RADAR ESTIMATION OF RAINFALL

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ABSTRACT

By calibration of the equipment, power back-scattered to the radar by precipitation can be measured. The power back-scattered depends on the reflectivity of the precipitation, which in turn depends on the form of the drop-size distribution. Rainfall rate is also a function of drop size distribution, and an empirical relationship exists between reflectivity and rainfall rate. Using this relation, measurements of received power permit an estimate of rainfall rate. Trials conducted overseas suggest that an accuracy of a factor of two can be achieved with suitable equipment on all occasions, and that the equivalent raingauge density is of the order of 1 to 2 per 100 square miles. Results of overseas trials and preliminary results of rainfall estimates made at Ohakea are presented.

METEOROLOGICAL RADAR

In the early days of microwave radar, it was discovered that precipitation scatters power back to the radar. In early applications, this was regarded as noise, but soon after the war, meteorologists began to use this source of weather information. A voluminous literature now exists, and a yearly international conference is held. Battan (1959) gives a very full treatment of theoretical and applied radar meteorology.

The radar transmitter produces short pulses of very high power at the aerial. These are reflected by a dish, shaped to give a beam of the required characteristics. A narrow beam, with a sharp peak in power along the axis, gives directional information on scattering bodies in its path. Measurement of time taken for a pulse to make the two-way journey between aerial and target gives its range.

In meteorological work, the reflector is usually a circular paraboloid, with a dipole aerial at the focus. As this is not a point source, the beam diverges. The beam half-width is defined as the angle between the axis and the direction where the radiated power falls to half that along the axis. In a well-designed system power radiated outside this cone is very small. The CR353, used in the N.Z. Meteorological Service, has a half-width of 1.4°.

Target information is displayed visually, on a cathode ray tube. There are several types of display; that with most application in rainfall measurement is the Plan Position Indicator. The

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P.P.I. (Fig. 1) shows echoes received from targets in plan form, with the station normally at the centre of the tube-face. The aerial rotates continuously, and an electron beam generates a radial sweep, which is initiated at the centre of the display, synchronously with the pulse leaving the aerial. Time taken for the sweep to travel to the edge of the tube is known, and hence range to any echo is found. Each successive sweep moves along a radius vector which rotates in phase with the aerial. The beam is intensity modulated; the amplified signal voltage is applied to the beam-generating electrode, so that the brightness of the 'paint' on the tube face increases as received power increases. Controls are set so that when no signal is received, the beam makes no trace on the tube-face.

Since the beam is 2.8° wide, it cannot discriminate between targets separated by less than this angle. Subject to this limitation on resolution, the position and extent of targets are shown in plan form on the P.P.I. Bearings are obtained from an azimuth scale at the edge of the tube, and range from rings (see Fig. 1), generated electronically.

Within limits, brightness of the paint is proportional to received power, but the dynamic range of most display systems is far less than that needed to handle the range of meteorological targets.

Relation Between Received Power and Rainfall Rate

Weather radars operate on wavelengths between 3 and 10 cm. The CR353 is in the 10cm. band. At these wavelengths, raindrops are Rayleigh scatterers. Power is not scattered isotropically, and there is a strong maximum on either side of the particle, along the direction of incident radiation. An equivalent back-scattering cross section of a particle is defined as the cross section of a perfectly reflecting sphere, which would scatter the same power back to the aerial as does the particle. The back-scattering cross section of a particle with diameter D less than about 1/5 of the wavelength l is given by

$$S = \frac{\pi^5 K^2 D^6}{l^4} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where S is back-scattering cross section and K is a refractive index parameter. The power received at the radar from an assemblage of particles which fill the cross section of the beam is

$$Pr = \frac{Pt\ h\ Ae}{8\pi\ r^2}\ k\ \Sigma\ S \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This is a simplified form of the radar equation. Pt is transmitted power, h the pulse length, Ae the effective aerial aperture, r the range to target, and k an attenuation factor. The summation is taken over unit volume.

