

B = "baseflow" = $b_f - \alpha.\,(t - 4000)$ with $b_f = 0.1$ and $\alpha = 0.000165$

The choice of the "baseflow" model was based simply on a visual fit, such that the recession limbs of the storm hydrographs fell back to something approaching the underlying baseflow B.

The linear reservoir model therefore has two additional parameters to be estimated, the runoff coefficient, a, and the storage constant, k. The *MaxH* model has three parameters, a and k, with the additional parameter, V_{max}. The modelling and parameter estimation was undertaken using the *ModelMaker* package (Cherwell Scientific, 1995). The model files are available from Jowitt (1998).

Results

The model was calibrated using data from 1992 and then independently validated using data from 1995. The chosen goodness of fit criterion was to maximise

$$R^2 = 1 - \frac{\text{var(errors)}}{\text{var(data)}} \qquad (36)$$

The model parameters and the R^2 values in calibration and validation are given in Table 2 for both the linear reservoir and *MaxH* models. In the validation exercise, exactly the same "baseflow" was used as in the calibration. In both calibration and validation, the rainfall and runoff data were rescaled to an hourly mean of unity.

The *MaxH* model produces a slightly better goodness of fit than the linear model in both calibration and validation, but this might be expected given that it does have three parameters rather than just the two of the linear res-

[3] *Ex post* baseflow separation and/or the use of runoff factors as model parameters is a weak point of many rainfall-runoff models. Finding a way of separating baseflow and specifying percentage runoff a *priori* remains the Holy Grail of catchment modelling.

Table 2. – Model Results Goodness of Fit Criterion: $R^2 = 1 - \frac{\text{var(errors)}}{\text{var(data)}}$

Model	Data Set	Parameter values	R^2
MaxH	1992 t=0 - 1600 OPTIMISATION	a = 0.707 k = 0.0386 Vmax = 212.9	0.724
	1995 t=0 - 4000 VALIDATION		0.710
Linear	1992 t= 0 - 1600 OPTIMISATION	a = 0.653 k = 0.0581	0.681
	1995 t=0 - 4000 VALIDATION		0.699

ervoir model. The performance in validation of both models is good (in fact the Linear Reservoir model produces a better fit than it did in calibration).

Figures 9 and 10 show portions of the rainfall and runoff data from the 1992 data used for calibration. Figures 11 and 12 show similar results from the validation using 1995 data.

In both sets of data, the actual variation in the percentage runoff varies from storm to storm. This poses an obvious difficulty for both the *MaxH* and linear models, which attempt to model the whole hydrograph (comprising a series of rainfall-runoff episodes) using a single runoff coefficient. Given the size of the Haast catchment, the problem is exacerbated in that both models use just a single rainfall gauge that might not be representative of the areal rainfall over the whole catchment. Fitting the models to the whole hydrograph - with isolated storm events separated by periods of recession with no rainfall - means that optimising on the R^2 goodness-of-fit measure is a compromise between large errors on a few peak flows and small errors on a large number of recession flows. Neither the linear nor MaxH models properly predict the long recession limbs, with the actual flows receding more slowly than the model values. For the larger storm

