Journal of Hydrology (NZ) 45 (1): 1-12, 2006

A stochastic spatial-temporal disaggregation model for rainfall

P. S. P. Cowpertwait[1], **T. Lockie**[2] **and M. D. Davis**[3]

[1] *Institute of Information and Mathematical Sciences, Massey University at Albany, Auckland, New Zealand. Corresponding author: P.S.Cowpertwait@massey.ac.nz*

[2] *Meritec Limited, Newmarket, Auckland City, New Zealand*

[3] *Metro Water Limited, Mt Roskill, Auckland City, New Zealand*

Abstract

A stochastic model for disaggregating spatial-temporal rainfall data is presented. In the model, the starting times of rain cells occur in a Poisson process, where each cell has a random duration and a random intensity. In space, rain cells have centres that are distributed according to a two-dimensional Poisson process and have radii that follow an exponential distribution. The model is fitted to seven years of five-minute data taken from six sites across Auckland City. The historical five-minute series are then aggregated to hourly depths and stochastically disaggregated to five-minute depths using the fitted model. The disaggregated series and the original five-minute historical series are then used as input to a network flow simulation model of Auckland City's combined and wastewater system. Simulated overflow volumes predicted by the network model from the historical and disaggregated series are found to have equivalent statistical distributions, within sampling error. The results thus support the use of the stochastic disaggregation model in the intended application.

Keywords

Point processes, rectangular pulses, urban hydrology, multivariate time series.

Introduction

A method based on a Neyman-Scott process was developed for simulating spatial-temporal rainfall time series data and was tested using historical records of hourly data from the Arno Catchment in Italy (Cowpertwait *et al.*, 2002). The results indicated that the model was able to preserve extreme values at the 1- and 24-hour levels of aggregation, thus supporting its use in hydrological catchment studies, e.g., flood studies. A special case of the model is considered here for disaggregating hourly data to five-minute data.

A range of stochastic models are used for disaggregating rainfall data. For example, there are models for downscaling the output from deterministic global circulation models (e.g., Skaugen, 2002; Venugopal *et al.*, 1999; Charles *et al.*, 1999; Mehrotra and Singh, 1998; Lebel *et al.*, 1998; Gao and Sorooshian, 1994). Some models are aimed more specifically at producing fine-resolution data, e.g., for urban catchment studies (e.g., Hingray *et al.*, 2002; Cowpertwait, 2001; Durrans *et al.*, 1999; Cowpertwait *et al.*, 1996a,b; Koutsoyiannis, 1994; Koutsoyiannis and Onof, 2001; Koutsoyiannis *et al.*, 2003; Kottegoda *et al.*, 2003; Ormsbee, 1989). The model that we consider here is in a class of conceptual-stochastic models, initially developed by Rodriguez-Iturbe *et al.*

(1987) and Cox and Isham (1988), which incorporate random variables to represent 'rain cells' that are understood to occur in physical rainfall processes. Furthermore, this model can be applied in both space and time, which increases the range of possible applications. For example, the model could be used for generating spatially representative data at sites lacking data, i.e., spatial-temporal infilling.

The methods used here are similar to those used by Glasby *et al.* (1995), Onof *et al.* (1996) and Bo *et al.* (1994), in that 'within storm' rain cells will have arrival times that occur in a Poisson process. However, these papers apply a Bartlett-Lewis process to univariate rainfall time series, whilst our focus here is on disaggregating spatial-temporal data to fine resolutions.

Auckland City catchment

The Auckland region is in the North Island of New Zealand, with mainland Auckland City occupying 15,300 hectares on an isthmus between the Waitemata and Manukau Harbours (Fig. 1). Auckland has considerable industrial activity and an extensive roadway system, and is the largest city in New Zealand, with a population of 368,000, extending to 1.2 million people in the greater Auckland area. Auckland City is forecast to grow to a population of 583,000 over the next 50 years (Auckland Regional Council, 1999).

There are three types of piped drainage networks in Auckland City: wastewater, stormwater and combined. Combined stormwater and wastewater networks remain in 18% of the city (Auckland City Council, 2003). The assemblage results in two interrelated drainage systems that remove: (i) stormwater from within Auckland City to streams and harbours where it is discharged, and (ii) wastewater and combined flows to the Mangere wastewater treatment plant. In all, approximately 2,500 outfalls discharge potentially impaired stormwater along 82 km of coastline and into numerous urban streams, including discharge from some 350 designed overflow structures on the combined and wastewater networks (Auckland City Council and Metro Water Limited, 2001).

