

Uncertainty analysis of reservoir sedimentation using Latin Hypercube sampling and Harr's method: Shahar Chai Dam in Iran

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Abstract

One of the problems that affects hydro-installations and reduces the useful life of dams is sedimentation in the reservoir. In estimating reservoir sedimentation and accumulation, a number of uncertainties arise, including the quantity of streamflow, sediment load, sediment particle size and specific weight, trap efficiency, and reservoir operation. To evaluate suspended sediment and bed load for some hydrometric stations, in addition to using the total time period, the field data were separated into wet and dry periods. Latin Hypercube sampling and Harr's probabilistic point estimation method were used to quantify the uncertainty of accumulated reservoir sedimentation through time. Sensitivity analysis was conducted to examine the importance of various factors on the uncertainty of accumulated reservoir sedimentation. The effect of each uncertain factor, both individually and in combination, on the uncertainty of accumulated reservoir sedimentation through time were examined for Shahar Chai Dam in Iran. The results show that using Latin Hypercube sampling, the uncertainty of accumulated reservoir sediment volume is 37% for total time periods and 31% for wet and dry periods, and in Harr's method the uncertainties are 38% and 23% respectively.

Key words

Reservoir sedimentation, Harr's method, Latin Hypercube sampling, uncertainty analysis

Introduction

Decision-making in engineering practice often involves uncertainties: natural variations in the phenomenon being considered, an incomplete understanding of mechanisms, and inaccurate characterization of important parameters or variables. Thus, physical or numerical models developed and used to simulate natural phenomena are often, in reality, probabilistic, and hence subject to analysis by rules of probability theory. Identifying the components of uncertainty relating to physical phenomenon and quantifying them can therefore improve decision-making (Huang, 1986; Mercer, 1975).

Reservoir sedimentation varies with several factors, including sediment production, sediment transport rate, sediment type, mode of sediment deposition, reservoir operation, reservoir geometry, and streamflow variability. Sediment is transported as suspended sediment and bed load by streams and rivers entering a reservoir. Once deposited, the sediment may consolidate through

time, compacting under its own weight and the weight of overlying water. Accurate prediction of the amount of accumulated sediment behind a dam is one of the most important problems in hydraulic engineering. Empirical models, based on surveys and field observations, have been developed and applied to estimate the annual reservoir sedimentation load (RSL), the accumulated reservoir sedimentation load, and the accumulated reservoir sedimentation volume (ARSV) after a given number of years of reservoir operation (Strand and Pemberton, 1982; Morris and Fan, 1998). Likewise, several mathematical models for predicting reservoir sedimentation have been developed, based on the equations of motion and continuity for water and sediment – see, for instance, Chen *et al.* (1978), Soares *et al.* (1982), and Morris and Fan (1998). However, empirical methods are still widely used in actual engineering practice (Butler, 1987; Ruddy 1987; Shen and Julian, 1993). Estimates of reservoir sediment inflow, reservoir sedimentation, and reservoir sediment accumulation involve a number of uncertainties: 1) quantity of streamflow, 2) quantity of sediment inflow into a reservoir, 3) sediment particle size, 4) specific weight of the deposits and 5) reservoir size and operation (U.S. Bureau of Reclamation, 1987, p.529). Fan (1988) obtained information on 34 streams, 18 watersheds, and 12 reservoir-sedimentations models and stated that different models may give significantly different results, even when using the same set of input data. Such an additional factor is known as ‘model uncertainty’ and may be quite a large component of the overall uncertainty (Salas and Shin, 1999).

Several methods of uncertainty analysis have been developed and applied in water resource engineering. The most widely used methods are first-order variables estimation, Harr’s probabilistic point estimation method, and Monte Carlo simulation (Ang and Tang, 1984).

First-order variables estimation is based on linearizing the functional relationship that relates a dependent random variable and a set of independent random variables by Taylor series expansion (Yen *et al.*, 1986). This method has been applied in several water resources and environmental engineering problems involving uncertainty. Examples include storm sewer design (Tang and Yen, 1972), groundwater flow estimation (Dettinger and Wilson, 1981), prediction of dissolved oxygen (Burgess and Lettenmaier, 1975; Chadderton *et al.*, 1982), estimation of subsurface flow and contaminant transport (Sitar *et al.*, 1987) and the water surface profile of a buried stream flowing under coarse material (Hansen and Bari, 2002).

In Harr’s method, average and variance of probabilistic variables and their correlations are used (more details are introduced in Tung, 1993). If there are N variables, the number of cases (points) will be $2N$, an important advantage compared to the point estimate method proposed by Rosenblueth (1981). In cases where obtaining the derivatives are too complicated, Harr’s method is considered a good substitute for the first-order variables estimation method. This method has been used in studying the spatial variation of river-bed scouring (Yeh and Tung, 1993) and for uncertainty analysis incorporating marginal distributions (Chang and Yang, 1997).

In Monte Carlo simulation, stochastic inputs are generated from their probability distributions and are then entered into empirical or analytical models of the underlying physical processes involved in generating stochastic outputs. Then, the generated outputs are analyzed statistically to quantify the uncertainty of the output. Several examples of uncertainty analysis using a Monte Carlo simulation can be found in water resources and environmental engineering (Salas, 1993; Hipel and Mcleod, 1994; Melching, 1995). Some of them include groundwater flow estimation

(Smith and Freeze, 1979; Jones, 1989) and water quality modeling (Warwick and Cale, 1986; Brutsaert, 1975), and a study of spatiotemporal stochastic open-channel flow (Gates and Al-Zahrani, 1996). Scavia *et al.* (1981) compared Monte Carlo simulation and first-order variables estimation for determining uncertainties associated with eutrophication model outputs such as plankton, zooplankton and nitrogen forms. They concluded that both first-order variables estimation and Monte Carlo simulation agree well in estimating the mean and variance of model estimates. However, the Monte Carlo simulation provided better information about the frequency distribution.

