Journal of Hydrology (NZ) 35(2): 239-258, 1996

Physical model study of stage-discharge relationships in a gorge of a braided river

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Abstract

A distorted rough turbulent flow, Froude-law mobile bed physical hydraulic model represents the passage of a flood hydrograph through the gravel-bedded Waimakariri Gorge, South Island, New Zealand, and allows the intrinsic variability of the stage-discharge curve at a recorder site in the Gorge to be assessed. Intrinsic variabilities equivalent to ± 300 m^3/s and ± 500 m^3/s were measured respectively on the rising limb and on the falling limb of the flood wave in the model at flows representing more than 1000 m^3/s in the prototype.

A theoretical analysis of flow and sediment transport in the river forms a basis for extending stage-discharge relationships to high flows at which direct field measurements are impracticable. The variability evident in the model stage-discharge relationship suggests that a simpler analysis might be equally useful.

Introduction

General

Flood forecasting is vital for reducing the damage and loss of life caused by river flooding; indeed, flood warning-evacuation systems are the most realistic way to cope with large floods (Davies and Hall, 1992). To forecast the time of passage and crest height of a flood wave at a critical section requires knowledge of the flood discharge at an upstream location, which in turn necessitates that a reliable stage-discharge relationship be defined at the upstream location.

Gravel-bed braided rivers are particularly inconvenient for measuring stage and discharge; if the river flow is distributed among, say, ten individual channels, each with a different water surface elevation, the practical problems of measuring the total discharge and of associating this with a particular 'stage height' are obvious. Therefore gauging stations are usually located at narrow sections such as gorges, where cableways or bridges can facilitate current meter gauging and a single water surface elevation can be defined. However, flood flows often cause a lot of movement of bed gravels at such sections, with the result that the bed level can change rapidly due to scour and fill, making the definition of a simple stage-discharge relationship impossible. Rating curves at such sections are highly variable and often exhibit complex loops.

In addition, it is not practicable to measure flow rates at high flood stages; flow velocities in these narrow sections are often very high, turbulence and bedload movement are intense and the data obtained are highly imprecise due to temporal and spatial variability of flow geometry and parameters. Hence, a procedure is needed for extending the more reliable and less variable rating curves obtained at lower flows to high flood stages for which direct measurements cannot be made. Given the complex processes occurring during a flood, it is unlikely that theory alone can form a satisfactory basis for such extrapolation of the rating curve, and physical modelling may be a useful complementary approach.

Waimakariri River

The city of Christchurch lies adjacent to the Waimakariri River on the east coast of the South Island of New Zealand (Fig.1). The Waimakariri flows for the last 60 km of its course across a suite of postglacial outwash fans, incised below their surface adjacent to the foothills of the Southern Alps, but flowing at close to the surface level for 30 km from Halkett to the sea. Before being confined by training banks over the last 130 years, the river was gradually building up the land surface in the vicinity of Christchurch by avulsion of its braided channels during high flows (generally due to intense rain on the main divide of the Southern Alps).

The lower Waimakariri is now controlled by artificial banks which should be overtopped only at flows greater than about 3500 m^3/s, which is about the 0.01 aep flow. This flow rate will be exceeded in the future, and the unarmoured control banks are also susceptible to breaching by scouring at lower flows, so it remains important to be able to give as much warning as possible of high flows in the lower river. Flow can be measured only where the State Highway One bridge crosses the river just north of Christchurch, and at the lower Gorge near Oxford where the river flows in a single channel through a bedrock outcrop. Elsewhere the river is extensively braided, of the order of 1 km wide with up to 10 separate, shifting flow channels, and flow measurement is impracticable. Thus the

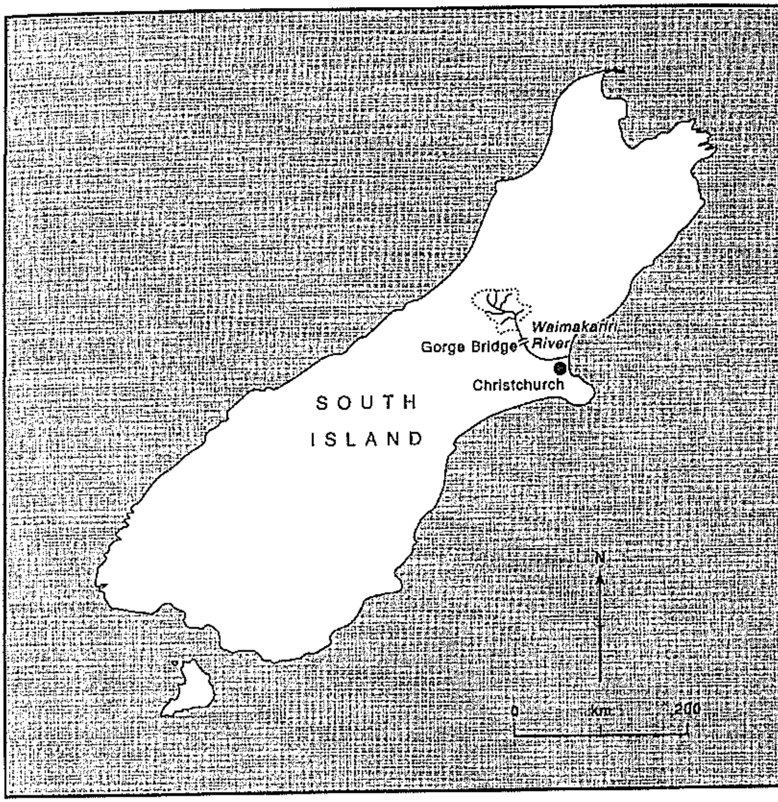


Figure 1 Location map, Waimakariri River, South Island, New Zealand.