Transmitted power in the pulses is very high. The pulse is short in relation to the interval between pulses. In the CR353 pulse power is 700 kw., whilst mean transmitted power is only about 0.5 kw. The 2 micro-second pulse gives a wave-train of length 600 metres. This limits the resolution of the system in range to half a pulse length.

Effective aerial aperture is determined by several factors and is very difficult to measure directly. It has analogies with lens aperture in photography. It is affected by beam width and by field strength distribution in the beam.

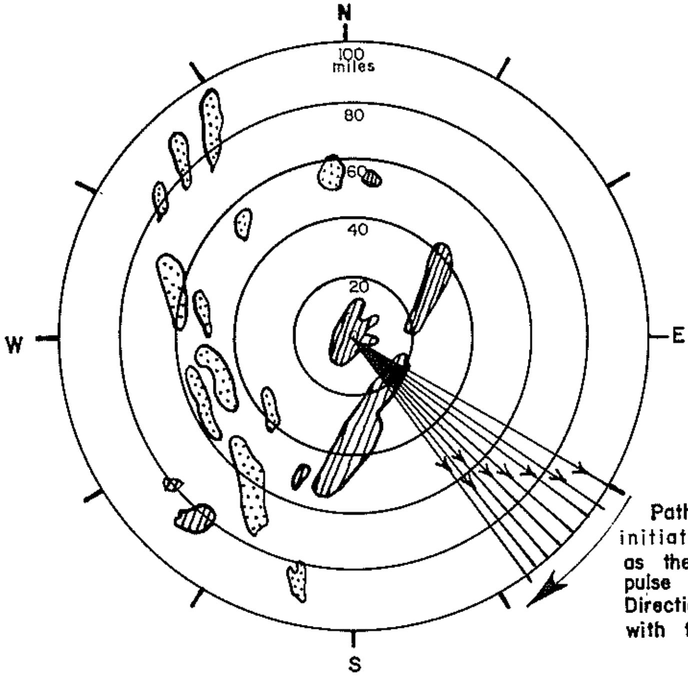
Pr is inversely proportional to r^2. For a 'point' target, Pr is proportional to $1/r^4$, but where the target fills the beam, there is no range attenuation on the outward path, as the precipitation intercepts all the radiation leaving the aerial. It is assumed that the attenuation factor, k is unity. For 10cm. wavelength, this introduces negligible error.

If the beam is not filled, Pr approximates to that from a point target, and quantitative work is then impossible.

For a given radar, all the factors in (1) and (2) are, or can be made, constants except for P, r, and S. We define a radar reflectivity factor, Z as the reflectivity per unit volume, $Z = \Sigma D^6$ and write the radar equation in the form

$$Pr = \frac{CZ}{r^2} \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

where the constant C combines refractive index and equipment parameters.



Path of the sweep, initiated at the centre as the transmitter pulse leaves the aerial. Direction synchronised with that of the aerial.

Fig. 1 — PLAN POSITION INDICATOR — Areas striped represent 'permanent echoes' present on the Ohakea scope, produced by local high ground close to the centre, and by the main ranges to the east, and Ruapehu in the north. The stippled areas show the distribution of echo in a typical cold front. The sweep spacing is much closer than is indicated here. The test area used for rainfall comparisons lies in the clear zone to the south-east of Ohakea, at about 15 miles range, to north-west of the northern end of the Tararuas.

By taking samples of rain, and counting the drops in size ranges, Z, and R, the rainfall rate can be found from the sample. R is proportional to Σ vt D^3, where vt is the terminal velocity of raindrops in each size range. There is no one-to-one correspondence between values of Z and R, since both are sensitive to the form of the size distribution. An empirical relation can be found from the values obtained from rain samples. This takes the form $Z = AR^b$, and both A and b may vary for different types of rain. An average of the many relations which have been obtained in different areas is $Z = 200R^{1.6}$, where Z is in mm^6/m^3 and R in mm/hr. Details of the Z-R relation found for Ohakea are given in the section headed **Results of Ohakea Trials.**

Using this relation, (3) becomes

$$Pr = \frac{C_1 \; R^B}{r^2} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

where C_1 now also includes the factor A. Range is known from the display. If Pr can be measured, an estimate of R is possible.

