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Recession of streamflow supplied from channel bed and bank storage

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Abstract

A streamflow recession formula for natural basins is derived by nonlinear hydrologic routing of inflow from channel bed and bank storage through a stream channel storage. The formula gives basin outflow as a two-parameter, inverse square function of time and is valid when the ratio of inflow to outflow is approximately constant. It provides a good fit to a master baseflow recession curve from each of 10 basins of differing areas in the greywacke geologic terrain of Canterbury, New Zealand. These basins have no significant lakes or springs and the formula applies to that section of a master recession curve between median flow and minimum recorded 7-day low flow, for the period January to March in a flow record.

The master recession curve in an ungauged basin within the same terrain may be predicted if the volume of water to be discharged during this recession can be estimated. This volume is taken to be the product of total stream length, average cross-sectional area of channel bed and bank storage and storage porosity. It can be calculated if there is a reference basin available, similar in character to the ungauged basin, with a measured master recession curve.

While further testing of the formula is needed, particularly in other geologic terrains, its application is relatively simple and it should be useful at basin scale for predicting master recession curves in basins where little is known about water storage behaviour.

Keywords

Streamflow recession: recession curve: bank storage: nonlinear model: low flow: baseflow

Introduction

Streamflow in many natural basins is sustained after a precipitation event by drainage of riparian subsurface water storages. The resulting basin outflow hydrograph or the distribution in time at a cross-section of contributions from upstream channel bed and bank storage is typically a single-valued function amenable to analysis. The form of this recession curve depends primarily on basin geology and geomorphology and the density of the drainage network (Wittenburg, 1999; Smakhtin, 2001).

The description and prediction of recession curves is a central problem in hydrology (Brutsaert, 2005; Botter *et al.,* 2009). Here, we are concerned with that section of a recession curve that lies between median flow and minimum recorded 7-day low flow. Moreover, to reduce complexity, this study is restricted to natural basins within a specific geologic terrain.

Quantitative analysis of streamflow recession began perhaps with the work of Dausse (1842) and Boussinesq (1877) and has centred on the problem of finding an analytic expression or mathematical model for the recession curve. Comprehensive reviews by Hall (1968), Tallaksen (1995) and Smakhtin (2001) reveal that most studies have dealt theoretically or empirically with baseflow recession where streamflow is generated largely from groundwater storage depletion.

Solution of these problems is of considerable importance to many aspects of water resource management including, for example, water allocation, irrigation, instream flow requirements for aquatic ecosystems, hydroelectricity generation, wastewater disposal and dilution of contaminant discharges (Fenemor, 1992; Tallaksen, 1995; Smakhtin, 2001; Pearson and Henderson, 2004). Recession curves also feature as components of other hydrological models and studies concerned with reservoir storage, frequency of low flows, regional low flows and rainfall-runoff forecasting, amongst others (Kachroo, 1992; Stedinger *et al.*, 1993).

The purposes of this study are: (1) to derive a basin-scale formula for streamflow recession resulting from stream drainage of channel bed and bank water storage, and (2) to develop an approach for predicting recession curves in ungauged basins. The analysis uses a non-linear model derived from the conservation of mass or continuity equation to determine the outflow hydrograph where inflow from channel bed and bank storage is prescribed. The aim is to gain further understanding of a complex problem in which highly variable field conditions are limited as much as practicable. The study is exploratory in nature and extends earlier work by Griffiths and Clausen (1996) which employed a linear model and treated multiple, different water storages.

Physical background

The rivers and streams considered in this study are located in the steep foothill and mountainous areas of the eastern Southern Alps of Canterbury, New Zealand. All have coarse boulder or gravel beds composed almost exclusively of greywacke and are incised in glacial outwash gravels or mass movement deposits. (Here, greywacke is the common name for grey, well-indurated, feldspathic sandstone). The watercourses traverse the one geological terrain in terms of lithology, structure and recent geomorphic history. Waugh (1970) also grouped recession curves in Northland, New Zealand, according to the dominant lithology in their respective catchments.

The selected basins have no significant lakes or springs; water supply to the rivers and streams is by exfiltration from channel bed and bank storages supplied largely by rapid subsurface flow and to a minor extent by saturation overland flow in the riparian area (Hayward, 1976; Pearce and McKerchar, 1979). To avoid effects of seasonality and the influence of snow and ice storage, only recessions occurring during the period January to March were selected to construct master baseflow curves. Despite these restrictions individual recession curves varied considerably, probably owing, at least, to changeable basin wetness and storage conditions before storms and variability in storm pattern.