To fulfill environmental regulatory requirements and meet the demands of the rising population, Auckland City Council and Metro Water Limited initiated an integrated catchment study. The objective of the study is to develop a comprehensive understanding of the Auckland City drainage system to enable engineers to redesign and upgrade the system to meet regulatory requirements that minimize the pollution to receiving watercourses and flooding due to excess loads.

A network model of the trunk and larger wastewater and combined pipes was assembled using the 'MOUSE' hydraulic model developed by the Danish Hydraulics Institute (2002a,b). The network model consists of some 3,122 manholes, 236 overflows (structures, pump stations, and lumped overflows) and 568 sub-catchments (Fig. 1; Healey and Carne, 2001).

An understanding of rainfall is crucial to the success of the integrated catchment study. Available records from sites in Auckland City include five-minute data from six sites for the period 1993-99 (Fig. 1). As the historical data is limited, a spatial-temporal stochastic model (Cowpertwait *et al.*, 2002) is proposed for simulation of long records of multi-site 1-hour data that could be used as input to the network model to assess system performance under a range of conditions. However, rainfall data at finer resolutions than 1 hour are needed because of the rapid response of urban drainage systems to rainfall events. Consequently, a stochastic disaggregation model is required to disaggregate simulated 1-hour series to five-minute series. A similar approach was adopted for single sites in the United Kingdom, where a disaggregator

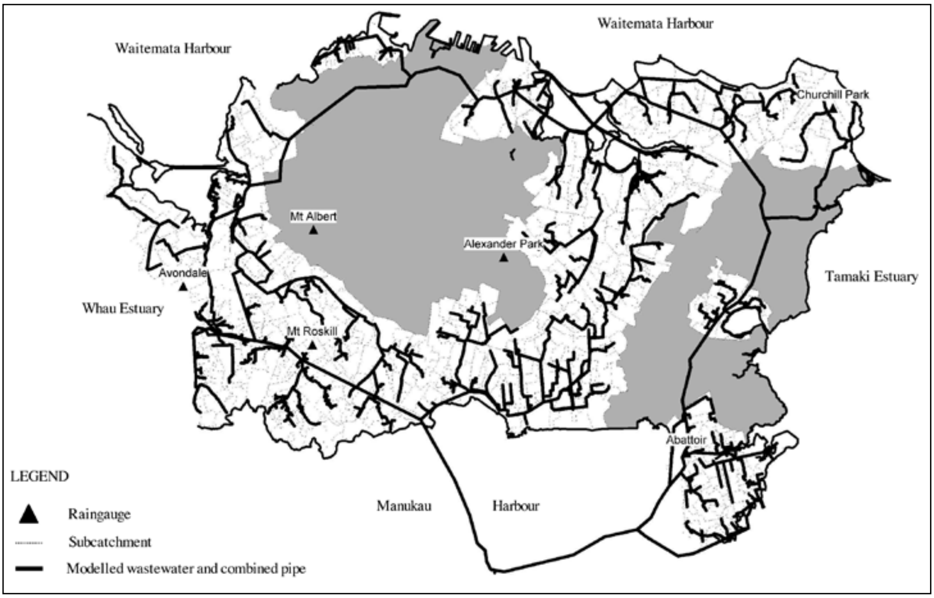

Figure 1 – Auckland city catchment: network model and location of rain gauges (records are for the period 1993–1999). Shaded areas correspond to areas in which only the main trunk pipes were modelled.

based on an algorithm developed by Ormsbee (1989) was used to disaggregate simulated hourly time series (see Cowpertwait *et al.*, 1996b). The focus in this paper is to propose a suitable model for stochastically disaggregating multisite 1-hour data and to validate the model using a wastewater and combined network model for Auckland City.

Model definition

Consider a stochastic process of rain cells:

$$\{(U_i, V_i), S_i, L_i, X_i, R_i\},$$

where for the i th cell (U_i, V_i) forms a two-dimensional Poisson process with rate φ (per km^2). (U_i, V_i) and R_i form discs in two-dimensional space, where (U_i, V_i) is the disc centre and R_i is an independent random variable representing the disc radius. S_i is the arrival time of the cell, which occurs in a Poisson process with rate λ. L_i is an independent random variable representing the cell lifetime, so that the cell terminates at a time $S_i + L_i$. X_i is an independent random variable representing cell intensity, which remains constant throughout the cell lifetime and over the area of the disc. Rain cells can thus be thought of as cylinders in three-dimensional space with heights given by X_i. Furthermore, the total intensity at time t and location $\mathbf{x} \in \mathbb{R}^2$, denoted as $Y(\mathbf{x}, t)$, is the sum of the intensities of all cells active at time t and overlapping point $\mathbf{x}$. The stochastic process of rain cells is similar to that used by Cowpertwait *et al.* (2002), with the exception that the cells arrive in a Poisson process instead of a Neyman-Scott process.