This presents difficulties, as Pr may be as low at 10^{-12} watts. At Ohakea, the radar has been calibrated by suspending a metal sphere of diameter one foot, from a tethered balloon. It is held high enough for the beam not to intercept the ground, and the target is aligned along the aerial axis. Its back-scattering cross section is equal to its geometric section, since it is large compared with the wavelength. The received signal is attenuated (that is, its power is reduced), by a known amount, until the echo at the display lies at the threshold of visibility. The range and attenuation are noted. The radar equations for point and distributed targets are compared and, after correcting for range, the reflectivity of the precipitation is found by measuring range and attenuation. This method avoids the need to make the difficult measurement of aerial gain. Errors are due to instability of the radar, and to the finite steps used in the attenuator. Calibration is not likely to be needed more than two or three times a year, and errors are thought to amount to about 20% of Z for single readings, but errors due to instability are likely to cancel out to some extent for consecutive readings.

Methods of rainfall estimation

To obtain rainfall accumulation over an area, results at the display must be integrated over both time and area. As the radar gives an instantaneous picture of reflectivity distribution, it is necessary to take readings frequently. For the time integration, it is convenient to use a grid system and total the amounts at suitable intervals before doing the areal integration. If the signal is attenuated by known steps, and attenuation required to bring the echo to threshold level is measured for each grid square, rainfall rates can be assigned to each attenuation level on the basis of the calibration, and these totalled to obtain period rainfall. Range attenuation can be corrected in the receiver circuitry, which is available in the CR353. This method of obtaining areal rainfall is accurate but expensive in man-hours.

Photographic techniques are also used. Polaroid transparencies can be superimposed to obtain an approximate time integration, or multiple exposures can be made. As the dynamic range of the display system is limited, the attenuation level must be selected to obtain information over the range of interest. This usually involves sacrifice of signal from low rainfall rates, to permit measurement at higher rates without saturating the system.

Although brightness depends on signal voltage the relation is a power law. Signal voltage is roughly proportional to $R^{0.8}$, and it is not possible to arrange for the tube power law to compensate for this. Hence the response is not linear with rainfall rate, and photographic techniques result in a loss of accuracy. The multiple exposures are evaluated using a densitometer to obtain equivalent isohyets.

Several methods of electronic integration are available.

Analogue computers are reliable, but expensive and cheaper methods, at the present stage of development, are less reliable.

Limitations

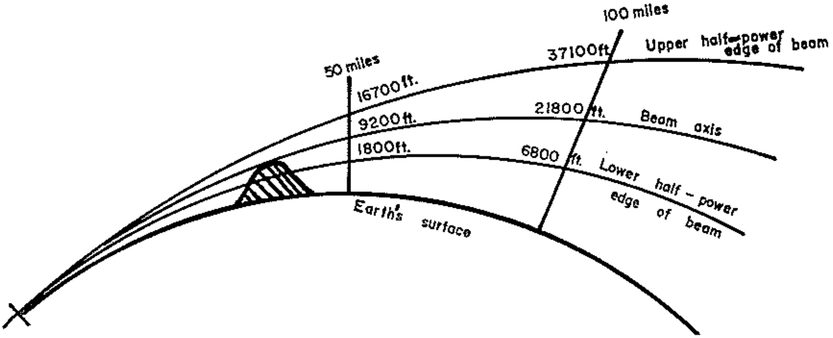
Apart from factors already discussed, beam geometry affects accuracy greatly. Fig. 2 shows a vertical section through the CR353 beam. At low elevations, high ground intercepts a portion of the beam, and areas of 'permanent echo' are found at the display (Fig. 1). Precipitation cannot be distinguished in such areas.

Beam filling is essential. A shower of average size in the Ohakea area, of diameter 3 miles, will fail to fill the beam in azimuth beyond about 53 miles. Average echo tops at Ohakea are about 13500 feet. The beam is not filled in the vertical, at this height, beyond about 50 miles. Hence, beam width restricts the useful range for hydrological work. A narrow beam is desirable, but requires a large and expensive aerial system.