Theory

An idealised basin-scale model is adopted owing to lack of knowledge of the geometry, porosity and hydraulic conductivity of the assumed channel bed and bank storage. This is presumed to contribute a net inflow to a river or stream, and streamflow response is assumed to depend on time elapsed rather than real time of input. Moreover, because storage, as opposed to dynamic effects, is of concern, storage inflow is routed using the conservation of mass equation through a single reach representing the stream channel. In short, the proposed recession model is conceptual, lumped and time invariant. In what follows, the outflow function or recession curve is obtained by solving the differential equation for continuity for a specified inflow function. Various properties of the recession curve are then derived to assist with its prediction for an ungauged basin.

Conservation of mass

The conservation of mass or continuity equation for water storage analysis may be expressed as

$$\frac{dS}{dt} = I - Q \tag{1}$$

where S is the volume of water in stream channel storage, I is inflow from the channel bed and channel bank storage to the stream, Q is stream outflow and t is time. Following Chow (1959) we adopt a storage function of the form

$$S = aQ^{0.5} \tag{2}$$

in which a is a constant. For a prismatic rectangular channel, the exponent in Eq. 2, based on the Manning formula, is 0.6 (Chow, 1959, p. 606): for mathematical convenience we adopt a value of 0.5.

Differentiation of Eq. 2 with respect to t and substitution for dS/dt into Eq. 1 yields

$$\frac{dQ}{dt} = \frac{2Q^{0.5}}{a} \quad (I - Q) \tag{3}$$

We now introduce the substitution

$$w^2 = Q \tag{4}$$

and use this to transform Eq. 3 to

$$a\frac{dw}{dt} = I - w^2 \tag{5}$$

which is a non-linear, first order Riccati-type differential equation.

Inflow

Inflow from channel bed and bank storage to the stream is assumed to be a positive, decreasing function of time. Following the findings of Griffiths and Clausen (1996) for streams similar to those considered herein, we express inflow as

$$I = I_0 / (1 + bt)^2 \tag{6}$$

where I_0 is initial inflow at $t = 0$ and b is a constant.

Outflow

Substitution of Eq. 6 into Eq. 5 gives

$$a\frac{dw}{dt} = \left[I_0 / (1 + bt)^2 \right] - w^2 \tag{7}$$

To solve Eq. 7 we introduce a change of variable by defining

$$w = 1 / z(1 + bt) \tag{8}$$

Differentiation of Eq. 8 with respect to t yields

$$\frac{dw}{dt} = -w^2 (1 + bt) \left[\frac{dz}{dt} + \frac{b}{w(1 + bt)^2} \right] \tag{9}$$

and substitution for w and dw/dt in Eq. 7 using Eq. 8 and Eq. 9 gives

$$a(1 + bt)\frac{dz}{dt} = 1 - abz - I_0 z^2 \tag{10}$$

the differential equation which defines the model.

A solution to Eq. 10 is given in the Appendix. As noted there, when the ratio of inflow to outflow (I/Q) is approximately constant, we define the outflow hydrograph by Eq. A(13) which is

$$Q = Q_m / (1 + bt)^2, \quad Q \geq 0, t \geq 0 \tag{11}$$

where Q_m is median flow. (In fact, Eq. 11 is a special case of the model used for recession curves by Brutsaert and Nieber (1977), which may be expressed as

$$\frac{dQ}{dt} = -cQ^d \tag{12}$$

in which c and d are constants. Equation 12 was obtained from the Dupuit-Boussinesq aquifer model (Brutsaert, 2005) and independently from concentrated storage models. Differentiation of Eq. 11 with respect to t gives

$$\frac{dQ}{dt} = -2bQ_m/(1+bt)^3 \quad (13)$$

and elimination of the term $(1+bt)$ using Eq. 11 yields

$$\frac{dQ}{dt} = -2bQ^{1.5}/Q_m^{0.5} \quad (14)$$

to be compared with Eq. 12.)