only site in reasonable proximity to Christchurch at which it is possible to make flow measurements in time to give adequate warning of a flood is at the lower Gorge. At this site, 40 km upstream of Christchurch, a stage recorder has been working since 1929; however, because the gravel bed of the Gorge may scour 5 m or more during floods, and because of the difficulty of flow gauging at the site, the stage-discharge relationship is not well known.

In order to obtain information about the variability of the stage-discharge relationship at the lower Waimakariri Gorge, a vertically exaggerated, Froude-law flume model with a rough turbulent mobile-bed and unsteady flow, was developed at the Department of Natural Resources Engineering, Lincoln University, under a contract with Canterbury Regional Council.

This model of the Waimakariri Gorge has been used to provide empirical data for extending the rating curve of the prototype river. It has also yielded valuable insights into the physics of flow behaviour, and data with which to test theoretical developments. This study forms the initial part of a project to develop further understanding of bed behaviour during the passage of a flood wave through a narrow section of a braided river, leading to a theoretical expression for stage-discharge relationships in the two-phase flow. This will ultimately lead to a semi-theoretical procedure for extending rating curves to flood conditions, and possibly for estimating rating curves in general where information on flow and sediment transport rates is lacking.

Theoretical Considerations

Under conditions of unsteady non-uniform flow in a rigid boundary channel the stage-discharge relationship is not unique. With the passage of a flood wave the water level at any cross-section traces out a loop in the stage-discharge plane because discharge is not a function of depth alone. Henderson (1966) analyses and gives theoretical-empirical formulae for this loop-rating curve. Here we are concerned with the same problem but with the addition of two complications; a contraction in channel width and a channel bed of non-cohesive sediment which scours and fills as a flood wave passes. There is unsteady, non-uniform flow (as treated by Henderson, 1966) and unsteady, non-equilibrium bedload transport; suspended sediment transport is ignored as its influence on stage-discharge relationships is minor.

A theoretical stage-discharge relationship for a wide rectangular channel is now developed. The one-dimensional shallow water flow equations or St Venant equations are, for momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial y}{\partial x} = g(S_o - S_f) \tag{1}$$

and for continuity of flow

$$u\frac{\partial y}{\partial x} + y\frac{\partial u}{\partial x} + \frac{\partial y}{\partial t} = 0 \tag{2}$$

in which u is mean flow velocity, y is flow depth, g is the acceleration of gravity, S_o is channel slope, S_f is the friction slope, x is the spatial co-ordinate in the downstream direction and t is time.

The continuity equation for bedload transport is

$$\frac{\partial g_v}{\partial x} + \quad \frac{\partial z}{\partial t} = 0 \tag{3}$$

where g_v is the bedload transport rate by volume per unit width, α is the ratio of particle volume to total volume in the bed and z is bed elevation above datum. Further details about the derivations of equations 1, 2 and 3 are given for example in Cunge *et al.* (1980).

The constitutive relations for flow resistance and equilibrium bedload transport are taken herein as respectively the Manning formula

$$u = \frac{1}{n} y^{0.67}\ S_f^{0.5} \tag{4}$$

in which n is the Manning roughness coefficient; and the Meyer-Peter and Muller (1948) formula

$$g_v = 8[g\,(S_S - 1)d^3]^{0.5}\left[\left(\frac{n_p}{n}\right)^{1.5} \frac{n^2u^2}{y^{0.33}\,(S_S - 1)d} - \tau_c\right]^{1.5} \tag{5}$$

where S_s is the specific gravity of sediment, d is mean sediment diameter, n_p is the Manning particle roughness coefficient and τ_c is Shields entrainment function at initial motion conditions. Equations 4 and 5 were developed for steady state conditions and their use here is problematic but at present there appears to be no reliable alternative (see, for example, van Rijn, 1989). Griffiths (1989) gives reasons for adopting the Meyer-Peter and Muller (1948) formula in theoretical models concerned with braided rivers in the laboratory or the field. Combination of Equations 1 and 2 yields

$$S_f = S_o - \frac{1}{g}\frac{\partial u}{\partial t} + \frac{u}{gy}\frac{\partial y}{\partial t} - (1 - F^2)\frac{\partial y}{\partial x} \tag{6}$$

in which $F = u/(gy)^{0.5}$ is the Froude Number. Partial differentiation of Equation 5 treated as g_v (u, y) gives with Equation 3

$$\frac{\partial g_v}{\partial x} = -\alpha\frac{\partial z}{\partial t} = 12[g\,(S_S - 1)d^3]^{0.5}\left[\frac{n_p^{1.5}n^{0.5}u^2}{y^{0.33}\,(S_S - 1)d} - \tau_c\right]^{0.5}\left[2\frac{\partial u}{\partial x} - \frac{u}{3y}\frac{\partial y}{\partial x}\right] \tag{7}$$