Behind permanent echoes, the beam must be elevated to avoid shading by high ground. If there is wind shear in the vertical, rainfall rates aloft and at the ground may differ greatly, and measurements must be made as low as possible. In Fig. 2 the lower edge of the beam rises with range, which imposes a further restriction on useful range. The drop size distribution is affected by many factors, and distributions at the ground and aloft may differ widely.

The power back-scattered by a particle is a function of the refractive index. When the precipitation does not consist of rain, different Z-R relations apply. As snowflakes pass through the melting level, they become coated with water, and there is a very large increase in Pr. Lower, they melt and Pr falls again, though not to such a low level as in the dry snow above. In winter, with low melting levels, this factor restricts the available range for reliable measurements. In mountainous country, information on melting level obtained in this way is often made available to the hydrologist.

With all these requirements satisfied, overseas results vary widely. Those cases showing poor agreement between gauge and radar measurements are mostly attributed to variations in the Z-R relation. No record can be found of any trials incorporating drop samples in the test area, simultaneous with the radar measurements. This is now being done for the Ohakea trials.



ig. 2 — BEAM GEOMETRY — The heights given relate to the 2.8° beam width of the CR353, vith the aerial axis elevated to 1.4° (i.e. the lower edge of the beam has zero elevation). Owing o refraction the beam is curved. Under average conditions, the curvature is such that it can be egarded as straight with respect to a fictitious earth radius of 5000 miles. High ground in he path of the beam obscures the volume behind it. In azimuth, the width of the beam is about 2.5 miles at 50 miles range, and 5 miles at 100 miles range.

OVERSEAS RESULTS

Most of these are from the U.S.A., using the WSR57, which was designed for quantitative weather surveillance work. The River Forecast Service works in cooperation with hydrologists. Information is given on areas of heavier rain, probably times of beginning and ending, speed of cells and movement and orientation of rain areas with respect to catchments, reservoirs and irrigation areas. Photographic superposition is used to locate areas of heaviest accumulation and to estimate amounts in heavier rains. The radar is also used for the severe storm (tornado) warning service, aircrew briefing and routine surveillance. This limits its availability for quantitative work (McCallister, 1962).

Direct observations on shape and motion are most useful in conjunction with the hydrologist's knowledge of the catchments and of antecendent precipitation. For instance, a heavy storm moving down stream at about the same speed as the peak stage produces a much higher peak than one moving up stream.

Transponder beacons (Rockney, 1960) have been developed for remote measurement of falls, using the tipping-bucket raingauge. When it tips, a 'pip' appears on the P.P.I. in the corresponding position. A second tip gives a further pip, on the same bearing, and one mile further out. This continues until the pips reach the edge of the display, when the beacon recycles to zero. Initial cost is said to be low and battery powered versions are being developed for use in remote areas. Several such beacons could be used to calibrate the radar for a particular storm.

Aoyagi (1963, 1964) in Japan, set up a network of 27 recording raingauges in about 250 sq. miles. Ten minute falls were estimated and isohyets drawn for both gauge and radar measurements· The radar estimates were statistically equivalent to a gauge density of 1 per 80 sq. miles. Over four hours, the difference in mean areal fall between radar and gauges was only -3.6%. Over the four hour period, the equivalent gauge density was one per 30 sq. miles. This was on an occasion of steady rain, but comparisons were also made in showers. Differences were up to 42% of the gauge rainfall for periods of about one hour, but Aoyagi considers that, in some of these cases, the equivalent gauge density was higher than that in the network, and consequently the radar estimate was better than that found from the raingauges. He suggests the use of a single recording raingauge, readily accessible, to obtain a value of A in the relation $Z = AR^b$, for a particular storm, with a corresponding improvement in accuracy of the radar estimate (b cannot be found in this way).

Harrold (1965) used the first 40 minutes of a heavy storm over a small gauge network to calibrate the radar for the remainder of the storm. For this portion, the radar estimate was 0.22 in./hr.

compared with 0·30 in. at the gauges. The equivalent gauge density was 1 per 6 sq. miles. The results obtained by this method were better than those which would have been found with an assumed Z-R relation. Consequently, he recommended use of a small calibrating-raingauge network in the area of interest.