To obtain the volume of water, V, under the recession curve, we integrate Eq. 11 with respect to t between the limits of 0 and t to obtain

$$V = Q_m t(1+bt) \quad (15)$$

Now, our concern is to predict the section of a recession curve beginning at the point $Q = Q_m$, $t = 0$ and ending at $Q = Q_f$, $t = t_f$, where Q_f is the minimum recorded 7-day low flow and t_f is its time of occurrence in a flow recession. Introduction of these limits into Eq. 15 yields

$$V_r = Q_m t_f/(1+bt_f) \quad (16)$$

where V_r is the volume under the recession curve between the specified limits. Similarly, Eq. 11 may be written as

$$Q_f = Q_m/(1+bt_f)^2 \quad (17)$$

Elimination of the constant b between Eq. 16 and Eq. 17 allows us to write

$$t_f = V_r/(Q_m Q_f)^{0.5} \quad (18)$$

where Eq. 18 may be used to determine t_f for given Q_m and Q_f when V_r can be estimated – a matter to be dealt with below. With t_f known, the recession curve can be determined by eliminating the constant b between Eq. 11 and Eq. 17 to obtain the non-dimensional formula

$$Q/Q_m = 1/\left\{1+(Q_m/Q_f)^{0.5}-1\,(t/t_f)\right\}^2 \quad (19)$$

To complete the analysis, V_r must be estimated. We assume that V_r is supplied from channel bed and bank storage and that

$$V_r = A_s L_s \sigma \quad (20)$$

in which A_s is the cross-sectional area of the riparian storage, L_s is its length, equal to the total length of the stream channel network, and σ is its porosity.

Application

The suitability and performance of the derived recession formulae are now tested to some extent with data from rivers and streams of Canterbury, New Zealand.

Data selection

Ten basins of differing size and hydrologic and physiographic characteristics were selected from greywacke geologic terrain in the foothills and mountains of Canterbury. Details of hydrological recording stations, or sites, record length and hydrologic and basin properties are given in Table 1. Water stage time series for all sites were checked for errors. Stage discharge rating cures were also checked to see that curves were consistent with one another and their definition was reasonably supported by gaugings. Mean annual precipitation was estimated using mean flow calculated from flow records, with an allowance averaging 0.5 m for evapotranspiration. Basin area was obtained from Walter *et al.* (2000) and mean basin elevation and total stream length above each site were estimated from computerised 1:50,000 scale topographic maps (NZMS 260 Series), on which the stream network is defined by blue lines.

Master baseflow recessions

For each of the 10 sites, at least eight flow recessions were selected from the January to March periods of the flow record, and master baseflow recession curves were constructed using the tabular method described by Toebes

Table 1 – Hydrologic and physiographic characteristics of selected Canterbury basins and flow recession data.

River and site name	Site number (Walter, 2000)	Record yrs	Mean annual flow m^3/s	Area km^2	Mean annual runoff mm/yr	Mean annual precipitation mm/yr	Mean elevation m	Stream length km L_s	Median flow m^3/s Q_m	Min 7-day low low m^3/s Q_f	Measured recession duration days t_f	Measured recession volume m^3*10^5 V_r	Storage volume per unit length of stream m^2 V_r/L_s	Estimated porosity of storage % σ	Bulk storage per unit length of stream m^2 $V_r/(L_s*\sigma)$	RMS values of fit of Eq. 19 to recession curves %
Acheron at Clarence	62103	1959-2009	23.2	973	751	1251	1320	1400	15.9	2.33	41	186	13.3	15	88.7	7.2
Ashley at Lees Valley	66210	1978-1999	3.90	121	1017	1517	1420	170	2.69	0.434	26	23.1	13.6	15	90.7	5.7
Waimakariri at Old Highway Bridge	66401	1967-2009	124	3210	1223	1723	850	4630	94.0	23.6	24	982	21.2	25	84.8	3.6
Camp Stm at Craigieburn	66405	1968-2009	0.0364	0.9	1276	1776	1590	2	0.025	0.008	41	0.762	38.1	35	109	4.8
Selwyn at Whitecliffs	68001	1964-2009	3.21	164	618	1118	680	250	2.02	0.499	42	31.8	12.7	15	84.7	5.1
Dry Acheron at Water Race	68529	1980-1991	0.214	6.19	1093	1593	1100	8.7	0.163	0.058	35	2.7	31	30	103	9.6
North Ashburton at Old Weir	68810	1983-2009	8.35	276	955	1455	1150	370	6.72	2.12	48	147	39.2	30	130	4.2
Orari at Gorge	69505	1982-2009	9.20	522	556	1056	930	760	6.2	1.73	50	133	17.5	20	87.5	5.3
Rocky Gully at Rockburn	69621	1965-2009	0.311	23	427	927	890	21	0.173	0.054	32	2.52	12	30	40.0	7.5
Forks at Balmoral	71129	1963-2009	3.15	98	1014	1514	1550	145	2.5	0.803	60	90.8	62.6	40	157	8.7