and combination of Equations 2, 6 and 7 then yields after some manipulation

$$S_f = S_o - \frac{1}{g}\frac{\partial u}{\partial t} + \left(\frac{6+F^2}{7u}\right)\frac{\partial y}{\partial t} - \alpha\frac{(1-F^2)}{A}\frac{\partial z}{\partial t} \tag{8}$$

where A is a sediment transport factor given by

$$A = 28[g(S_S - 1)d^3]^{0.5}\left[\frac{n_p^{1.5}n^{0.5}F^2gy^{0.66}}{(S_S-1)d} - \tau_c\right]^{0.5}\left[\frac{n_p^{1.5}n^{0.5}F^2g}{(S_S-1)dy^{0.33}}\right] \tag{9}$$

Alternatively Equation 8 may be expressed in terms of $\partial y/\partial x$ rather than $\partial y/\partial t$.

$$S_f = S_o - \frac{1}{g}\frac{\partial u}{\partial t} - \left(\frac{6+F^2}{6}\right)\frac{\partial y}{\partial x} + \frac{7\alpha F^2}{6A}\frac{\partial z}{\partial t} \tag{10}$$

With Q = uyB, where Q is water discharge and B is channel width, the depth-discharge relationship from Equation 4 is

$$y = \left(\frac{nQ}{BS_f^{0.5}}\right)^{0.6} \tag{11}$$

and if h is stage or water level above datum where h = y + z then from Equations 8 and 11

$$h = z + \left\{nQ/B\left[S_o - \frac{1}{g}\frac{\partial u}{\partial t} - \left(\frac{6+F^2}{7F(gy)^{0.5}}\right)\frac{\partial y}{\partial t} - \alpha\frac{(1-F^2)}{A}\frac{\partial z}{\partial t}\right]^{0.5}\right\}^{0.6} \tag{12}$$

which is the required stage-discharge relationship. The version of Equation 12 corresponding to Equation 10 is

$$h = z + \left\{nQ/B\left[S_o - \frac{1}{g}\frac{\partial u}{\partial t} - \left(\frac{6+F^2}{6}\right)\frac{\partial y}{\partial t} - \frac{7\alpha F^2}{6A}\frac{\partial z}{\partial t}\right]^{0.5}\right\}^{0.6} \tag{13}$$

Now, in uniform flow $S_0 = S_f$ so it is the terms to the right of S_0 in Equation 8 or 12 which produce a looped rating curve. Loop geometry will differ from the rigid boundary case noted before because the second and third terms to the right of S_0 in Equation 12 are different from their counterparts in Equation 6. Alternatively, compare Equation 8 (sediment bed) and Equation 6 (rigid boundary) directly.

Finally, Equation 12 provides a theoretical basis for interpreting results from the physical model and for deriving a procedure for extending stage-discharge relationships beyond directly measured values.

Model Requirements

The model must represent as closely as possible the stage-discharge relationship at the lower Waimakariri Gorge. Canterbury Regional Council maintains a water-level recorder at this site, but the rating curve is indeterminate because of bed level variation due to deep scouring of the gravel bed by flood flows. It is this bed scouring that the model is intended to represent, so that the stage-discharge relationship in the model can be confidently scaled up to the prototype, and the degree of variation inherent in the Waimakariri's stage-discharge relationship can be assessed.

The rate of scour in a given gravel river bed depends upon the flow. If the flow rate is varying, as is the case with flood flows, the scour geometry depends on the rate of scour in relation to the rate of change of flow. It is thus important that the model bed scours at the same rate, relative to the time characteristics of the flood hydrograph, as in the prototype. The *time scale of bed deformation* is crucial to such a model. The relationship between this time scale and linear scale in such a model is not well-defined, so an experiment was carried out to investigate this.

The other scale ratios of the model are reasonably well-defined so that extrapolation of model behaviour to the prototype is not a major problem in principle. It is, however, necessary to demonstrate the capability of the model to reproduce known aspects of prototype behaviour.

Model Scales

The scale ratio between corresponding model (m) and prototype (p) quantities is represented by the symbol λ. In this report

λ_l is the linear scale ratio $= l_p/l_m$

similarly $\lambda_x = x_p/x_m$ = linear scale in the horizontal plane

$\lambda_y = y_p/y_m$ = linear scale in the vertical plane.

(In a vertically exaggerated or distorted model $\lambda_x \neq \lambda_y$; in a non-distorted model $\lambda_x = \lambda_y = \lambda_l$)

and λ_{Gs} = volumetric sediment transport rate scale

λ_V	=	volume scale
λ_Q	=	discharge scale
λ_d	=	sediment grain diameter scale = λ_y
λ_y	=	flow depth scale
λ_{tw}	=	time scale of water motion
λ_{tb}	=	time scale of bed deformation

In order to represent a significant length of river upstream and downstream of the Gorge (total river length approximately 3.5 km), and fit this into the 20 m length of the available modelling flume, a horizontal scale of 1 : 180 was chosen. The vertical scale needed to be less than this so that

(a) water surface elevations in the model could be measured with useful precision,
(b) surface tension effects were negligible, and
(c) rough turbulent flow occurred in the model at times and places at which sediment was in motion.