Latin Hypercube sampling is used to generate random stochastic inputs in a stratified manner from the probability distributions. In this way the number of generated inputs can be reduced considerably, compared to Monte Carlo simulation (see McKay *et al.*, 1979). Chang *et al.* (1993) used Latin Hypercube sampling to analyse sensitivity and uncertainty in his research. Yeh and Tung (1993) applied first-order variables estimation, the point estimate method proposed by Rosenblueth (1981), and Latin Hypercube sampling to analyze the uncertainty of migration of a pit. They pointed out that the point estimate method yields a larger mean and variance than those obtained by first-order variables estimation and Latin Hypercube sampling methods. Furthermore, in studying the importance of stochastic inputs on the output by sensitivity analysis, Latin Hypercube sampling yields more information than the other two methods.

In this study, uncertainty analysis based on Latin Hypercube sampling and Harr's method is conducted to obtain the accumulated reservoir sedimentation volume. Then sensitivity analysis is carried out to show the relative importance of stochastic inputs in estimating the volume. In addition,

uncertainty analysis of the accumulated reservoir sedimentation volume over time is undertaken for single and combinations of factors using Latin Hypercube sampling.

Methods

Annual sediment loads and accumulation rates

The incoming sediment load and the streamflow discharge are usually measured at hydrometric gauging stations, and a sediment rating curve is constructed. Incoming sediment includes suspended sediment and bed load. Where the bed load cannot be measured, it can be estimated by formulas (Vanoni, 1975). The annual sediment rating curve is the relation between annual sediment load and annual streamflow discharge. Several methods can be used to derive the annual sediment rating curve from the daily sediment rating curve (Colby, 1956). To evaluate suspended sediment and bed load for a number of hydrometric stations, the field data are divided into wet and dry time periods. This data separation improves the results compared to the one in which the entire data set is used. The division into wet and dry time periods is based on the incoming daily streamflow discharges and their monthly average. Wet time periods are those with a daily streamflow discharge that is bigger than the monthly average, while dry time periods are those with a daily streamflow discharge that is smaller than average. Then the other calculations have been performed for wet and dry time periods, as well as the total time periods. Several different methods – USBR (1987), Average classes (Abul-Ghasmi and Morid, 1995), and Food and Agricultural Organization method (1981) – were used for estimating reservoir sediment inflow in the Urmia Basin. The Food and Agricultural Organization method (Food and Agricultural Organization, 1981) and its coefficients gives the best estimate compared

Table 1 – Estimates of reservoir useful volume for Baroon Dam using the USBR, Average classes and Food and Agricultural Organization (FAO) methods

FAO Method	Average Method	USBR Method	year	
useful volume (ton)	useful) volume (ton)	useful volume (ton)		
150	150	150	1995	
149.2	149.63	149.8	1996	
148.03	149.2	149.5	1997	
147.71	148.82	149.06	1998	
146.2	148.36	148.68	1999	
145.35	147.95	148.2	2000	
144.98	147.53	147.83	2001	
144.46	147.2	147.5	2002	
Real data 144.45	143.75	146.74	147.26	2003
142.07	146.39	146.64	2004	
141.42	145.05	146.2	2005	

to real data for annual average suspended and bed load. For example Tables 1 and 2 show the result using different methods for Baroon Dam in Urmia Basin and Ekbatan Dam in Hamedan. Shahar Chai Dam is 49 km far from Baroon and 400 km from Ekbatan Dam, which is in another basin in Iran.

The accumulated reservoir sedimentation volume $ARSV_v$, is estimated using the following steps:

- 1) Calculate daily rating curve of suspended sediment and bed load by:

$$\log_{10} QSD = a'_1 + b'_1 \log_{10} QWD \quad (1)$$

$$\log_{10} QBD = a'_2 + b'_2 \log_{10} QWD \quad (2)$$

where QSD is daily suspended load (tons/day); QBD is daily bed load (tons/day); QWD is daily average streamflow discharge (m^3/s); a'_1 , b'_1 and a'_2 , b'_2 are rating curve coefficients for suspended sediment and bed load respectively.

Table 2 – Estimates of reservoir useful volume for Ekbatan Dam using the USBR, Average classes and Food and Agricultural Organization (FAO) methods

FAO Method	Average Method	USBR Method	year	
useful volume (ton)	useful) volume (ton)	useful volume (ton)		
7.84	7.84	7.84	1964	
7.74	7.81	7.83	1965	
7.53	7.75	7.81	1966	
7.45	7.73	7.81	1967	
7.28	7.68	7.79	1968	
6.58	7.50	7.75	1969	
6.55	7.49	7.74	1970	
6.27	7.41	7.72	1971	
5.99	7.33	7.70	1972	
5.88	7.30	7.69	1973	
5.76	7.26	7.68	1974	
5.69	7.24	7.67	1975	
5.44	7.18	7.65	1976	
5.38	7.16	7.64	1977	
5.37	7.15	7.64	1978	
5.32	7.14	7.64	1979	
5.26	7.12	7.63	1980	
5.16	7.10	7.62	1981	
5.07	7.07	7.61	1982	
Real data 5.23	4.92	7.03	7.60	1983
4.89		7.02	7.60	1984
4.77		6.98	7.59	1985
4.66		6.95	7.58	1986
4.56		6.93	7.57	1987
4.44		6.89	7.56	1988
Real data 4.45	4.38	6.87	7.55	1989
4.34		6.86	7.55	1990
4.30		6.85	7.54	1991
4.13		6.81	7.53	1992
4.13		6.81	7.53	1993
Real data 3.92	4.06	6.78	7.52	1994

2) Calculate the corresponding annual rating curves of suspended sediment and bed load:

$$\log_{10} QS_t = a_1 + b_1 \log_{10} QW_t \quad (3)$$

$$\log_{10} QB_t = a_2 + b_2 \log_{10} QW_t \quad (4)$$

where QS_t is annual average suspended load (tons/day) in year t , QB_t is annual average bed load (tons/day) in year t ; QW_t is annual average streamflow discharge (m^3/s) in year t , and a_1 , b_1 and a_2 , b_2 are rating curve coefficients for annual average suspended and bed load, respectively.