and Strang (1964). In applying this method, mean daily discharges were employed and a check was made to ensure that there was no significant rainfall in a basin during a selected recession. Indeed the occurrence of rainfall events in the basins severely restricted the number of recessions eligible for analysis. Q_m and Q_f were calculated from flow records and t_f and V_r values were obtained from the master baseflow recession curves (Table 1).

Equation 19 provided a good fit to the master baseflow recession curve for each of the 10 basins, as demonstrated by the small root mean square value of 100 $(Q_a - Q_p)/Q_a$, where Q_a is the measured value of mean daily discharge and Q_p is the value predicted by Eq. 19 (Table 1). This result was to be expected, as Griffiths and Clausen (1996) found that Eq. 19 gave the best performance of a range of two-parameter formulae for recession curves in Canterbury rivers and streams, including the Hurunui, Jollie and Ohau Rivers and Mary Burn, all of which drain basins within greywacke geologic terrain. An example of a fitted recession curve is shown in Figure 1.

Recession curve prediction

Within the defined geologic terrain, a master baseflow recession curve may be predicted for an ungauged natural basin using Eq. 19, in which t_f is found from Eq. 18, and in turn, V_r is estimated using Eq. 20. To undertake this calculation, the values of A_s, L_s, Q_m, Q_f and σ must be known for the basin. Of these, L_s is obtainable from topographic maps, as previously described. Q_m can be estimated from information given in, for example, Gabites (2006), and for practicable purposes the mean annual 7-day low flow, Q_{fm}, may be substituted for Q_f, as Q_{fm} may be available from maps of the spatial variability of this parameter given in publications such as Scarf (2002). Methods of predicting Q_{fm} based on statistical or correlative approaches are detailed in McKerchar and Dymond (1981), Hutchinson (1990) and Pearson (1995). Estimation of σ is more difficult, but it may be predicted reasonably accurately in a relative sense, that is, between rivers and streams rather than absolutely, using information in Chow (1964, p. 13-14) supported by field

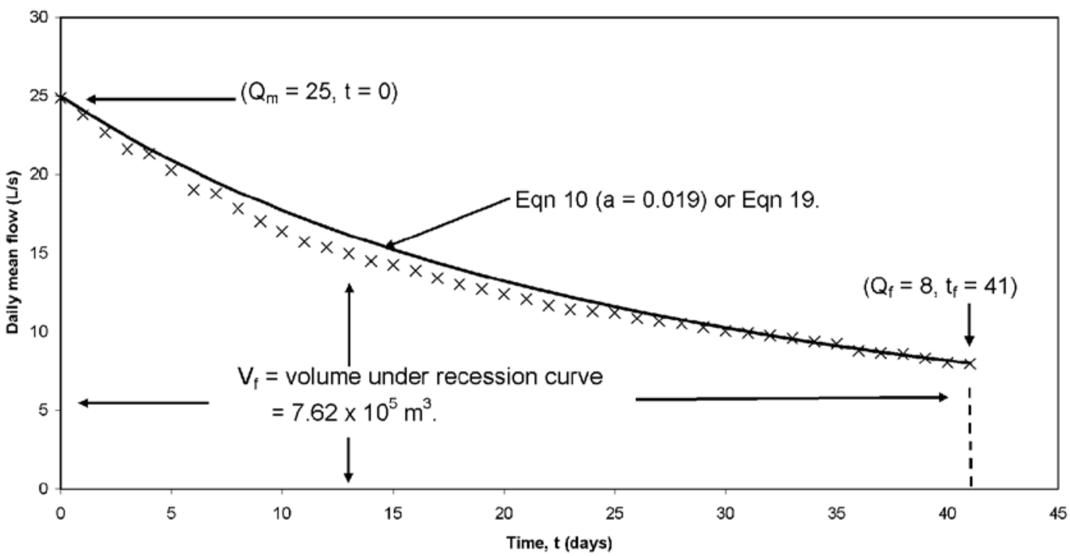

Figure 1 – Master recession curve (x) for Camp Stream at Craigieburn (Table 1) and fitted recession curve (Eq. 19) showing definitions of variables.

inspection. Our results are listed in Table 1 for the 10 basins.