Accordingly a vertical scale of 1 : 70 was chosen, giving a relatively low vertical exaggeration of 2.6. The alignment of the river bed downstream of the Gorge was changed by approximately 15° in order to fit the model within the 3 m width of the flume. Figure 2 shows the modelled reach in plan.

Table 1 shows the Froude law scales of other quantities that result from this choice of linear scales.

TABLE 1

MODEL SCALES

λ_x	180
λ_y	70
λ_Q	105,400
λ_{Gs}	105,400
λ_s	2.6 (Channel slope - see below)
λ_d	70

Although the model slope ought, in theory, to be exaggerated by a factor of 2.6, it was found in practice that this gave unacceptably high model Froude numbers in the range 0.95-1.4 compared with field volumes of 0.7-0.8. The reason for this problem is not clear, but it is perhaps simplistic to assume that in models with a vertical exaggeration of n, the model slope should also be exaggerated by n. Clearly, to obtain Froude number similarity, model slope needs to be less than the theoretical value. Model slope was chosen by running a prototype hydrograph through the model at various slopes, and choosing the slope that gave the best representation of

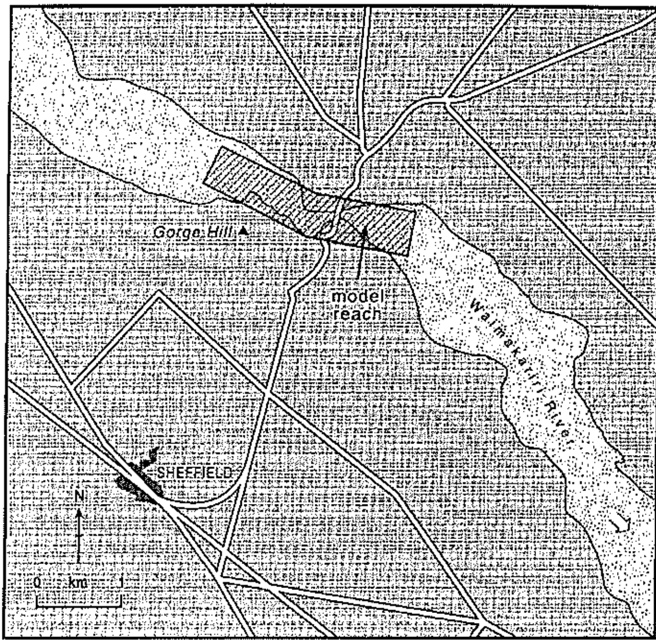


Figure 2 Map of the Waimakariri River at Lower Gorge, showing the modelled reach (hatched box). Flow is from northwest to southeast (left to right).

the measured prototype stage-time curve. By this empirical method the overall model bed slope was chosen to be 0.9%, a vertical exaggeration of 2.25 times rather than 2.6.

Time-scale of Bed Deformation

Kobus (1980) and Novak and Cabelka (1981) both deduce that in a distorted model

$$\lambda_{tb} = \frac{\lambda_x^{2.5}}{\lambda_y^2}$$

implying that in an undistorted model where $\lambda_x = \lambda_y = \lambda_l$

$$\lambda_{tb} = \lambda_l^{0.5}$$

which is the same as λ_{tw} in a Froude-law model. Breusers and Raudkivi (1991), however, report laboratory data on local bedscour that imply

$$\lambda_{tb} = \lambda_l^2 \, \lambda_{(V-Vcr)}^{-4.3}$$

where $\lambda_{(V-Vcr)}$ is the scale of excess flow velocity over that needed to move sediment. If V is large with respect to V_{cr} then

$$\lambda_{(V-Vcr)} \approx \lambda_V \approx \lambda_l^{0.5}$$

in a Froude-law model and

$$\lambda_{tb} = \lambda_l^2 \, (\lambda_l^{0.5})^{-4.3} = \lambda_l^{0.15}$$

which is very close to λ_l^0 implying that $\lambda_{tb} \simeq 1$ when flow conditions are well above motion threshold, i.e., scouring times will be the same in model and prototype.

Clearly there is significant doubt as to which, if any, of these λ_{tb} values are correct. To try to resolve this doubt an experiment was undertaken to measure the rate of bed deformation under a series of geometrically and hydraulically identical conditions at different scales.

A 50 mm wide, 5 m long perspex-walled channel was used, fed with mains water through an adjustable valve. Local scour conditions were established at five different linear scales, each of which corresponded to a different grain size of sediment, so that the development of scour behind a lateral sill could be measured as a function of time at five different scales. Flow quantities in each case were scaled according to the Froude law, and a solid upper flow boundary was used to suppress surface waves. The rigid flume bed upstream of the sill, and the upper flow boundary, were roughened with sediment identical to that being used in each case. The only flow quantity not scaled was the flow width : depth ratio. Figure 3 illustrates the results. T_c is a characteristic time of scour development - in this case the time taken for the maximum scour depth to increase from five to ten grain diameters - and d is the grain diameter, or characteristic linear scale. Figure 3 indicates behaviour which is apparently well-defined but clearly complex. Several interpretations are possible, but this seems to be a field requiring further detailed research.