For model fitting and simulation, some assumptions are made about the distributions of the random variables used for the rain cells: X_i is taken to be an independent Weibull random variable with parameters θ and α and survivor function $P(X_i > x) = e^{(x/\theta)^{\alpha}}$; the cell lifetime L_i and radius R_i are taken to be independent exponential random variables with parameters η and ϕ respectively. Under these assumptions, $\phi^2 = 2\pi\varphi^2$, which reduces the number of parameters in the model to five: λ, ϕ, η, θ, and α.

Rainfall data are usually available as discrete time-series, so that it is necessary to consider the aggregated stochastic process:

$$Y_k^{(h)}(\mathbf{x}) = \int_{(k-1)h}^{kh} Y(\mathbf{x},t)\,dt \tag{1}$$

so that $Y_k^{(h)}(\mathbf{x})$ is the rainfall depth in the k th time interval of duration h at location $\mathbf{x}$.

Statistical properties of $Y_k^{(h)}(\mathbf{x})$, up to third-order, follow directly by taking C ≡ 1 (Cowpertwait *et al.*, 2002, equations (5)-(8)). These properties then follow as:

$$\mu_h = E\left\{Y_k^{(h)}(\mathbf{x})\right\} = \lambda E(X)h/\eta \tag{2}$$

$$\gamma_{\mathbf{x},\mathbf{y},h,l} = \mathrm{cov}\left\{Y_k^{(h)}(\mathbf{x}), Y_{k+l}^{(h)}(\mathbf{y})\right\} = P(\phi,d)\,\gamma_{\mathbf{x},\mathbf{x},h,l} \tag{3}$$

$$E\left[\left\{Y_k^{(h)}(\mathbf{x}) - \mu_h\right\}^3\right] = 6\lambda E(X^3)(\eta h - 2 + \eta h e^{-\eta h} + 2e^{-\eta h})/\eta^4 \tag{4}$$

where:

$$\gamma_{\mathbf{x},\mathbf{x},h,l} = \gamma_{\mathbf{y},\mathbf{y},h,l} = 2\lambda E(X^2)A(h,l)/\eta^3 \tag{5}$$

$$A(h,l) = \begin{cases} (h\eta + e^{-\eta h} - 1) & \text{for } l = 0 \\ \frac{1}{2}(1 - e^{-\eta h})^2 e^{-\eta h(l-1)} & \text{for } l > 0 \end{cases} \tag{6}$$

$$P(\phi,d) = \frac{2}{\pi}\int_0^{\pi/2}\left(\frac{\phi d}{2\cos y} + 1\right)e^{-\phi d/(2\cos y)}\,dy \tag{7}$$

$P(\phi, d)$ is the probability that a cell overlaps some point $\mathbf{x}$, given that it overlapped a point $\mathbf{y}$, at spatial separation $d = \|\mathbf{x} - \mathbf{y}\|$. The moments of the Weibull distribution for the X_i are given by: $E(X^r) = \theta^{r/\alpha}\Gamma(1 + r/\alpha)$.

In the above, spatial-temporal stationarity is assumed, e.g., $\gamma_{\mathbf{x},\mathbf{x},h,l} = \gamma_{\mathbf{y},\mathbf{y},h,l}$ in equation 5.

Fitted model

When fitting the model to five-minute time series (i.e., with h = 5 minutes in equations 2–4), it is convenient to work with the following dimensionless functions, which do not depend on the cell intensity scale parameter θ:

Coefficient of variation: $v(\lambda, \eta, \phi, \alpha) = \gamma_{\mathbf{x},\mathbf{x},5,0}/\mu_5$

Autocorrelation (lag 1): $\rho(\lambda, \eta, \phi, \alpha) = \gamma_{\mathbf{x},\mathbf{x},5,1}/\gamma_{\mathbf{x},\mathbf{x},5,0}$ (8)

Cross-correlation: $\rho_{\mathbf{x},\mathbf{y}}(\lambda, \eta, \phi, \alpha) = \gamma_{\mathbf{x},\mathbf{y},5,0}/\gamma_{\mathbf{x},\mathbf{x},5,0}$

Coefficient of skewness: $\kappa(\lambda, \eta, \phi, \alpha) = E\left[\left\{Y_k^{(5)}(\mathbf{x}) - \mu_5\right\}^3\right]/\gamma_{\mathbf{x},\mathbf{x},5,0}^{3/2}$

To obtain the sample estimates of these functions, data from the six sites (Fig. 1) were pooled and the equivalent sample estimates calculated. Hours containing zero rainfall at all sites were removed before the calculation of the sample statistics so that the estimates were for 'wet' hourly sequences only. (The sample autocorrelation was calculated using adjacent values from 'unbroken' wet hourly series, using the sample mean and variance from all the pooled data.)