The key to the predictive process is the estimation of V_r in Eq.20. Observe that values of the water storage per unit stream length V_r/L_s, and the bulk water storage per unit stream length $V_r/L_s\ \sigma = A_s$ (Eq. 20) in Table 1, divide into two classes. For Acheron, Ashley, Waimakariri, Selwyn and Orari Rivers, A_s is reasonably constant. Why this holds is not clear, but the modal water cross-section (determined from a random sample of cross-sections above each site) of all of these rivers at low flows is wide and shallow and is incised in a bed of coarse, river-sorted gravels. In contrast, the remainder are pool-and-riffle mountain streams. Camp Stream traverses scree and mass movement deposits, mostly of cobble and boulder sizes. Dry Acheron Stream (Griffiths and Hicks, 1980) and the North Ashburton River are similar, but the bed material contains fewer boulders. Rocky Gully is deeply incised, with bedrock often very close to the channel bed, and the lower section of Forks Stream flows through extensive and porous moraine deposits and outwash gravels.

While the values of A_s in Table 1 are generally quite variable, they can serve as the basis for prediction using a reference or analogue basin approach. For this method, one selects a reference basin, where V_r is known, that is similar in character to the ungauged catchment of interest, that is, with similar geology, geomorphology, drainage density and hydrological regime. Then, with estimated values of Q_m, Q_f (or Q_{fm}), L_s, and σ, in the ungauged basin combination of Eq. 18 and Eq. 20 yields

$$t_f = L_s\, \sigma\, A_{sr}\, /\, (Q_m\, Q_f)^{0.5} \qquad (21)$$

in which A_{sr} is the value of A_s in the reference basin. The predicted master baseflow recession curve can then be constructed for the ungauged basin by using the value of t_f from Eq. 21 in Eq. 19. This process is now illustrated using ungauged basins to show the performance of the method.

Example 1: Jollie at Mt Cook Station (Site no. 71135)

From flow records (1965-2009) we find Q_m = 6.43 m^3/s, Q_{fm} = 2.89 m^3/s and from the master base flow recession curve for January to March, t_f = 26 days. From topographic maps L_s = 175 km, and we estimate that σ = 35% because of the extent of porous glacial deposits. We adopt North Ashburton at Old Weir as the reference catchment, for which A_{sr} = 130 m^2 (Table 1). North Ashburton at Old Weir was selected because it is the most similar catchment in character to Jollie at Mt Cook Station, amongst those catchments with suitable flow records. Substitution of these values into Eq. 21 gives t_f= 1.90 × 10^6 s, or 22 days, which allows construction of the predicted recession curve using Eq. 19, as shown in Figure 2.

Example 2: Waiau at Marble Point (Site no. 64602)

From flow records (1967-2009) we find Q_m = 72.1 m^3/s, Q_{fm} = 31.9 m^3/s, and from the master baseflow recession curve for January to March, t_f = 16.0 days. Now L_s = 2990 km, and we estimate that σ = 0.25. We adopt Waimakariri at Old Highway Bridge as the reference catchment, for which A_{sr} = 84.8 m^2 (Table 1). Substitution of these values into Eq. 21 gives t_f = 1.32 x 10^6 s, or 15.3 days, which allows construction of the predicted recession curve using Eq. 19, as shown in Figure 3.