It is concluded that, as recommended by Kobus (1980) and Novak and Cabelka (1981), the time scale of bed deformation is best adjusted empirically in an unsteady-flow model; in other words, once the linear scales have been chosen, the model should be run at various hydraulic time scales until its behaviour best represents the known prototype behaviour. This was done in the present study; the time-scale of the input hydrograph, t_w, was varied and the time-scale that best represented prototype behaviour chosen. This was $t_w = 90$. (It is interesting that, using the present model scales of $\lambda_x = 180, \lambda_y = 70$, the theoretical value of λ_{tb} by Kobus (1980) and Novak and Cabelka (1981) is 88.7.)

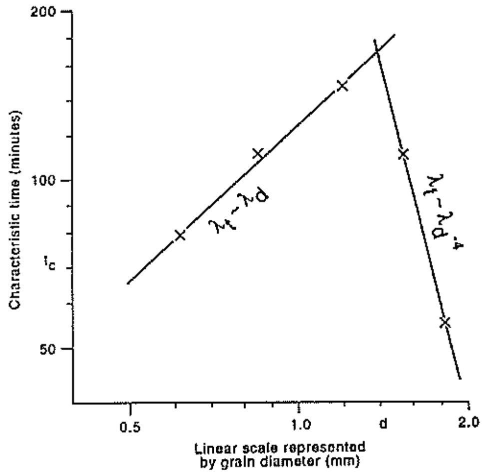


Figure 3 Results of the experiment to determine the characteristic scour time as a function of linear scale. The characteristic scour time is the time taken for the maximum scour depth to increase from five to ten grain diameters and the linear scale corresponds to the grain-size of the bed sediment.

Model Flows

Model hydrograph flows were generated at the upstream water inlet by manually adjusting the pump discharge valve to follow a precalculated time-series of readings on a mercury manometer connected to a calibrated venturi meter in the discharge line. Both sediment and water were recirculated by a 150 mm centrifugal pump.

Bed Sediment

This was scaled down according to λ_y from the prototype grain-size distribution; the model sediment was truncated at about d = 0.6 mm to avoid ripple development which would be unrepresentative of prototype behaviour.

Model Construction

The bedrock Gorge walls, and major upstream terraces, were modelled using expanded polystyrene covered with rough mortar. The Gorge bridge piers were accurately modelled. A perspex window in one wall allowed bed scour at the bridge section to be seen.

General

There is considerable uncertainty about what river flows are capable of moving bed sediment in the Waimakariri Gorge. Hence we do not know how long sediment motion continues after a hydrograph peak; this is something that is probably not well represented in the model since the cessation of sediment motion involves relatively low depths and bed shear stresses, and therefore low model grain-size Reynolds' numbers. It is thus possible that the initial conditions of bed topography in the Gorge at the onset of a flood event, which result from bedload motion since the last event, might not be well represented in the model.

Representation of Prototype Behaviour

Prototype data are scarce in this case. There is a large quantity of stage-time data from the Gorge water-level recorder, but in only one case is discharge information available. Discharge information was generated by back-routing a hydrograph measured at the SHW 1 bridge site, 40 km downstream. This calculated hydrograph is used in the model study, and calibrating the model consists of obtaining the correct scaled stage-discharge curve when the scaled hydrograph is run through the model.

Model Tests

A series of 38 tests was undertaken in order to

(i) establish the optimum time-scale for the model hydrograph;

(ii) establish the optimum model bed slope. The criteria for this were, firstly, correspondence between scaled model water level difference between 1000 m^3/s and the maximum flow of 2457 m^3/s and the water level difference between these flows given by the computed back-routed Gorge hydrograph; and, secondly, the general appearance and Froude number of flow during the passage of the hydrograph;

(iii) establish, by repeated runs at the same time-scale and slope, the degree of variability in the model stage-discharge relationship at the Gorge. Thus 15 of the tests (nos.23-37) were run at a time scale of 90 and a bed-slope of 0.9%.

The initial bed conditions for these tests were those resulting from the previous test, and so were effectively random; it is thought that much of the variability of the stage-discharge relationship results from variation of initial conditions.

Measurements

Water surface level and bed level were measured to a precision of ± 1 mm using a dip-scale at the model water level recorder section and at a section upstream of the bridge. A stilling well at the recorder site was also read. Video films were taken to record changes in flow pattern during the hydrograph.

Results

Table 2 summarises the significant aspects of the test data. Representative stage-time and discharge-time curves are shown in Figure 4. For comparison, the stage-time and discharge-time curves for the prototype backrouted hydrograph are shown in Figure 5.

The model did reproduce one peculiarity of prototype behaviour, viz., the large oscillations in water surface level recorded at flood peaks (Goring, 1988), indicating that bed configurations and their motion were adequately modelled.

A number of points arise from scrutiny of the data.

(a) There is a significant degree of scatter in the relationship between water level and flow rate in the model. Peak flow stages range from 230 to 240 mm, a range of 10 mm which is about ± 8% of the average flow depth at peak flow (Table 2).