The sample estimates were calculated using an approach similar to that in Cowpertwait *et al.* (2002), with the exception that one sample estimate was used for all seasons, so that the same fitted disaggregation model would be applied over all calendar months. On first sight this may seem an unwarranted over-simplication. However, it is reasonable to assume that most of the seasonal variation in rainfall data is captured at higher levels of aggregation (e.g., hourly or daily levels), which will be preserved in the disaggregated series.

The parameters λ, α, η, ϕ were estimated by minimising the following sum of squares:

$$SS = \left(1 - \frac{v}{\hat{v}}\right)^2 + \left(1 - \frac{\hat{v}}{v}\right)^2 + \left(1 - \frac{\rho}{\hat{\rho}}\right)^2 + \left(1 - \frac{\hat{\rho}}{\rho}\right)^2 + \left(1 - \frac{\kappa}{\hat{\kappa}}\right)^2 + \left(1 - \frac{\hat{\kappa}}{\kappa}\right)^2 + \sum_{(\mathbf{x},\mathbf{y}) \in \mathbf{A}} \left[\left(1 - \frac{\rho_{\mathbf{x},\mathbf{y}}}{\hat{\rho}_{\mathbf{x},\mathbf{y}}}\right)^2 + \left(1 - \frac{\hat{\rho}_{\mathbf{x},\mathbf{y}}}{\rho_{\mathbf{x},\mathbf{y}}}\right)^2 \right] \quad (9)$$

where **A** is the set of (15) pairs of points corresponding to the locations of the 6 sites.

After substituting the pooled sample mean for μ_h in Equation (2), θ can be estimated directly from:

$$\hat{\theta} = \left\{ \frac{\hat{\mu}_5 \hat{\eta}}{\hat{\lambda}\Gamma(1 + 1/\hat{\alpha})} \right\}^{\hat{\alpha}} \quad (10)$$

Using the above procedure, the following parameter estimates were obtained for the Auckland data: $\hat{\lambda}$ = 0.0721/min, $\hat{\eta}$ = 0.838/min, $\hat{\phi}$ = 0.407/km, $\hat{\alpha}$ = 0.673, and $\hat{\theta}$ = 0.123 mm/min. The sample and fitted statistics are shown in Table 1 and Figure 2, where it can be seen that a reasonable fit is obtained, but with a slight over-estimation of skewness and autocorrelation (Table 1). There was also some slight under-estimation of the sample cross-correlations for those sites with greater spatial separation (Fig. 2). However, these discrepancies were only slight and a very parsimonious model parametrization had been used, so no further improvement in fit was sought.

Table 1 – Fitted Statistics

Statistic	Sample Value	Fitted Value
Mean	0.0708	0.0708
Standard Deviation	3.69	3.77
Skewness	8.95	10.1
Autocorrelation	0.138	0.151

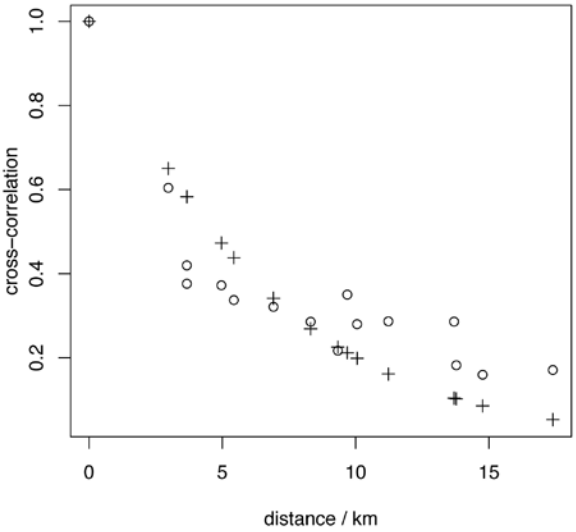

Figure 2 – Fitted (+) and historical (○) cross-correlations against distance.

Disaggregation algorithm

There are many possible implementations of the stochastic model presented which would enable the disaggregation of rainfall data. The approach adopted here attempts to reduce computional demands. Disaggregation is carried out for each wet hour by applying an algorithm which is summarised below. A schematic summary of the algorithm is given in Figure 3.

Constants:

N	Number of sites (6).
NH	Number of hours to disaggregate.
M	Number of intervals in each hour (12 for five-minute data).
$XTOL$	Tolerance parameter (0.5 mm).
WTH	Width of catchment (50 km; the catchment is treated as a square of area WTH^2).