3) As in the Food and Agricultural Organization method, coefficient (a) replaces (a"), where (a") is defined as:

$a'' = \frac{\bar{Q}_s}{(\bar{Q}_w)^b}$, where \bar{Q}_s is daily average suspended load (tones/day) for suspended load and daily average bed load (tones/day) for bed load. \bar{Q}_w is daily average streamflow discharge (m^3/s) and therefore, QS_t and QB_t are calculated with a new (a).

4) Calculate total sediment inflow in year t :

$$QT_t = QS_t + QB_t$$

5) Calculate the trap efficiency using Brune's (1953) data; several empirical methods have been developed to estimate trap efficiency (Churchill, 1948; Brune, 1953; Brown, 1958):

$$TE_t = a_3 + b_3 [\log_{10} (C_{t-1}/IW_t)]^2 \quad (5)$$

where TE_t is trap efficiency (%) in year t , C_{t-1} is useful reservoir capacity (m^3) at the beginning of year t , IW_t is $31.536 \times 10^6 QW_t$ stream flow (m^3) in year t and a_3 , b_3 are regression coefficients.

6) Calculate the total sediment load trapped in a reservoir in year t :

$$RSL_t = 3.65 QT_t \times TE_t \quad (6)$$

where RSL (annual reservoir sedimentation load) is in tonnes; and the accumulated sediment in reservoir after t years is calculated as:

$$\begin{aligned} ARSL_t &= ARSL_{t-1} + RSL_t \quad t=1, 2, \dots \\ \text{Where } ARSL_0 &= 0 \end{aligned} \quad (7)$$

7) Estimate the average specific weight of sediments deposited after t years, using the Miller (1953) formula:

$$W_t = W_1 + 0.4343K \left[\left(\frac{t}{t-1} \right) Lnt - 1 \right], t > 1 \quad (8)$$

where W_t is average sediment specific weight (kg/m^3) after t years; W_1 is specific weight of sediment in the first year; and K is a consolidation parameter.

Both W_1 and K are functions of the type of reservoir operation and the size of sediment (Lane and Koelzer, 1943); Table 3 shows values of W_1 and K for various reservoir operating conditions and sediment types such as clay, silt and sand. For a mixture of sediment, a weighted average of specific weights and a consolidation constant (9) and (10) must be used (Lara and Pemberton, 1965):

$$W_1 = 0.01 [W_1(c) P(c) + W_1(m) P(m) + W_1(s) P(s)] \quad (9)$$

$$K = 0.01 [K(c) P(c) + K(m) P(m) + K(s) P(s)] \quad (10)$$

where $W_1(c)$, $W_1(m)$ and $W_1(s)$ are initial specific weight; $K(c)$, $K(m)$ and $K(s)$ are consolidation constants; and $P(c)$, $P(m)$ and $P(s)$ are percentages of clay, silt, and sand, respectively.

8) Calculate ARSV_t:

$$ARSV_t = 1000 ARSL_t / W_t \quad (11)$$

9) Estimate C_t :

$$C_t = C_0 - ARSV_t \quad (12)$$

where C_0 is the initial useful reservoir capacity.

Table 3 – Values of W_1 in kg/m^3 (initial specific weight) and K (consolidation constant) for various reservoir operating conditions and sediment types such as clay, silt and sand. (Strand and Pemberton, 1982).

Type of reservoir operation	Clay		Silt		Sand	
	$W_1(c)$	$K(c)$	$W_1(m)$	$K(m)$	$W_1(s)$	$K(s)$
Sediment always submerged or nearly submerged	416	256	1120	91	1500	0
Moderate to considerable reservoir drawdown	561	135	1140	29	1550	0
Reservoir normally empty	641	0	1150	0	1550	0
Riverbed sediments	941	0	1170	0	1550	0

Stochastic inputs

In the empirical models, various uncertain factors affect reservoir sedimentation:

- 1) Inputs associated with annual sediment inflows such as regression coefficient a_1 , b_1 and a_2 , b_2 of steps (3) and (4) respectively;
- 2) inputs associated with the type of incoming sediment, such as the percentage of clay, silt and sand;
- 3) inputs associated with the regression equation for estimating the trap efficiency of the reservoir; and
- 4) inputs associated with the variability of the water inflows to the reservoir.

In Latin Hypercube sampling, well-known results from regression analysis also indicate that a'_1 and b'_1 are bivariate normally distributed (Mood *et al.*, 1974, p.489). For predicting the fraction of sediment type (clay, silt and sand), assume that such fractions are uniformly distributed, with lower and upper bounds that are obtained from the measurements and soil texture diagrams. These fractions must add up to 100%. a_3 and b_3 may be assumed to be bivariate normally distributed too. But Harr's method does not take into account the probability distribution

of variables, which may be considered a disadvantage (Soleimani, 2003).

The uncertainty of annual stream flow is an important factor affecting the uncertainty of reservoir sedimentation. Stochastic time series models have been widely used for many water resources problems (Loucks *et al.*, 1981; Salas 1993). Auto-regressive models have been the most commonly used models for annual streamflow simulation (see, for instance, Mcleod and Hipel, 1978, and Salas *et al.*, 1980). Auto regressive (1) model is defined as:

$$QW_t = \mu + \phi_1(QW_{t-1} - \mu) + \varepsilon_t \quad (13)$$

where μ is the mean; ϕ_1 is the lag-1 autoregressive coefficient; and ε_t is a normal random variable with mean zero and variance σ_t^2 . We then have 10 sources of uncertainty for studying reservoir sedimentation: $\{a_1, b_1, a_2, b_2, a_3, b_3, P(c), P(m), P(s) \text{ and } Q_w\}$.

Harr's method is a simple, effective, and precise method. It uses the two first-order moments of stochastic variables and not the probability distribution, but it is easy to calculate and is considered a good substitute for other methods. Different stages of this method can be summarized:

- Identify input physical parameters of each of the relationships and calculate its correlation matrix.
- Decompose the correlation matrix to an eigen vector matrix and diagonal eigen values matrix (with MATLAB software)

$$CO = VLV^t \quad (14)$$

where $V = (v_1, v_2, \dots, v_n)$ is the eigen vector matrix and $L = \lambda_1, \lambda_2, \dots, \lambda_n$ is the eigen value diagonal matrix.