(b) The degree of variation in mean water depth at peak discharge is similar to that of water surface elevation - ± 10% approximately. Hence flow depth is not a better measure of peak flow than is water surface elevation.

(c) While flow is *increasing*, i.e., on the rising limb of the hydrograph, the variation in water surface elevation is remarkably small, given the lack of control on initial conditions. At 1322 m^3/s, model stage ranges from 224 mm to 230 mm; at 2000 m^3/s, from 228 mm to 236 mm. These are variations of up to about ± 5% of flow depth; strangely, on the rising limb, water surface elevation at a given flow rate is more consistent than flow depth. Variations in water surface elevation are generally twice as great on the falling limb and at the peak as on the rising limb (Table 2).

(d) Water surface elevations at the existing water level recorder site downstream of the bridge are very significantly less than those at the model section upstream of the bridge. The combined variation of the two sites suggests that the use of local water surface slope as an additional variable would not clarify the situation. Findings (a)-(c) indicate that the degree of imprecision in flow rates estimated from stages is not greater than that of flows estimated from measured depths. Hence the effort involved in measuring flow depth rather than water surface elevation may not be justified.

(e) Flows can be estimated from water surface elevations on the rising limb of a flood with about half the imprecision associated with estimation of peak flows. Since the slope of the approximately linear stage-discharge relationship (Fig.4) is about 1 mm of model stage for every 100 m^3/s ($Q > 1000$ m^3/s), then the imprecision of flow estimate from water level on the rising limb is about ± 300 m^3/s and on the falling limb and at peak flow about ± 600 m^3/s. Estimates based on water depth have possible errors of ± 500 m^3/s or greater.

(f) On the evidence of the present tests this imprecision cannot be reduced significantly by measuring further variables. Bed level is the most likely variable to refine the estimation of flow rate, since one would expect flow rate to relate better to flow depth than to water surface elevation; however, this does not seem to be the case. Presumably the complexities of local variations of bed topography, channel bends and the passage of gravel bars contribute to this unexpected result.

(g) The variability shown by repeated model tests is a combination of intrinsic variation and measurement error. It is estimated that ± 1 or 2 mm could be error in measuring water surface level; the rest is intrinsic variation. The corresponding measurement error in the prototype would be ± 70 or 140 mm, which seems reasonable. Thus there seems no reason to expect variability in the prototype stage-discharge curve to be markedly different to that in the model.

Application of Theory

Data from the physical model indicate that the order of magnitude, 0[ ] of each of the terms in Equation 8 has a typical value of:

$$S_o = 0[10^{-3}], \quad \frac{1}{g}\frac{\partial u}{\partial t} = 0[10^{-6}], \quad \frac{6+F^2}{7u}\frac{\partial y}{\partial t} = 0[10^{-5}]$$

$$\frac{\alpha(1-F^2)}{A}\frac{\partial z}{\partial t} = 0[10^{-3}]$$

For practical purposes the second and third of these terms may be neglected as small. With this approximation Equation 8 becomes

$$S_f \approx S_o - \frac{\alpha(1-F^2)}{A}\frac{\partial z}{\partial t} \tag{14}$$

Table 2

VARIATIONS IN STAGE DURING HYDROGRAPH

	Water Surface Level (mm) at (discharge m^3/s equivalent)					
Test No.	1322↑	2000↑	Peak	2000↓	1300↓	
23	229	231	236	233	230	
24	228	233	238	231	224	
25	230	234	240	237	226	
26	227	234	238	234	228	
27	228	233	238	228	221	
28	228	-	238	-		(Data missing)
29	228	236	240	236	224	
30	225	230	234	233	225	
31	225	229	236	229	220	
32	226	230	233	226	223	
33	228	234	233	227	216	
34	228	234	230	226	223	
35	24	228	231	229	221	
36	226	231	233	228	221	
37	227	231	233	230	219	
Range	± 3 mm	± 3 mm	± 5 mm	± 5.5 mm	± 7 mm	
% Depth	± 5%	± 5%	± 8%	± 9%	± 12%	

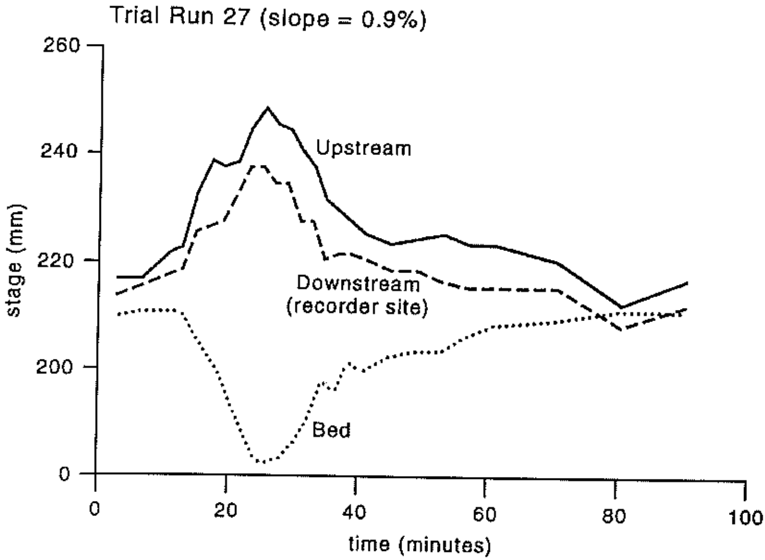
and using Equation 14 we may express Equation 12 as

$$h = z = \left\{ nQ / B \left[S_o - \frac{\alpha(1 - F^2)}{A} \frac{\partial z}{\partial t} \right]^{0.5} \right\}^{0.6} \qquad (15)$$

which is an approximate formula for the stage-discharge relationship for flood wave passage through a channel contraction with scour and fill of the sediment bed.