Variables:

H	A counter for the hour being disaggregated.
$Z(i, j, k)$	Simulated data in the kth (five-minute) interval for the jth hour in an adjacent pair ($j = 1, 2$) at the ith site ($N \times 2 \times M$ array).
$Y(i, j, k)$	Simulated data, due to a single rain cell, in the kth (five-minute) interval for the jth hour in an adjacent pair ($j = 1, 2$) at ith site ($N \times 2 \times M$ array).
$T(i, j)$	Total historical rainfall in the jth hour at the ith site (array of size $N \times \{NH + 1\}$).
$G(i, j)$	Simulated total rainfall in the jth hour of an adjacent pair ($j = 1, 2$) at the ith site (array of size $N \times 2$).
$A(i)$	Scaling factor for each site.
S	Starting time of a rain cell.
L	Lifetime of a rain cell.
W	Intensity of a rain cell.
X	x-coordinate for the centre of a rain cell.
Y	y-coordinate for the centre of a rain cell.

Initialise:

$Z(i, j, k) = 0$	$i = 1, \ldots, N$; $j = 1, 2$; $k = 1, \ldots, M$.
$Y(i, j, k) = 0$	$i = 1, \ldots, N$; $j = 1,2$; $k = 1, \ldots, M$.
$G(i, j) = 0$	$i = 1, \ldots, N$; $j = 1, 2$.
$T(i, NH + 1) = 0$	$i = 1, \ldots, N$.

$H = 0$.

Algorithm:

1. Read in the hourly totals $T(i, j)$ for each site ($i = 1, \dots, N$; $j = 1, \dots, NH$).
2. Set $H = H + 1$. If $H = NH + 1$ terminate the algorithm.
3. Set $Z(i, 1, k) = Z(i, 2, k)$ for $i = 1, \dots, N$; $k = 1, \dots, M$. (The five-minute series in the first hour is set to zero plus the rain due to previous overlapping cells.)
4. Set $Z(i, 2, k) = 0$ for $i = 1, \dots, N; k = 1, \dots, M$. (The five-minute series in the second hour is set to zero.)
5. Set $Y(i, j, k) = 0$ for $i = 1, \dots, N$; $j = 1, 2$; $k = 1, \dots, M$. (The five-minute series due to a single cell starts as zero.)
6. Generate variables for a rain cell:
   (a) A starting time S taken from a uniform distribution over the interval $(0,M)$, i.e., $S \sim U(0,M)$. (Note that for a Poisson process the distribution of arrival times over a fixed time interval is uniform.)
   (b) A cell lifetime, $L \sim exp(\hat{\eta})$.
   (c) A cell intensity, $W \sim \text{Weibull}(\hat{\alpha},\hat{\theta})$.
   (d) A cell radius, $R \sim exp(\hat{\phi})$.
   (e) A cell centre, (X, Y);
   $X \sim U(-WTH/2, WTH/2)$, $Y \sim U(-WTH/2, WTH/2)$.
7. Calculate a simulated series $Y(i, j, k)$ at each site due to the generated rain cell ($i = 1, \dots, N$; $j = 1, 2$; $k = 1, \dots, M$). Note that the cell contributes to the simulated series for the ith site if, and only if, the distance from the cell centre to the site is less than (or equal to) the cell radius R. In addition, note that the starting time $S \leq M$ of the cell occurs in the first hour ($j = 1$), but the cell may overlap into the next hour ($j = 2$).
8. Aggregate the simulated (five-minute) series to a pair of hourly totals $G(i, j)$: Set $G(i, j) = \sum_{k=1}^{M} Y(i, j, k)$, for $i = 1, \dots, N$, $j = 1, 2$.
9. If the either of the resultant pair of hourly totals ($j = 1, 2$) obtained by adding the series $G(i, j)$ to the hourly total $\sum_{k=1}^{M} Z(i, j, k)$, exceed either of the historical hourly totals $T(i,H)$ or $T(i,H+1)$ (to within $XTOL$; excluding $H+1$ when $H = HT$) at any site, then discard the generated cell and return to step 5; i.e., return to step 5 if $G(i, j) + \sum_{k=1}^{M} Z(i, j, k) - T(i, H - 1 + j) > XTOL$ for any site i or any $j = 1, 2$.
10. Update the simulated (five-minute) series $Z(i, j, k)$ at each site by adding $Y(i, j, k)$, i.e., set $Z(i, j, k) = Z(i, j, k) + Y(i, j, k)$ for all i and $j = 1, 2$.

11. Repeat steps 5–10 until the hourly series $T(i,H)$ and the aggregated simulated (five-minute) series $\sum_{k=1}^{M}(Z(i, 1, k))$ are closely matched at each site, i.e., repeat steps 5–10 until $|\sum_{k=1}^{M} Z(i, 1, k) - T(i,H)| < XTOL$ for all $i = 1, \ldots, N$.