- Calculate 2N intersection points where these pairs of points are calculated from the following equation (Hosseini, 2000; Soleimani, 2003):

$$X_{i\pm} = \mu \pm \sqrt{N} \begin{bmatrix} \sigma_1 \dots\dots\dots \\ \sigma_2 \dots\dots\dots 0 \\ \dots - \dots \\ 0 \dots - \dots \\ \dots\dots\dots \sigma_N \end{bmatrix} V \quad (15)$$

where μ is the mean; σ_i is the standard deviation of the i th stochastic input; N is the number of inputs, and V is the eigen vectors matrix.

- Calculate $Y_{i\pm} = g(X_{i\pm})$ and $Y_{i\pm}^2 = g^2(X_{i\pm})$ for ($i=1, 2, \dots, N$) where Y_i is the model output, and then calculate $\bar{Y}_i = \frac{Y_{i+} + Y_{i-}}{2}$ and $Y_i^2 = \frac{Y_{i+}^2 + Y_{i-}^2}{2}$.
- Calculate the average and variance of different model outputs:

$$E(Y) = \frac{\sum_{i=1}^N \bar{Y}_i \lambda_i}{\sum_{i=1}^N \lambda_i} = \frac{\sum_{i=1}^N \bar{Y}_i \lambda_i}{N} \quad (16)$$

$$E(Y^2) = \frac{\sum_{i=1}^N \bar{Y}_i^2 \lambda_i}{N} \quad (17)$$

$$\text{Var}(Y) = E(Y^2) - E^2(Y) \quad (18)$$

- Compute the model uncertainty with coefficient of variation – for an elaborate discussion on Harr's method the reader is referred to Hosseini (2000).

In Harr's method, the set of stochastic inputs ($a_1, b_1, a_2, b_2, a_3, b_3, P(c), P(m), P(s)$) are generated 2N times, but annual flows Qw_t are separated from the correlation matrix, then generated 2N*t times using SIMLAB Software.

Latin Hypercube sampling is an effective method for dealing with nonlinear relationships. Its main disadvantage is the need for a probability distribution for variables. The probability distribution of variables can be estimated as follows: 1) obtain a random set of size n of the stochastic inputs from the corresponding probability distribution; 2) use the procedure outlined in 'Annual sediment loads and accumulation rates' section; 3) analyze the output statistically to determine its probability distribution and basic statistics such as mean, standard deviation, coefficient of variation, and coefficient of skewness.

In using Monte Carlo simulation to generate the stochastic input referred to in step 1 above, normally a large data set – for instance, $n=1000$ – is generated from the probability of each input. An alternative to Monte Carlo simulation sampling, which reduces the number of sets of generated inputs and consequently the number of generated outputs, is the Latin Hypercube sampling method. The basic concept of Latin Hypercube sampling lies in generating random numbers of a stochastic input over its range in a stratified manner, so that the overall variability of the given stochastic input can reasonably be delineated for a limited sample size. The properties of Latin Hypercube sampling are discussed by McKay (1988) and McKay *et al.* (1979). In this method all stochastic inputs are assumed to be independent. If a large number of stochastic inputs are involved in determining

the output, sensitivity analysis may be carried out to determine the degree of influence of each stochastic input on the output uncertainty, C_i .

In Harr's method and Latin Hypercube sampling, a linear regression relationship between x 's, input parameters, and output, Y can be considered, as the following:

$$Y = a_0 + \sum_{i=1}^N a_i x_i + e \quad (19)$$

where a_0 is the intercept value of the line with y axis, a_i refers to regression coefficients that show the sensitivity coefficients, and e indicates the model error. Due to the dimensional problems, it is recommended to centralize the output parameter, and then, by standardizing ($Y-Y$) and input parameters, the regression can be conducted. In this case, the coefficients will indicate the output variation for a variation of input parameter equal to one standard deviation. Then C_i values, which indicate the uncertainty of the input parameter, can be calculated from the following relationship:

$$C_i = \frac{SSR_i}{SSR} R^2 \quad \text{For } i = 1, 2, \dots, N \quad (20)$$

in which SSR_i is the summation of square values of the i th input stochastic parameter from the regressed line and SSR is the summation of SSR_i for the independent input parameters. For more details the reader is referred to McKay (1988).

By sensitivity analysis, the stochastic inputs that are more important to output uncertainty are selected for detailed analysis. Sensitivity analysis can be carried out based on Harr's method, and a more complete analysis based on Latin Hypercube sampling can be undertaken.

Procedural steps

In this section the uncertainty of the estimate of accumulated sediment in the reservoir

throughout a number of years of operation is considered by Latin Hypercube sampling and Harr's methods. The steps of the procedure are summarized here (Latin Hypercube sampling and Harr's methods are discussed separately where needed):

- 1) Generate annual flows $Q_{w_t}(t=1, \dots, M)$ from the eq.(13) model, where M =simulation run years; this is repeated n times, where n varies according to the stochastic sampling method.
- 2) Generate the other set of stochastic inputs ($a_1, b_1, a_2, b_2, a_3, b_3, P(c), P(m), P(s)$) n times, based on Latin Hypercube sampling described previously, and generate $2N$ intersection points based on Harr's method described in the previous section.
- 3) Using the obtained stochastic inputs, determine the stochastic output, namely, the $ARSV_t$ and useful capacity at the end of year t , C_t , from (6), (7), (11) and (12), respectively.
- 4) Obtain an array of n output for each t .
- 5) Determine the statistical characteristics of the array $ARSV_t$, such as the mean, variance, coefficient of variation, and coefficient of skewness.

Example application – Shahar Chai reservoir

Data

The Shahar Chai reservoir is located in the Urmia Lake basin and was constructed in 2005; the reservoir capacity is $221 \times 10^6 \text{ m}^3$. Basic information on streamflow and sediment data were obtained from West Azarbayjan Water Bureau in Iran (West Azarbayjan Water Bureau, 2002). The incoming suspended sediment load and the streamflow discharge are usually measured at hydrometric gauging stations and the bed load is calculated as 10–30% of suspended load. The nearest gauging station to Shahar Chai Reservoir is

the Band gauging station in the Shahr Chai River; streamflow data are available for 1949–2000 and suspended sediment load for 1964–2001. Table 4 shows basic statistics for the streamflow data at the Band station; the basic statistics for the suspended sediment load are listed in Table 5.