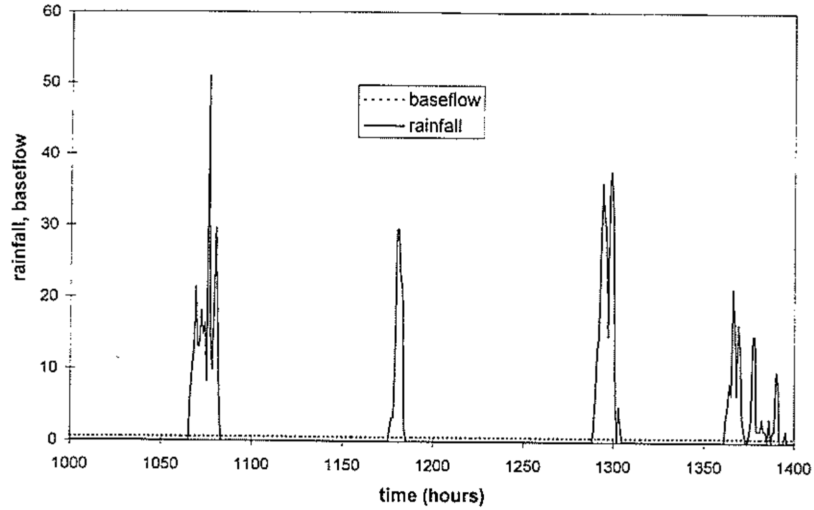

Figure 9. – Haast 1992: rainfall and "baseflow" (calibration).

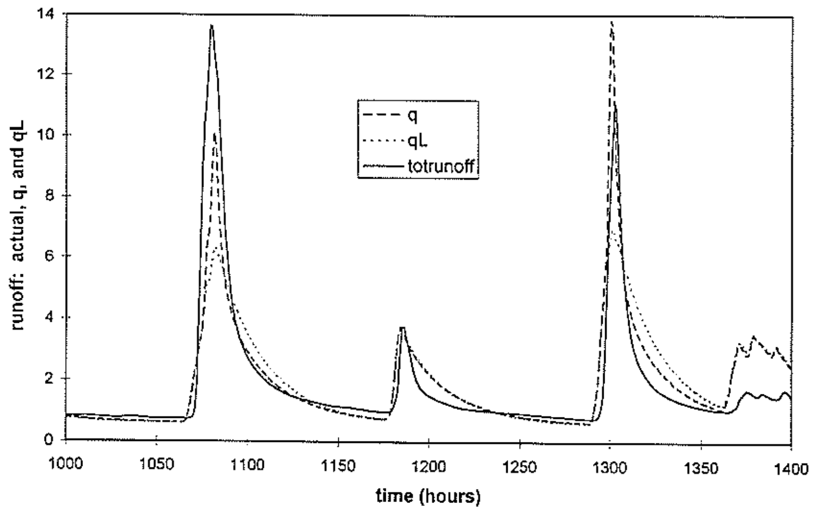

Figure 10. – Haast 1992: actual runoff and model predictions (calibration).

events, as the catchment storage in the *MaxH* model approaches V_{max} (see Figure 13), the nonlinear runoff effect starts to exert itself and the rate of rise of the hydrograph increases. No such effect occurs in the linear model. The effect is not as pronounced as that displayed during the simulation using synthetic data, but nevertheless, the general result is that the *MaxH* models such large storm events slightly better.

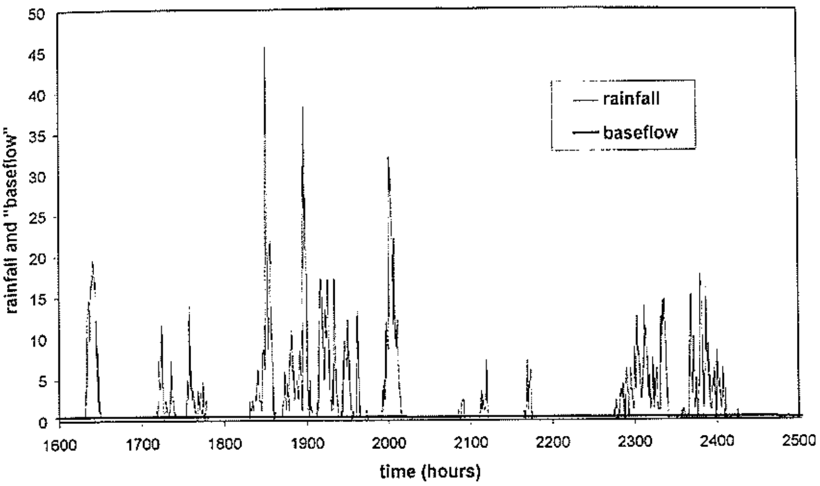

Figure 11. – Haast 1995: rainfall and "baseflow".