In Equation 15, $\partial z/\partial t < 0$ on the rising stage of a flood and $\partial z/\partial t > 0$ on the falling stage. Hence for a given discharge the depth will be less on the rising stage than on the falling stage, other parameters being more or less equal; and a loop-rating curve will be traced out in the stage-discharge plane. This loop-rating will vary from flood to flood with changing parameter values, notably of $\partial z/\partial t$ and A for a given depth. At some field sites the amount of variation may be small; nevertheless Equation 15 defines a family of loop ratings.

In flood forecasting from an upstream flow gauging station, water



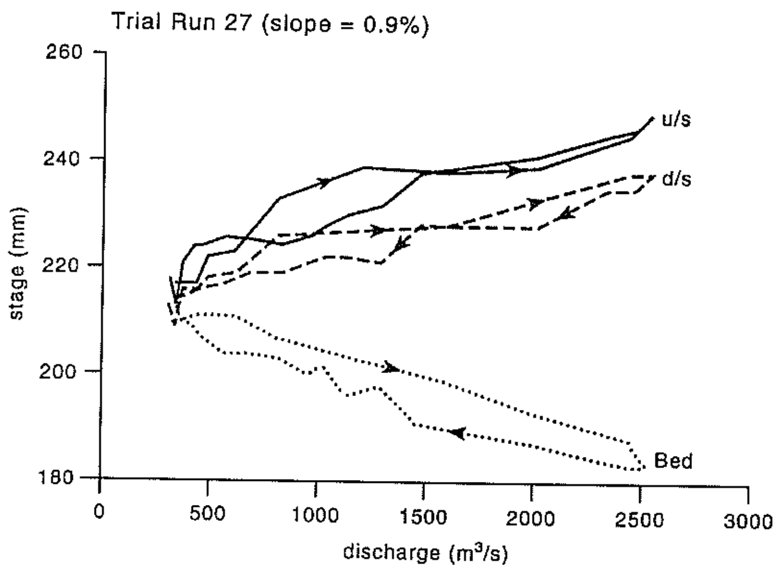
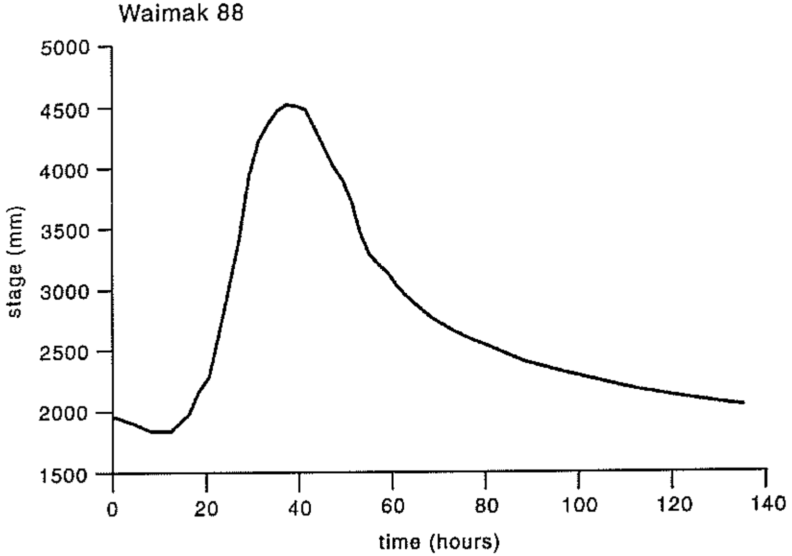


Figure 4 Representative stage-time and stage-discharge curves for the model results. Results for Trial 27.



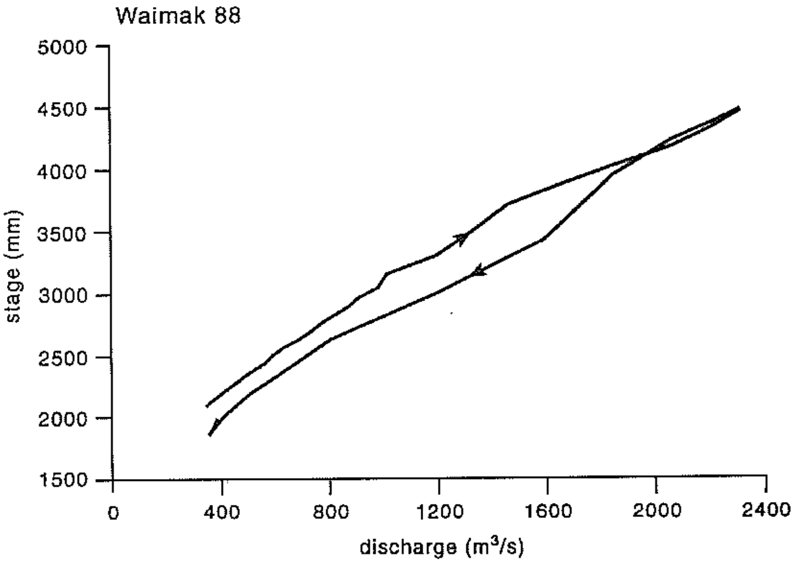


Figure 5 Stage-time and stage-discharge curves for the prototype backrouted hydrograph.