Table 4 – Basic statistics for streamflow data at the Band Station

Sample of streamflow datapoints	18250
Frequency of measurement	Every day
Standard deviation	2.095
Mean flow (m ³ /s)	5.325
Length of record	1949-2000
Station	Band

Table 5 – Basic statistics for suspended sediment load at the Band Station

Frequency of measuring	Two or three per month
Maximum and Minimum	30013.7 and 0.19
Mean (tons/day)	2906.21
Length of record	1964-2000
Station	Band

Results

For the Latin Hypercube sampling method, the corresponding annual rating curve equations are listed in Table 6. Mean values and standard deviations of the annual rating curve parameters (a₁, b₁) and (a₂, b₂) in dry and wet periods and over the total period are listed in Table 7. In this evaluation wet and dry time periods have the same rating, which is different from that of the overall period. In addition, the annual rating curves for both suspended sediment and bed load in wet and dry time periods and for the total

Table 6 – Annual rating curve for wet and dry time periods and total time periods and trap efficiency curve

Types	Regression equations
Annual suspended sediment (wet and dry periods)	QSt=23.381QWt2.2019
Annual bed load (wet and dry periods)	QBt=11.785QWt1.7082
Annual suspended sediment (total periods)	QSt=41.863QWt2.0275
Annual bed load (total periods)	QBt=15.978QWt1.6277
Trap efficiency	TEt=99.508-13.547{log10(Ct-1/IWt)}2

undifferentiated periods are derived in logarithmic coordinates, as shown in Figure 1. The correlation coefficient between regression coefficients are -0.9393 and -0.974 for suspended sediment and bed load in total time periods and -0.9177 and -0.9597 in wet and dry time periods. These coefficients are relevant for determining the relationship between two data sets and finding the best fit. The lower and upper bounds for each fraction, which are denoted as P(c), P(m) and P(s), listed in Table 7, were taken from field measurements and a soil texture diagram. Using Brune’s (1953) data, the trap efficiency curve was constructed with two regression coefficients (a₃, b₃) that are assumed to be bivariate normally distributed. The estimated mean and standard deviation of each regression coefficient are also listed in Table 7 (Salas and Shin, 1999). The correlation coefficient between a₃ and b₃ is -0.694. In summary, the 9 stochastic inputs have been characterized. The results show that 100 generating sets for Latin Hypercube

sampling gave stable statistics (Salas and Shin, 1999). Inputs were then generated using Latin Hypercube sampling with 100 sets.

For Harr’s method, the correlation matrix is shown in Table 8. For calculating accumulated reservoir sedimentation volume, the algorithm described in the previous sections was used. Moreover, the parameter uncertainty of the annual flows has been considered.

For uncertainty, the uncertain factors affecting the accumulated reservoir sedimentation volume are: 1) the annual

Table 7: Mean values and standard deviations of the annual rating curve parameters (a_1 , b_1) and (a_2 , b_2) in dry and wet periods and over the total period.

Inputs	Lower bound	Upper bound	Mean	Standard deviation	Distribution
a_1 (wet and dry periods)	–	–	29.831	12.460	Bivariate
b_1 (wet and dry periods)	–	–	2.104	0.316	Normal
a_1 (total periods)	–	–	42.015	20.613	Bivariate
b_1 (total periods)	–	–	2.106	0.352	Normal
a_2 (wet and dry periods)	–	–	12.942	3.406	Bivariate
b_2 (wet and dry periods)	–	–	1.669	0.179	Normal
a_2 (total periods)	–	–	15.530	4.147	Bivariate
b_2 (total periods)	–	–	1.668	0.178	Normal
a_3	–	–	99.508	1.5414	Bivariate
b_3	–	–	-13.547	0.5168	Normal
P(c)	12	28	26	–	Uniform
P(m)	50	72	54	–	Uniform
P(s)	20	50	20	–	Uniform
QW	–	–	5.325	2.095	Log normal

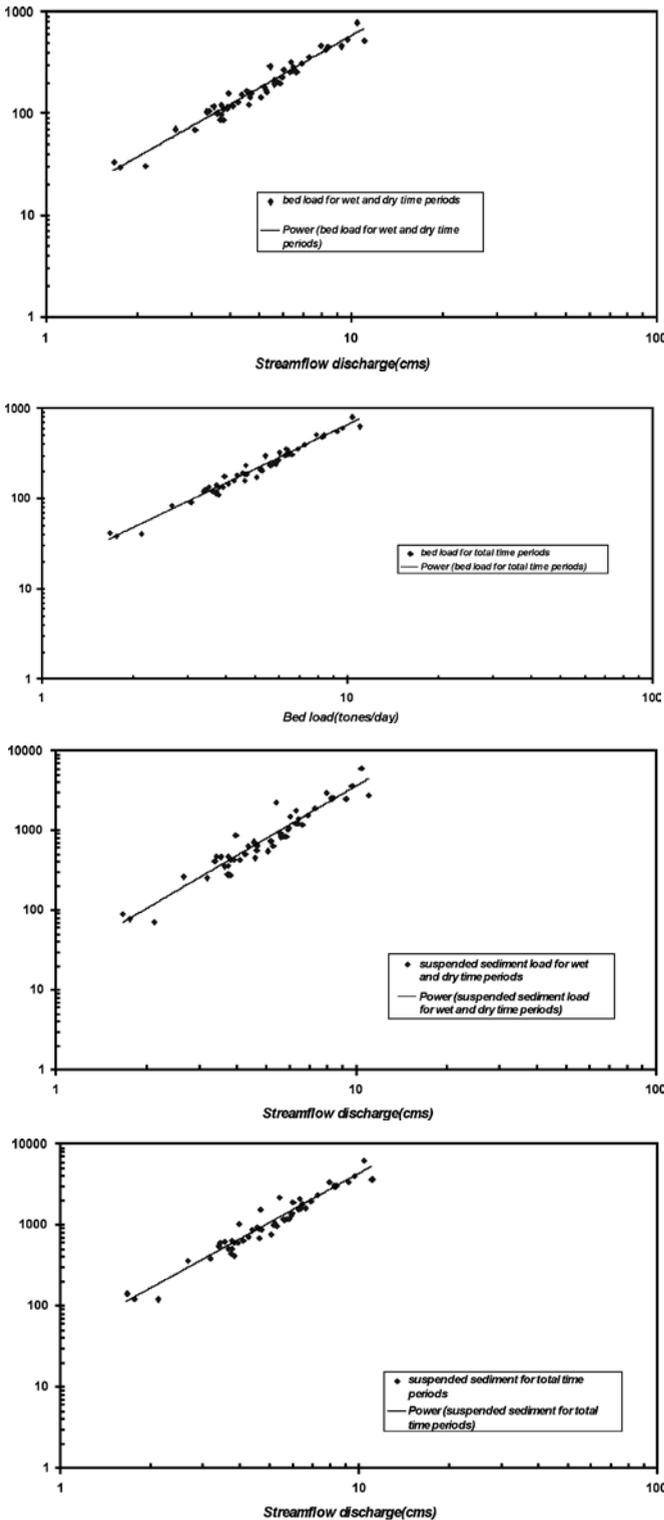
Table 8 – Correlation matrix (Harr's method)