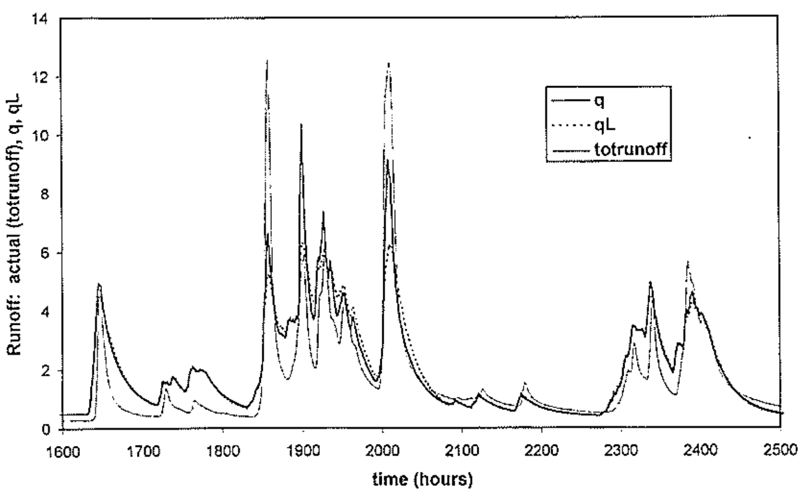

Figure 12. – Haast 1995: actual versus predicted runoff (validation).

Conclusion

The dynamic characteristics of the *MaxH* rainfall runoff have been described and compared to the response of the familiar linear reservoir model. The barriers to implementing the model noted in an earlier paper have been resolved. The *MaxH* model displays some interesting and potentially useful characteristics. Its nonlinear response is capable of simulating an in-

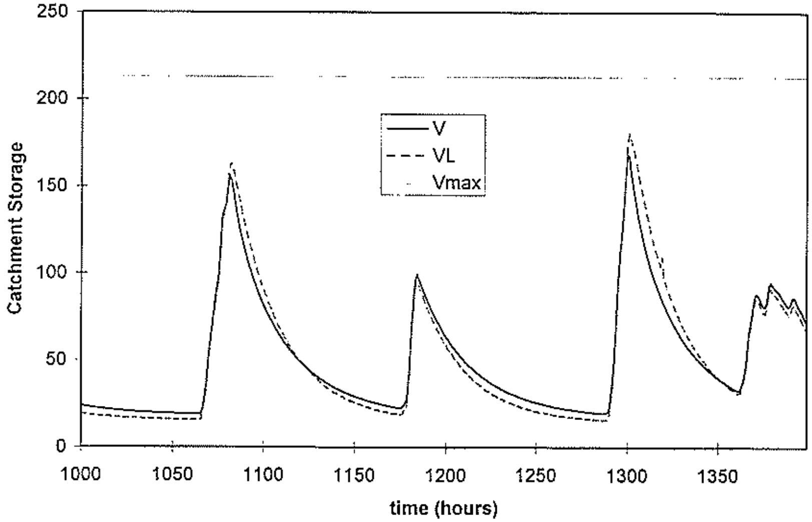

Figure 13. – Haast 1992: catchment storage (calibration).

creasingly rapid rate of rise of the hydrograph with catchment wetness. Both model types have been explored using rainfall-runoff from the Haast River. Despite the large size of the catchment and even though only one rainfall gauge is used as an input, the performance of both models is reasonable. Overall, the *MaxH* model produces a marginally better performance than its linear reservoir counterpart. This has to be balanced against its more complex structure, which makes it less easy to implement.