12. Scale the simulated (five-minute) series to achieve an exact match to the hourly data: Set $Z(i, 1, k) = A(i)Z(i, 1, k)$ where $A(i) = T(i,H)/\sum_{k=1}^{M} Z(i, 1, k)$.

13. Save the disaggregated data $Z(i, 1, k)$ to a file.

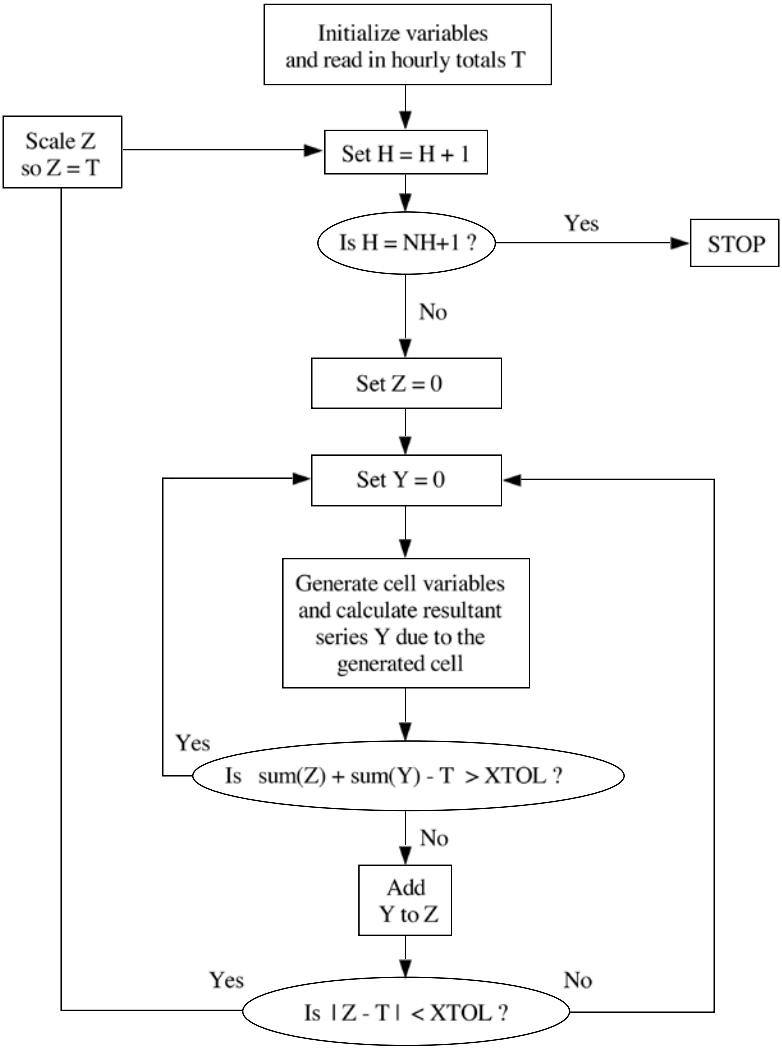

Figure 3 – Flow diagram of stochastic disaggregation algorithm.

In the above algorithm, cells are generated over the catchment area WTH^2. This area is chosen to be larger than the actual drainage area being studied to allow for cells that may have centres outside the drainage area but which overlap the drainage area.

The algorithm does not use $\hat{\lambda}$ and therefore introduces a small bias due to discarding larger rain cells. A more precise algorithm involves repeatedly simulating a random number of cells over the catchment using $\hat{\lambda}$ until a close match to the hourly totals at each site is obtained. However, as this was computationally too demanding, the above algorithm was adopted for its practicality.

Model validation

The historical five-minute series were aggregated to hourly series. These hourly series were then disaggregated to five-minute series using the fitted stochastic model and the disaggregation algorithm.

To verify the disaggregation algorithm, and to assess the bias, sample standard deviations of the simulated five-minute series were compared with the standard deviations of the historical five-minute series at each site (Table 2). A slight underestimation is apparent, which is probably due to the tendency for the retention of smaller cells to achieve a match to the hourly totals (Table 2).

Table 2 – Standard Deviation of Five-Minute Rainfall Series

Site	Historical Series	Disaggregated Series
1	0.0989	0.0965
2	0.109	0.102
3	0.108	0.0991
4	0.0974	0.0930
5	0.108	0.0986
6	0.104	0.0974

However, as the bias is small, the fitted model and implementation were retained for further validation against properties of interest in the intended application.