a) for wet and dry time periods

	a1	b1	a2	b2	a3	b3	p(c)	p(m)	p(s)
a1	1.000	-0.968	0.963	-0.974	0.456	0.385	-0.405	0.405	-0.405
b1	-0.968	1.000	-0.922	0.970	-0.481	-0.385	0.471	-0.471	0.471
a2	0.963	-0.922	1.000	-0.983	0.358	0.314	-0.363	0.363	-0.363
b2	-0.974	0.970	-0.983	1.000	-0.358	-0.287	0.401	-0.401	0.401
a3	0.456	-0.481	0.358	-0.358	1.000	0.984	-0.513	0.513	-0.513
b3	0.385	-0.385	0.314	-0.287	0.984	1.000	-0.453	0.453	-0.453
p(c)	-0.405	0.471	-0.363	0.401	-0.513	-0.453	1.000	-1.000	1.000
p(m)	0.405	-0.471	0.363	-0.401	0.513	0.453	-1.000	1.000	-1.000
p(s)	-0.405	0.471	-0.363	0.401	-0.513	-0.453	1.000	-1.000	1.000

b) for total time periods

	a1	b1	a2	b2	a3	b3	p(c)	p(m)	p(s)
a1	1.000	-0.966	0.988	-0.965	0.026	-0.067	-0.643	0.643	-0.643
b1	-0.966	1.000	-0.972	0.994	0.101	0.177	0.596	-0.596	0.596
a2	0.988	-0.972	1.000	-0.983	0.026	-0.055	-0.570	0.570	-0.570
b2	-0.965	0.994	-0.983	1.000	0.060	0.129	0.551	-0.551	0.551
a3	0.026	0.101	0.026	0.060	1.000	0.984	0.338	-0.338	0.338
b3	-0.067	0.177	-0.055	0.129	0.984	1.000	0.451	-0.451	0.451
p(c)	-0.643	0.596	-0.570	0.551	0.338	0.451	1.000	-1.000	1.000
p(m)	0.643	-0.596	0.570	-0.551	-0.338	-0.451	-1.000	1.000	-1.000
p(s)	-0.643	0.596	-0.570	0.551	0.338	0.451	1.000	-1.000	1.000



stream flow; 2) the annual sediment inflow, with uncertainty in the suspended sediment and bed load rating curve parameters (a_1, b_1) and (a_2, b_2) respectively; 3) the trap-efficiency curve, with uncertainty in regression coefficient (a_3, b_3); and 4) the fractions of clay, silt and sand: $P(c), P(m)$ and $P(s)$ respectively. First, the uncertainty of the accumulated sedimentation volume, considering the single effect of each uncertain factor listed above, is determined using the Latin Hypercube sampling method (see Table 9). Then, the uncertainty of the accumulated sedimentation volume is again determined using the Latin Hypercube sampling method considering the combined effect of various uncertain factors: $A=(1)$ is annual stream flow; $B=(1)+(2)$ is annual streamflow + annual sediment inflow; $C=(1)+(2)+(3)$ is annual stream flow + annual sediment inflow + trap efficiency curve; $D=(1)+(2)+(3)+(4)$ is the overall uncertainty (see Table 10). Finally, the uncertainty of the accumulated sedimentation volume is determined using Harr's method, with and without wet and dry periods considered separately (see Table 11).

Figure 1 – Annual sediment rating curve (for suspended sediment and bed load in wet and dry time periods and total time periods).

Table 9 – Statistical characteristics of accumulated reservoir sedimentation volume (ARSV) for 15, 30 and 45 years of reservoir operation, considering the effect of individual factors (Latin Hypercube sampling method) for wet and dry time periods and total time periods.

Time in years	Uncertain factors	Mean ($\times 10^6$)	Standard deviation ($\times 10^6$)	Coefficient of variation	Coefficient of skewness
15	1	13.65	4.4	0.325	0.950
	2 (total)	7.40	1.01	0.137	0.044
	2' (wet and dry)	5.47	0.5	0.104	-0.819
	3	15.97	0.3	0.019	-0.767
	4	14.77	0.5	0.004	0.383
30	1	25.79	6.15	0.239	0.855
	2 (total)	17.29	2.42	0.140	0.041
	2' (wet and dry)	12.78	1.39	0.109	-0.870
	3	26.07	0.49	0.019	-0.767
	4	24.21	0.87	0.004	0.383
45	1	39.51	9.79	0.248	1.143
	2 (total)	26.937	3.84	0.143	0.056
	2' (wet and dry)	19.89	2.29	0.115	-0.915
	3	52.09	0.99	0.019	-0.767
	4	48.44	0.18	0.004	0.383

Considering the individual effect of the following uncertain factors: 1) annual stream flow; 2) annual sediment inflow, with uncertainty in suspended sediment and bed load rating curve parameters (a_1, b_1) and (a_2, b_2) respectively; 3) trap-efficiency curve, with uncertainty in regression coefficient (a_3, b_3); and 4) percentage of sediment particles $P(c)$, $P(m)$ and $P(s)$.