Acknowledgments

This work was undertaken within the Department of Civil Engineering at the University of Canterbury, Christchurch, New Zealand, whilst the author was an Erskine Fellow. The support of the University in providing this Fellowship and the collegiality of all the staff there are most gratefully acknowledged. The author would like to thank Dr Bruce Hunt for his timely help in providing a neat and accurate numerical approximation for the zeta function used in this paper just at the moment when the prospects looked bleak. Thanks also to Dr Bente Clausen for providing access to the student project report describing the Haast catchment. Finally, thanks to NIWA for permission to use the Haast catchment rainfall and flow data used in this study.

References

Appleby, F. V. 1974: A determinate model for runoff as a nonlinear system. Proceedings, IAHS Warsaw Symposium: Mathematical Models in Hydrology. *IAHS Publication No 101:* 667-687.

Carson, D.; O'Riley, F.; Brook, T.; Carter, R.; Chung, K.; and McWhinnie, A. 1997: *Catchment Water Balance Report ENCI 443: Hydrology,* Department of Civil Engineering, University of Canterbury, Christchurch.

Cherwell Scientific 1995: *ModelMaker* 2.0, Cherwell Scientific Publishing, Oxford

Dooge, J. C. I. 1973: *Linear Theory of Hydrologic Systems,* Technical Bulletin No 1468, Agricultural Research Service, US Department of Agriculture, Washington D.C.

Gradshtein, I. S.; Ryzhik, I. M. 1980: *Tables of Integrals, Series and Products* (4th edition), Sections 325, 330, 1073, 1075. Academic Press, London, UK.

Hunt, B. 1997: *Personal communication,* Department of Civil Engineering, University of Canterbury, NZ.

Ibbitt, R. P. 1976: Prediction of flow in the Manungaparerua River, paper presented to the *NZ Hydrological Society Annual Symposium,* Rotorua, 23-26 November 1976.

Jaynes, E. T. 1983: *Papers on Probability, Statistics and Statistical Physics* (R. D. Rosenkrantz), D Reidel, Dordrecht.

Jessop, A. 1995: *Informed Assessments: An Introduction to Information, Entropy and Statistics,* Ellis Horwood, UK.

Jowitt, P. W. 1979: The extreme value type 1 distribution and the principle of maximum entropy, *Journal of Hydrology 42 :* 23-38.

Jowitt, P. W. 1980: Discussion: The extreme value type 1 distribution and the principle of maximum entropy, *Journal of Hydrology 47 :* 385-390.

Jowitt, P. W. 1989: Entropy + Rain = Floods, *Maximum Entropy and Bayesian Methods (ed. J. Skilling),* 8th International Maximum Entropy Workshop, St John's College, Cambridge, 1988, Kluwer Academic Publishers, pp 437-445.

Jowitt, P. W. 1991: A maximum entropy view of probability-distributed catchment models, *Hydrological Sciences Journal 36* (2): 119-131

Jowitt, P. W. 1997: *Tabulated values of* $\zeta(2,z)$. Contact: p.w.jowitt@hw.ac.uk

Jowitt, P. W. 1998: *MaxH* model of Haast River: *ModelMaker* files. Contact: p.w.jowitt@hw.ac.uk

Kalinin, G. P.; Milyukov, P. I. 1957: O raschete neustananovishegosya dvisheniya vody v otkrytykh Ruslakh (On the computation of unsteady flow in open channels), *Meteorologiya i Gidrologiya Zhuzurnal 10 :* 10-18, Leningrad.

Lambert, A. O. 1972: Catchment models based on ISO-functions. *Journal of the Institution of Water Engineers 26* : 413-422.

Moore, R. J.; Clarke, R. T. 1981: A distribution function approach to rainfall runoff modelling. *Water Resources Research, 17* : (5): 1367-1382.

Moore, R. J. 1985: The probability distributed principle and runoff production at point and basin scales, *Hydrological Sciences Journal, 30* : (2), 273-297.

Shannon, C. E. 1948: The mathematical theory of communication. *Bell System Technical Journal, 27:* 379-428, 623-656.

Strang, G. 1986: *Introduction to Applied Mathematics,* Wellesley-Cambridge Press, USA.