Future work

Refinement of the presented model is desirable in at least three areas. First, further testing of the model is needed using data from basins within the same greywacke geologic terrain and to explore the effect of using different reference catchments. Second, it would be useful to apply the model to other geologic terrains to assess the influence of this

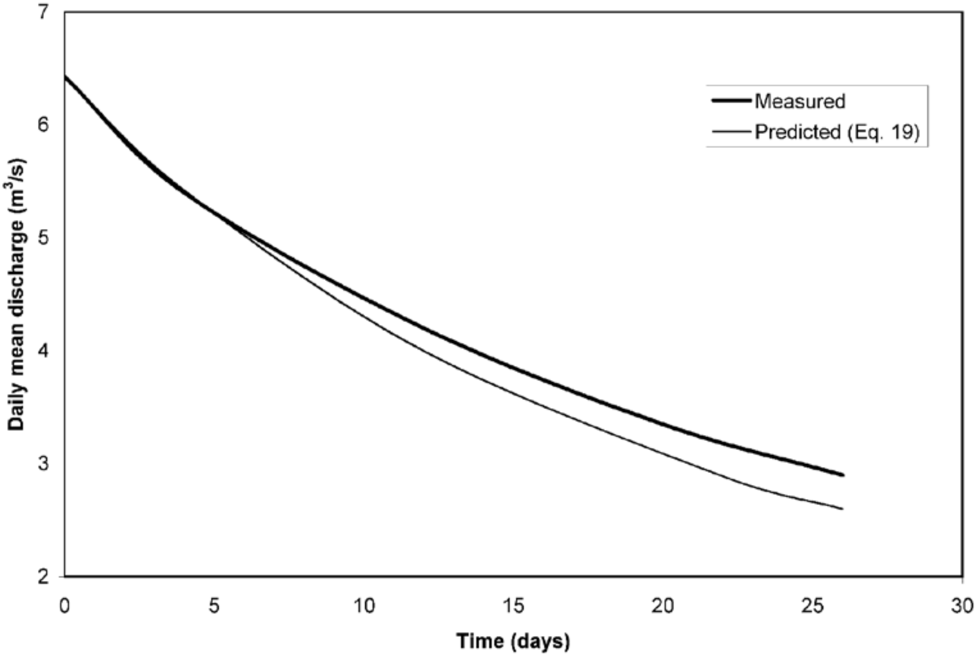

Figure 2 – Predicted and measured baseflow recession curves for Jollie at Mt Cook Station (Site no. 71135).

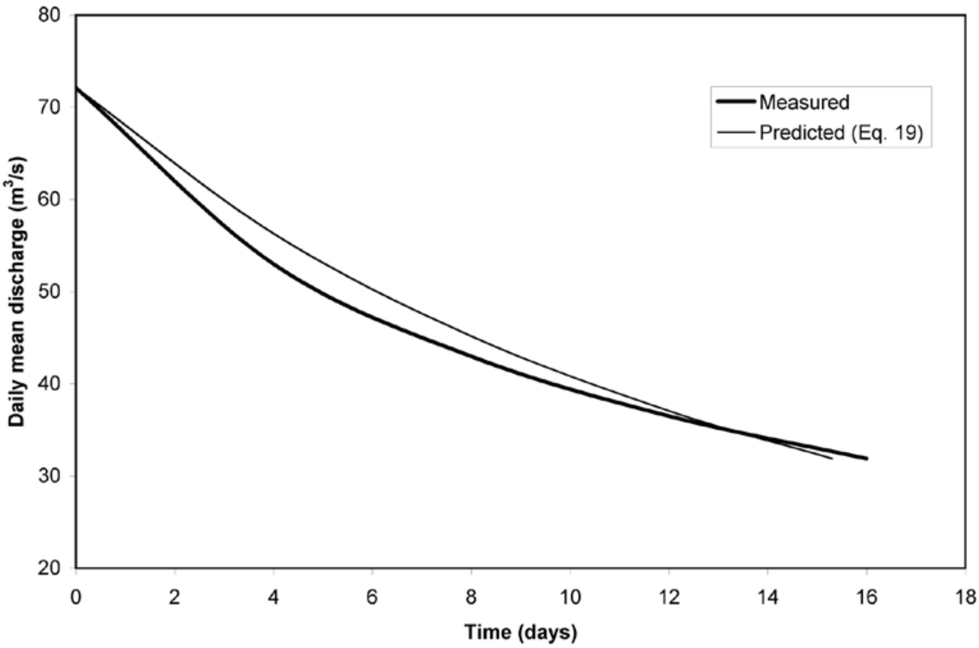

Figure 3 – Predicted and measured master baseflow recession curves for Waiau River at Marble point (Site No. 64602).

factor. Third, field work is needed to better understand the structure and properties of channel bed and bank storage.

Conclusions

Application of nonlinear hydrologic routing techniques to a prescribed inflow from stream channel bed and bank storage allows the derivation of a recession formula which models part of the falling limb of the outflow stream hydrograph from a basin.