For calculating accumulated reservoir sedimentation volume, $t=1, \dots, 45$, that is for each t , 100 values were used for the Latin Hypercube sampling method and 9 values were used for Harr's method, (except annual streamflow, which generated 45×100 values).

Conclusions on Shahar Chai analysis

The results of uncertainty analysis of the accumulated reservoir sedimentation volume (ARSV) are shown in Table 9 for the total period and for wet and dry time periods, considering the effect of each of the factors. Table 10 shows the combined effect of several factors in the Latin Hypercube sampling method. Table 11 shows the result of uncertainty analysis of the accumulated reservoir sedimentation volume for $t=1, \dots, 45$, from Harr's method. In addition, the

result of the sensitivity analysis for each inputs based on Harr's method are shown in Figure 2.

Table 9 shows that trap efficiency is the most significant factor affecting the mean of the accumulated reservoir sedimentation volume, and annual streamflow is the most significant factor affecting the standard deviation. The largest coefficient of variation (which is considered as the indicator of uncertainty) is associated with annual streamflow; it is 32% for $t=15$ years and decreases to 25% for $t=45$ years. The factor with smallest effect on the accumulated reservoir sedimentation volume is the fraction of sediments – its coefficient of variation is smaller than 0.5%. In annual sediment inflow, the coefficient of variation increases, and the coefficient of skewness shows no increase or decrease with the time, but the coefficient

Table 10 – Statistical characteristics of the accumulated reservoir sedimentation volume (ARSV) considering the combined effect of several factors (Latin Hypercube sampling method).

a) for wet and dry time periods

Time in years	Uncertain factors	Mean ($\times 10^6$)	Standard deviation ($\times 10^6$)	Coefficient of variation	Coefficient of skewness
15	A	13.65	4.44	0.325	0.950
	B	8.83	2.63	0.298	0.209
	C	9.31	2.73	0.294	0.414
	D	8.58	2.63	0.307	0.321
30	A	25.79	6.15	0.239	0.855
	B	17.11	4.52	0.264	0.384
	C	17.62	3.79	0.215	-0.162
	D	16.67	3.28	0.197	-0.229
45	A	39.51	9.79	0.248	1.143
	B	26.35	7.10	0.270	0.687
	C	26.94	6.08	0.226	0.250
	D	25.73	5.19	0.202	0.148

b) for total time period

Time in years	Uncertain factors	Mean ($\times 10^6$)	Standard deviation ($\times 10^6$)	Coefficient of variation	Coefficient of skewness
15	A	13.65	4.44	0.325	0.950
	B	6.92	2.28	0.330	0.687
	C	6.95	2.32	0.333	0.658
	D	6.80	2.53	0.371	0.285
30	A	25.79	6.15	0.239	0.855
	B	13.09	3.52	0.269	0.899
	C	13.17	3.59	0.273	0.897
	D	12.63	3.27	0.259	0.776
45	A	39.51	9.79	0.248	1.143
	B	20.01	5.55	0.278	1.131
	C	20.11	5.61	0.279	1.157
	D	19.26	4.88	0.253	1.201

Considering the combined effect of various uncertain factors: A=(1) = annual stream flow; B=(1)+(2) = annual streamflow + annual sediment inflow; C=(1)+(2)+(3) = annual stream flow + annual sediment inflow + trap efficiency curve; D=(1)+(2)+(3)+(4) = overall uncertainty.

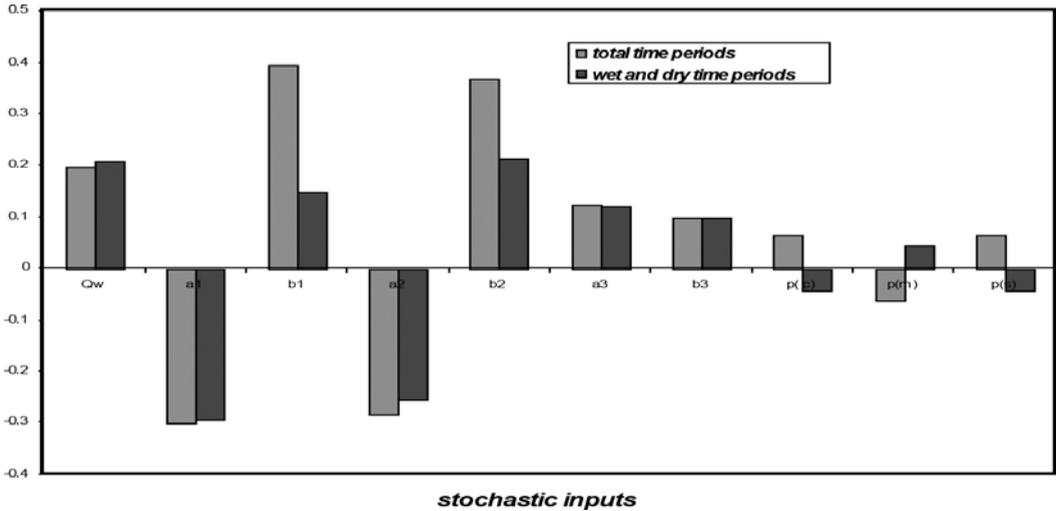


Figure 2 – Comparison of sensitivities for all inputs (Harr’s method). Considering the result of the sensitivity analysis for each inputs: QW= annual stream flow; a1,b1= rating curve coefficients for annual average suspended load; a2,b2= rating curve coefficients for annual average bed load; a3,b3= regression coefficients for trap-efficiency curve and P(c), P(m) and P(s) = percentage of clay, silt, and sand, respectively.

of skewness is negative for trap efficiency and for annual sediment inflow during wet and dry time periods, and in other cases is positive. This suggests that the probability distribution function of the accumulated reservoir sedimentation volume may well fit for all t’s, small and large t. In summary, the coefficient of skewness for annual sediment inflow for total time periods is less than for wet and dry time periods for each t, and