Jones (1963) measured rain over Illinois, with 50 gauges in about 400 sq. miles. Radar and network isohyet patterns correspond well but there were some cases of large differences in the areal estimates. The radar measurements varied between 18% and 158% of network rainfall. The variations were probably due to departures from the Z-R relation used. The wavelength used rendered the results rather sensitive to the drop-size distribution.

Stout et. al. (1953) in a similar trial, report differences of up to 59% between radar and gauge estimates.

Wilson (1963), at Atlantic City, compared hourly rain with that found at 60 recording gauges within range of the radar· For ranges up to 60 miles, almost 100% of rains of more than .01in./hr. were detected at the radar, and between 60 and 100 miles, 100% of falls greater than .04in/hr. were also detected; 80% of all estimates of point rainfall were within a factor of two of the gauge measurement. No figures were given for areal rain owing to inadequacy of the gauge network.

Austin (1963), for 44 storms in New England, compared hourly falls at several gauges with the radar point rainfall estimates. For 30 of the storms, estimates differed from gauge rain by a factor less than two at all the gauges . Consideration of synoptic type might have led the observer to suspect that the assumed Z-R relation might not apply in many of the remaining 14 storms.

Radar climatological studies have been made in the U.S.A., Canada, India and elsewhere· A limited study has been done at Ohakea (Ryan, 1966). Most of these studies are aimed at information of use to aviation forecasters, but it has been found that there is, in some cases, a useful relation between radar echo frequencies and period rainfall. Results have been used as an aid in placing isohyets on seasonal and annual rainfall maps in areas of low gauge density and over the sea and large lakes.

RESULTS OF OHAKEA TRIALS

The CR353 was commissioned in April, 1965, and is in daily operational use for windfinding and weather surveillance. Preliminary work on the rainfall trials began in May 1966. The radar has been calibrated three times and stability has been found satisfactory both for short and long periods. A secondary calibration, which can be done in a few minutes, has been devised to provide a check on changes in performance of the radar.

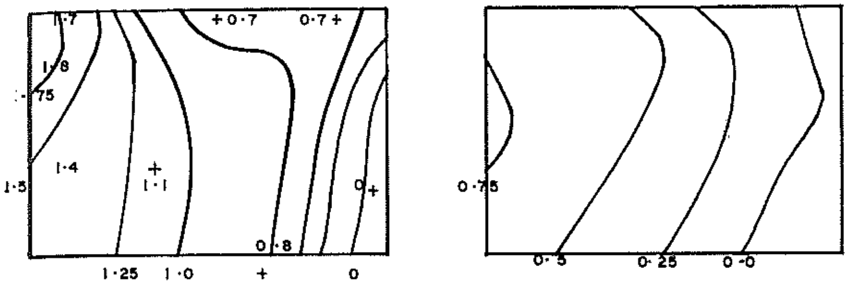
Ninety drop-size samples have been taken, using the stained filter-paper technique. From these, values of Z and R were obtained, and used to obtain the relation $Z = 239R^{1.55}$. The correlation coefficient between log Z and log R is 0.97, and the standard error of estimate of R from Z is $+30$ and -23% of R. In this relation, R is measured in mm/hr. As the technique samples volumes of the order of 1 m^3, the sample is not representative for the larger drops, which are present in concentrations well below 1 per m^3, and there is sime inaccuracy in the values of both Z and R due to this. Z is very strongly affected by the presence of the larger drops, owing to the dependence on D^6.

The standard error was not improved by separating the samples into occasions of steady rain and of showers. However, all the samples were of winter precipitation, generated above the freezing level in all cases. Summer rains may well give different results.

Eight recording raingauges have been set up in an area of about 24 sq. miles in and around Palmerston North .The area is about 14 miles from the radar, and there is good line-of-sight visibility. Observers were asked to time-mark the charts accurately. Contours of rain areas were drawn over the test area for various attenuation settings, and after range correction, the Z-R relation already quoted was used to obtain equivalent rainfall rates. The observations are repeated at 5-10 minute intervals, and used to obtain rainfall accumulation. As the network is rather small, the results are rather sensitive to wind drift and, as far as possible, tests will be done on occasions of light winds. Raindrop samples are to be taken in the test area during measurement runs and these will be used to obtain a Z-R relation for the particular storm, in the hope of explaining differences between radar and gauge estimates.