The formula is limited in its application to natural catchments in Canterbury in greywacke geologic terrain having no significant lakes or springs; and to that section of a master recession curve between median flow and mean annual 7-day low flow for the period January to March in a flow record. Under these conditions it provides a good fit to measured master recession curves.

Employment of the formula to predict the master recession curve in an ungauged basin within the same geologic terrain depends upon the ability to estimate the volume of water discharged during this recession. In turn this depends upon the availability of a reference basin of similar character to the ungauged basin, with a measured master recession curve. Where these prerequisites can be met, good results may be obtained.

Further testing of the prediction method is needed especially in different geologic terrains, and more knowledge of the operation of channel bed and bank storage is required if the method is to be substantially refined.

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Appendix: Solution of differential equation

The differential equation defining the model (Eq. 9) is

$$a(1+bt)\frac{dz}{dt} = 1 - abz - I_0 z^2 \qquad (9)$$

Separation of variables allows us to write

$$\int \frac{dz}{1 - abz - I_0 z^2} = \int \frac{dt}{a(1+bt)} = (1/ab)\ln(1+bt) + k \qquad (A1)$$

in which k is a constant of integration.

Now the left-hand side of Eq. A1 is a standard integral listed in Petit-Bois (1991, p. 4).

$$\int \frac{dz}{1 - abz - I_0 z^2} = \frac{1}{(4I_0 + a^2b^2)^{0.5}} \ln \frac{2I_0 z + ab + (4I_0 + a^2b^2)^{0.5}}{2I_0 Z + ab - (4I_0 + a^2b^2)^{0.5}} \qquad (A2)$$

Writing

$$h = (4I_o + a^2b^2)^{0.5} \quad \text{(A3)}$$

and combining Eqs A1, A2 and A3 gives

$$(1/h)\ln\frac{2I_o z + ab + h}{2I_o z + ab - h} = (1/ab)\ln(1+bt) + k \quad \text{(A4)}$$

We may evaluate k using the initial condition, $Q = Q_m$ at $t = 0$. From Eqs. 4 and 8 at $t = 0$ we find that $z(t=0) = 1/Q^{0.5}$. Substitution of this value of z into Eq. A4 yields

$$k = (1/h)\ln\frac{2I_o + Q_m^{0.5}(ab+h)}{2I_o + Q_m^{0.5}(ab-h)} \quad \text{(A5)}$$

Returning to Eq. A4, we now exponentiate this expression to obtain

$$\frac{2I_o z + ab + h}{2I \;\; + ab - h} = (1+bt)^{(h/ab)}e^{kh} \quad \text{(A6)}$$

Writing $D = (1+bt)^{(h/ab)}e^{kh}$ (A7)

and solving Eq. A6 for z gives

$$z = \left[h(D+1) - ab(D-1)\right]/2I_o(D-1) \quad \text{(A8)}$$

From Eqs 4 and 8 we have

$$z = 1/Q^{0.5}\ (1+bt) \quad \text{(A9)}$$

and substitution of Eq. A9 into Eq. A8 gives the outflow function sought

$$Q\left\{\frac{2(D-1)}{h(D+1) - ab(D-1)}\left[\frac{I_o}{(1+bt)}\right]\right\}^2 \quad \text{(A10)}$$

Using data given in McKerchar *et al.* (1998) for the upper Ashley River, we find that D is at least of the order of 10, as storage is large, so that D-1 ≈ D +1 ≈ D. (Further data are needed for other basins in the same terrain to check this finding.) As h, a, b are constants, then the moderating term in Eq. A10 may, for practical purposes and ease of use, be taken as constant, that is:

$$2(D-1)/\left[h/(D+1) - ab(D-1)\right] = k_1 \quad \text{(A11)}$$

where k_1 is a constant. Thus Eq. A10 may be approximated by

$$Q = \left[k_1 I_o/(1+bt)\right]^2 \quad \text{(A12)}$$

where k_1 is a constant. Note that this approximation is only valid when the ratio of inflow to outflow is nearly constant. If not, then Eq.A10 must be employed. Using the initial condition $Q = Q_m$ at $t = 0$ in Eq. A12, the recession curve model may be expressed simply as

$$Q = Q_m/(1+bt)^2 \quad \text{(A13)}$$

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