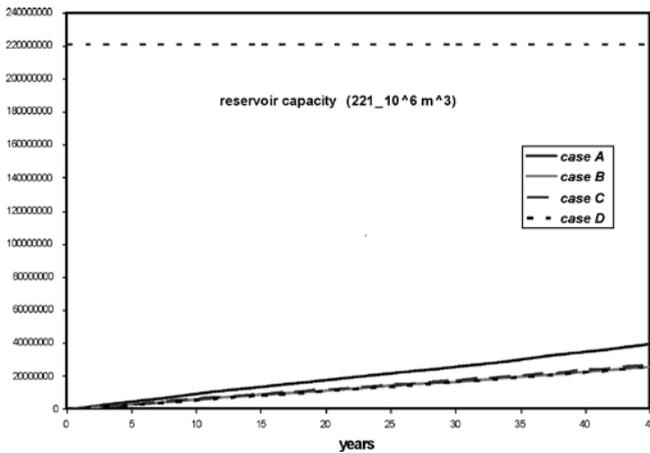
the largest skewness is related to annual streamflow for t=15 years. In addition the coefficient of variation in annual streamflow for total time periods is larger than for wet and dry time periods for each t.

Table 10 compares the results for the various combinations of uncertain factors, A-D,, where A is annual stream flow; B is annual streamflow + annual sediment inflow; C is annual stream flow + annual sediment

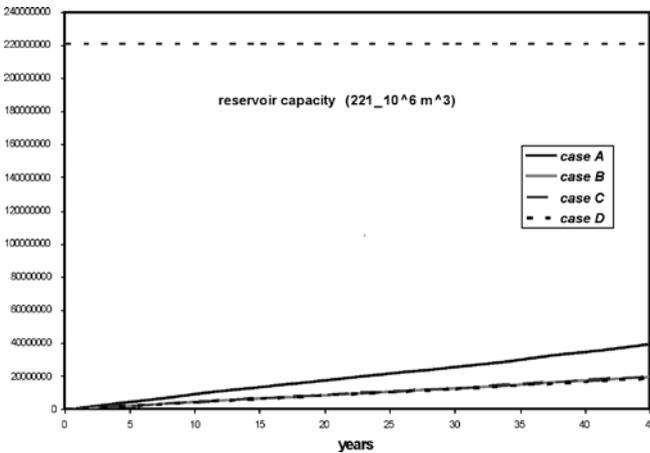
Table 11 – Statistical characteristics of the accumulated reservoir sedimentation volume ARSV for 15, 30 and 45 years of reservoir operation (Harr’s method) for wet and dry time periods and total time periods.

Time in years	Periods	Mean ($\times 10^6$)	Standard deviation ($\times 10^6$)	Coefficient of variation
15	wet and dry	6.98	1.58	0.226
	total	6.26	2.41	0.384
30	wet and dry	11.60	2.42	0.209
	total	13.92	3.70	0.266
45	wet and dry	22.58	5.51	0.244
	total	24.15	6.06	0.251

a) for wet and dry time periods



b) for total time periods



inflow + trap efficiency curve, and D is the overall uncertainty. Case (A) – annual streamflow – has a larger mean and standard deviation than other cases. The coefficient of variation of $ARSV_t$ for Case (D) – overall uncertainty – is 37% for $t=15$ in total time periods and 31% in wet and dry time periods, and decreases to 25% and 20% for $t=45$, respectively. The coefficient of skewness of $ARSV_t$ increases with time when considering the total time periods but decreases when wet and dry periods are differentiated. Also, total time periods show larger coefficients of variation (uncertainty) than do wet and dry

Figure 3 – Mean ARSV obtained from uncertainty analysis as referred to in table 8 (Results include mean ARSV obtained from historical data).

time periods. In this method, annual streamflow and annual sediment inflow are the most important factors that affect the uncertainty of the accumulated reservoir sedimentation volume, especially annual streamflow. The other factors are less important. Based on Figure 3 it is concluded that in case (A) the probability that the reservoir would be 80% full after 45 years is practically zero, but in case (D) it is much less probable than case (A). This result illustrates that a complete uncertainty analysis can provide a much more realistic evaluation and better optimization of reservoir design life.

Harr's method (Table 11) shows that the coefficient of variation is 38% for $t=15$ years in total time periods and 23% in wet and dry time periods and its magnitude decreases to 25% for $t=45$ years for all periods. In addition, sensitivity analysis shows that suspended sediment and bed load, followed by annual streamflow, are the most important factors influencing the accumulated reservoir sedimentation volume, in both total and wet and dry time periods, and trap efficiency and percentage of sediments are the next most important. In this study percentage of silt is the least significant factor.

Discussion and conclusions

Two types of conclusions can be drawn. First are those related to the case study:

- 1) Annual streamflow and annual sediment inflow in Shahar Chai Dam are the most significant factors influencing accumulated reservoir sedimentation; trap efficiency and percentage of sediments are less important factors.
- 2) In Latin Hypercube sampling method, the uncertainty of accumulated reservoir sediment volume is 37% for total time periods and 31% for wet and dry time periods, and in Harr's method the uncertainties are 38% and 23% respectively.
- 3) Sensitivity analysis shows that suspended sediment and bed load, followed by annual streamflow, are the most important factors influencing the accumulated reservoir sedimentation volume, for both the total period and the wet and dry time periods, and trap efficiency and percentage of sediments are the next most important. In this study the percentage of silt is the least significant factor.

The general conclusions are:

- 1) In estimating reservoir sediment inflow, the Food and Agricultural Organization method (Food and Agricultural Organization, 1981) and its coefficients show a reliable accumulated sediment volume.
- 2) The coefficient of variation is less when wet and dry periods are considered compared to using the total time.
- 3) Harr's method is a very simple method, but in estimating uncertainty it does not take into account the probability distribution of variables, which may be considered a disadvantage.
- 4) Harr's method is used in water resources problems, but the Latin Hypercube

sampling method is an easy and precise method that calculates the effect of each uncertain factor, individually and in combination, which is not the case in Harr's method. Therefore the method of Latin Hypercube sampling is recommended for design applications in all cases.

- 5) The probability distribution function of the accumulated reservoir sedimentation volume for all t , may be well fitted by a normal distribution.

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