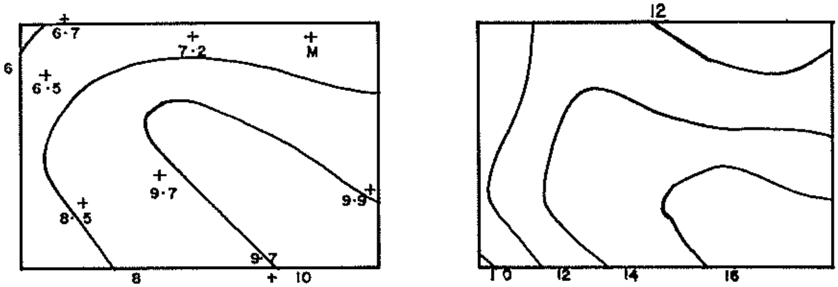
To date (August 1966) three comparisons have been made, and the results are given in Fig. 3. No simultaneous drop samples were available in the test area at this time, but samples were taken at Ohakea. On 14 August, when the radar estimate was .0037in. for the mean areal rainfall, compared with the gauge estimate of .0095in., use of three samples to obtain a new Z-R relation improved the radar estimate to .0061in. On 24 and 25 August, the results were rather better. Use of Ohakea samples to make a new estimate resulted in an improvement on the 25th, but not on the 24th.

All three trials were done on occasions of patchy, mainly light rain, and individual rain areas were small. Better results are to be hoped for when the rain is more continuous, or covers larger areas.



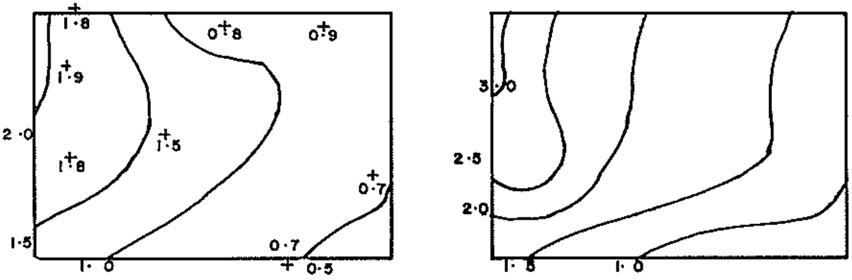
RAINGAUGE 14/8/66; 1935 — 2035· **RADAR**

Mean areal rainfall .009 ins. Estimated mean areal rainfall .004ins.



RAINGAUGE 24/8/66; 0834 - 0959. **RADAR**

Mean areal rainfall .089ins. Estimated mean areal rainfall .140ins.



RAINGAUGE 25/8/66; 0904 - 1048. **RADAR**

Mean areal rainfall . 012 ins. Estimated mean areal rainfall .018ins.

ig. 3 — RADAR-RAINGAUGE COMPARISONS MADE AT OHAKEA — Times are N.Z.S.T. Isohyets re in hundredths of an inch, for the periods stated. The area, in and near Palmerston North, 6 nautical miles by 4 and north is to the left of the figure. The raingauge sites, with rainfall shown, are indicated by crosses.

CONCLUSIONS

It has been shown by many investigators that quantitative radar rainfall measurements are feasible, with an accuracy at least as good as that of conventional raingauge networks in most countries. Both the radar site and the equipment must be selected to minimize the effects of the several limitations on the accuracy of the method.

Even in the case where the radar estimate is less accurate than that obtained from the existing raingauge network, it has the advantage that it becomes available as the rain falls, and includes also valuable data on size and movement of rain areas, with obvious application to flood forecasting. Staffing and equipment are expensive, but often part of the cost can be set against other uses of the radar, such as operational weather surveillance, storm warning, and wind finding. Radar studies are also of great value in the investigation of the structure of storms and the mechanisms of precipitation formation.

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