To validate the model for the intended application, the historical five-minute rainfall time series were input into the pipe network model (Fig. 1) and the spill volumes from each of 236 overflows calculated for each year. In addition, the stochastically disaggregated (simulated) five-minute series was also input into the same network model and the overflow volumes resulting from these series also found for each year. This resulted in 236 spill volumes for each year for both the historical and simulated five-minute rainfall series. Summary statistics for these series are given in Table 3, from which it is evident that the disaggregated series produces results comparable to the historical series.

The null hypothesis that there was no difference between spill volumes generated from the historical series and those generated from the simulated (disaggregated) series was tested using a paired sample t-test on the data for the 236 overflows for the 7-year period (a total of 1652 pairs of differences). The mean difference in spill volumes was 21.1m^3, with a corresponding p-value of 0.19, indicating that there was no statistical evidence for a difference in spill volumes due to the historical and simulated rainfall data.

A Kolmogorov-Smirnov (KS) test was used to compare the historical and simulated distributions of overflow volumes for each

Table 3 – Overflow Statistics (m^3 per year)

Statistics	Historical Series	Disaggregated Series
Largest	4.007×10^5	4.088×10^5
Second Largest	1.693×10^5	1.686×10^5
Mean	1.358×10^3	1.379×10^3
Standard Deviation	1.132×10^4	1.160×10^4

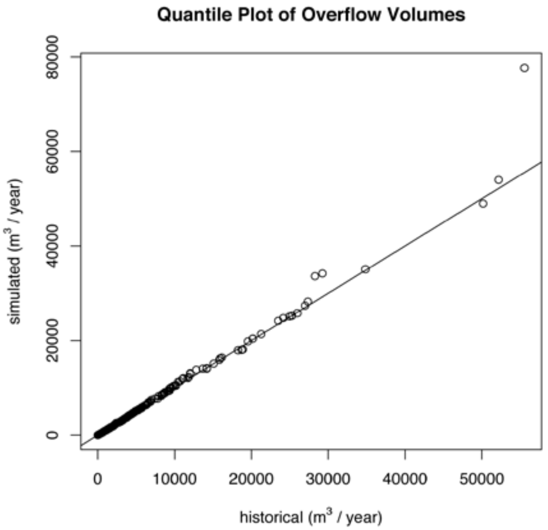

Figure 4 – Quantile plot for historical and simulated overflow distributions; p-value of Kolmogorov-Smirnov test statistic is 0.26.

year (i.e., 7 KS tests, one for each year). Each KS test gave a non-significant p-value, with the worst fit (for 1997) having a smallest p-value of 0.73, which is not statistically significant. In addition, when combining the overflow volumes for all the years together, the KS p-value was 0.26, which is also not significant. Consequently, there was no statistical evidence to reject the null hypothesis that the historical and simulated overflow distributions were the same, providing good support for the use of the stochastic model in the Auckland catchment study. A quantile plot, excluding the largest two values in Table 3, is given in Figure 4, from which it is clear that the simulated overflow distribution closely fits the overflow distribution based on historical rainfall data.

Conclusions

The results indicate that the stochastic model and disaggregation algorithm can be used with confidence for the intended application, as there were no significant differences between the historical and simulated overflow distributions.

There is scope for improving the disaggregation algorithm through using the Poisson arrival rate. However, this is computationally demanding and was not necessary for this application.

The method readily extends to the problem of infilling missing values: the disaggregation algorithm could be modified so that cells are generated until any available data are approximately matched.

A further development of the method would include disaggregating daily records to hourly records. This aspect is currently under investigation.

Acknowledgements

Auckland City Council (ACC) and Metro Water Limited (Metrowater) funded the work as part of the Integrated Catchment Study. Team members from both organisations have contributed to the Integrated Catchment Study, environmental programmes, and work contained in this paper. Several members are thanked for comments to improve the paper. The authors thank ACC and Metrowater for permission to present the paper. Discussion and interpretations are those of the authors of work in progress and should not be construed as those of ACC or Metrowater.

References

Auckland City Council 2003: Stormwater asset management plan 2003/2004. Technical report, Utility Planning, Auckland City Council, Auckland City, New Zealand.

Auckland City Council and Metro Water Limited 2001: Auckland city drainage system resource consents: Assessment of environmental effects. Technical report, Metrowater Ltd, Auckland City, New Zealand.

Auckland Regional Council 1999: Auckland regional growth strategy: 2050. A vision for managing growth in the Auckland region. Technical report, Auckland Regional Council, Auckland City, New Zealand. Available online at http://www.arc.govt.nz.

Bo, Z.; Islam, S.; Eltahir, E. 1994: Aggregation-disaggregation properties of a stochastic model. *Water Resources Research 30(12)*: 3423-3435.