discharges at high stages up to the time of flood peak, or shortly thereafter are of most concern. The question here is what guidance does Equation 15 provide for extrapolating stage-discharge relationships or loop-rating curves from, say, medium rising stage to the flood crest, and perhaps beyond. Obviously one must continue, as in current practice, to measure stage with time h(t). Also, z(t) must be monitored, at least at discrete intervals, and a relationship of, for instance, h(A) needs to be developed from low to medium stage flow gauging data so that A may be predicted for given h by extrapolation. Data from the physical model also indicated that both F and n increase systematically and slowly with h: the relationships h(F) and h(n) should also be determined empirically from gauging data in the channel reach.

In practice then, the known or readily measured constants are α, τ_c, B, d, g, n_p, S_0, and S_s; values of A, n and F for given h are predicted from h(A), h(n) and h(F); and h(t) and z(t) are measured directly during flood passage at the relevant gauging station. With this information Equation 15 will provide an estimate of Q for given h at high stage. Iteration will be necessary in the solution of Equation 15 between the value of Q implied by an estimate of $F = Q^2/g^{0.5}By^{1.5}$ and that given by Equation 15.

This suggested procedure requires further model testing and field trials. During the physical model experiments it was not feasible to measure g_v because of the small scale of the flow and the large spatial and temporal variation of g_v evident from observation. Accordingly the bedload transport formula (Equation 5) was not calibrated, which raises the question of whether the constant multiplier of 8 is the optimum value for the experiments. Without this calibration Equation 15 could not be fully tested using physical model data.

An alternative procedure for stage-discharge prediction would be to employ Equation 13. Order of magnitude analysis using the experimental data results in the practical formula

$$h \approx z + \left\{ nQ/B \left[\left(\frac{6+F^2}{6} \right) S_w + \frac{7\alpha F^2}{6A} \frac{\partial z}{\partial t} \right]^{0.5} \right\}^{0.6} \qquad (16)$$

where $S_w = -\partial h/\partial x$ is the water surface slope, a quantity which can normally be readily measured in a flow gauging reach (Hicks and Mason, 1991). The important difference in application between Equations 15 and 16 is that Equation 15 requires measurement of S_0, as opposed to S_w in Equation 16. Whether S_0 can be estimated from a channel reach at low flow stage and taken as a constant (whose value would be checked from time to time) from flood to flood, and during a flood when scour and fill occurs, is an open question requiring more model and field testing. Use of Equation 16 and measurement of S_w during floods of course averts this problem.

Conclusions

(a) The hydraulic model of the Waimakariri River was able to reproduce the inferred behaviour of the prototype during the passage of a flood wave through the lower Gorge. Due to difficulties in reconciling theory and actuality in selecting model bed slope and time scale, the model results might not be very precise indicators of prototype behaviour. A great deal more prototype data on stage and discharge variation would be needed to improve this situation.

(b) The hydraulic model tests revealed significant variability in the stage-discharge relationship at the Waimakariri Gorge. This was up to about ± 300 m^3/s for a given stage on the rising limb above 1000 m^3/s, and up to about ± 500 m^3/s at the peak and on the falling limb.

(c) Measuring flow stage on the rising limb of a flood wave at the Waimakariri Gorge is the best practical way of quantifying the event for predicting flood wave characteristics further downstream. Measuring flow depth at the Gorge is a less reliable way of quantifying the event. Eq.(16), however, suggests that measuring flow depth, water surface slope and bedload would allow prediction of the stage-discharge relationship to the degree of precision indicated above.

(d) The intrinsic variability evident in the physical model results suggests that significant simplifications could be made to the theory with little loss of predictive power.

(e) Physical hydraulic modelling, in conjunction with theoretical analysis, has been shown to be of value in ascertaining the variability intrinsic to the stage-discharge relationship of a braided river gorge, especially at high flows. This information is very useful for flood warning systems.

Future Work

The immediate future requirement is to clarify the nature of the sediment transport relationship in flow constrictions, i.e., the bedload formula (Eq.5) needs to be calibrated. Work is presently under way to investigate this point in the laboratory using a symmetrical contraction in a rectangular flume under steady flow. This apparatus will also be used to study the variation of S_0 during unsteady flow, and field measurements at the Waimakariri Gorge will provide an interesting comparison.

Acknowledgements

Model development and testing has been carried out by Rein Visser and Min Jian, whose very substantial contributions are gratefully acknowledged.

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