Charles, S.; Bates, B.; Hughes, J. 1999: A spatiotemporal model for downscaling precipitation occurrences and amounts. *Journal of Geophysical Research 104(D24)*: 31657-31669.

Cowpertwait, P. 2001: A continuous stochastic disaggregation model of rainfall for peak flow simulation in urban hydrologic systems. *Research Letters in the Information & Mathematical Sciences 2*: 81-88.

Cowpertwait, P.; Kilsby, C.; O'Connell, P. 2002: A space-time Neyman-Scott model of rainfall: Empirical analysis of extremes. *Water Resources Research 38(8)*: 1-14.

Cowpertwait, P.; O'Connell, P.; Metcalfe, A.; Mawdsley, J. 1996a: Stochastic point process modelling of rainfall: I. Single-site fitting and validation. *Journal of Hydrology 175*:17-46.

Cowpertwait, P.; O'Connell, P.; Metcalfe, A.; Mawdsley, J. 1996b: Stochastic point process modelling of rainfall: II. Regionalisation and disaggregation. *Journal of Hydrology 175*: 47-65.

Cox, D.; Isham, V. 1988: A simple spatial-temporal model of rainfall. *Proceedings of the Royal Society of London A 415*: 317-328.

Danish Hydraulics Institute 2002a: MOUSE Pipe Flow. Reference Manual. Danish Hydraulics Institute, Water and Environment, Hørsholm, Denmark.

Danish Hydraulics Institute 2002b: MOUSE Surface Runoff Models. Reference Manual. Danish Hydraulic Institute, Water and Environment, Hørsholm, Denmark.

Durrans, S.; Burian, S.; Nix, S.; Hajji, A. 1999: Polynomial-based disaggregation of hourly rainfall for continuous hydrologic simulation. *Journal of the American Water Resources Association, 35(5)*: 1213-1221.

Gao, X.; Sorooshian, S. 1994: A stochastic precipitation disaggregation scheme for GCM applications. *Journal of Climate 7(2)*: 238-247.

Glasby, C.; Cooper, G.; McGechan, M. 1995: Disaggregation of daily rainfall by conditional simulation from a point process model. *Journal of Hydrology 165*: 1-9.

Healey, M.; Carne, S. 2001: Project backbone: Phase 1 – wastewater modelling. Technical report, prepared by AWT New Zealand, Meritec Limited, and GHD Limited, Metro Water Limited, Auckland City, New Zealand.

Hingray, B.; Monbaron, E.; Jarrar, I.; Favre, A.; Musy, A. 2002: Stochastic generation and disaggregation of hourly rainfall series for continuous hydrological modelling and flood control reservour design. *Water Science and Technology 45(2)*: 113-119.

Kottegoda, N.; Natale, L.; Raiteri, E. 2003: A parsimonious approach to stochastic multisite modelling and disaggregation of daily rainfall. *Journal of Hydrology 274*: 47-61.

Koutsoyiannis, D. 1994: A stochastic disaggregation method for design storm and flood synthesis. *Journal of Hydrology 156*: 193-225.

Koutsoyiannis, D.; Onof, C. 2001: Rainfall disaggregation using adjusting procedures on a poisson cluster model. *Journal of Hydrology 246*: 109-122.

Koutsoyiannis, D.; Onof, C.; Wheater, H. 2003: Multivariate rainfall disaggregation at a fine time scale. *Water Resources Research 39(7)*: 1-18.

Lebel, T.; Braud, I.; Creutin, J. 1998: A space-time rainfall disaggregation model adapted to Sahelian mesoscale convective complexes. *Water Resources Research 34(7)*: 1711-1726.

Mehrotra, R.; Singh, R. 1998: Spatial disaggregation of rainfall data. *Hydrological Sciences Journal 43(1)*: 91-101.

Onof, C.; Faulkner, D.; Wheater, H. 1996: Design rainfall modelling in the Thames catchment. *Hydrological Sciences Journal 41(5)*: 715-729.

Ormsbee, L. 1989: Rainfall disaggregation model for continuous hydrologic modelling. *Journal of Hydraulic Engineering 115(4)*: 507-525.

Rodriguez-Iturbe, I.; Cox, D.; Isham, V. 1987: Some models for rainfall based on stochastic point processes. Proceedings of the Royal Society of London, Series A 410: 269-288.
Skaugen, T. 2002: A spatial disaggregation procedure for precipitation. *Hydrological Sciences Journal 47(6)*: 943-956.
Venugopal, V.; Foufoula-Georgiou, E.; Sapozhnikov, V. 1999: A space-time downscaling model for rainfall. *Journal of Geophysical Research 104(D4)*: 19705-19721.

Manuscript received 13 October 2005; revised; accepted for publication 3 February 2006