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# Interaction of surface seiche and recording frequency in lake level records (note)

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## Introduction

The surface of a lake does not usually remain still and flat. As well as the wind-generated surface waves, the entire lake surface may oscillate. The vertical movement at a point caused by these oscillations can be quite significant. Maori mythology attributes such oscillations on Lake Wakatipu to the beating of a giant Taniwha's heart. This phenomenon, which is known as seiching, is initiated when the lake surface is tilted, usually by wind friction or barometric gradient. The seiche period is a function of the lake size, geometry and bathymetry.

While seiches of more than one period can act on a lake at a given time, it is likely that the first or fundamental period will cause the greatest fluctuations in the lake surface. Spectral analysis on New Zealand lakes has shown that the fundamental period is likely to be the only significant one (Carter and Lane, *submitted*; Heath, 1975). However on some of the larger and/or more complex lakes (e.g. Te Anau) interactions with higher order seiches may be significant and cause problems in the lake level record.

The fundamental period of seiche for a lake can be estimated; the lake is approximated as a homogeneous body of water of unit width and constant depth. Then using the Merian formula (Wilson, 1972):

$$T = 2L / \sqrt{gh}$$

where $T$ = fundamental period, $L$ = lake length, $g$ = gravitational acceleration, and $h$ = lake depth. More complex mathematical procedures (e.g. Carter and Lane, *submitted*) are available for estimating higher modes of oscillation and for lakes of complex geometries.

In this note, errors in lake level records caused by the interaction of lake seiche and recording frequencies are considered. Records from lakes Coleridge and Ohau (South Island, New Zealand) are used to illustrate the interference, and moving averages are considered as a method for eliminating the problem.

## Aliasing

Lake level recorders record the effect of seiche on the surface of lakes, as seiching involves a tilting of the whole lake surface and is less likely to be damped by the intake pipes of a stilling well recorder than surface waves. Digital recorder measurements are taken at regular intervals. This recording cycle interacts with the seiche process to give beating, in which the two

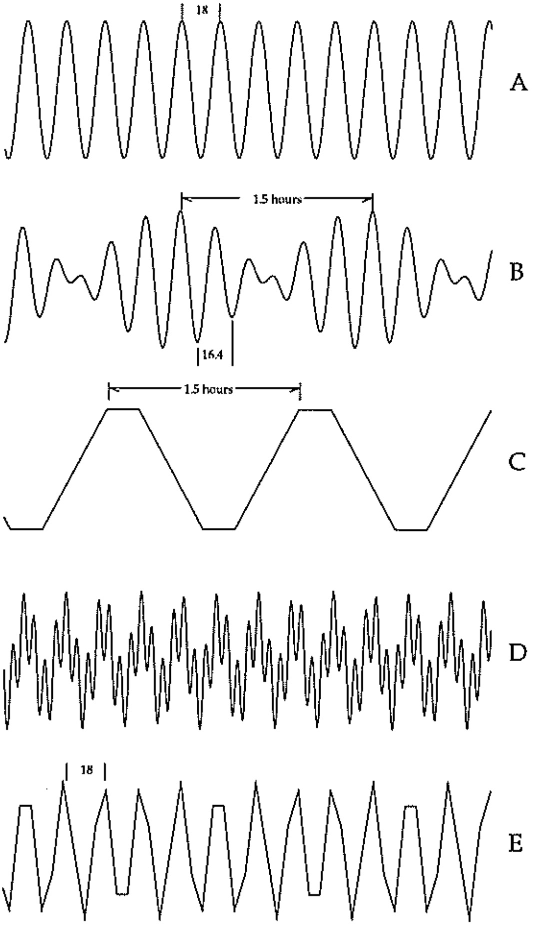

Figure 1 - A) An 18-minute sine wave. B) Addition of an 18-minute oscillation with a 15-minute oscillation results in a 16.4-minute oscillation with a 1.5-hour (90-minute) oscillation in the amplitude. C) If an 18-minute sine wave is sampled at 15-minute intervals, a 1.5-hour alias oscillation is recorded resulting in the loss of information on the 16.4-minute wave. D) Adding a 5-minute sine wave to an 18-minute wave results in complex wave form. E) If an 18-minute sine wave is sampled at 5-minute intervals, no information on the cycle period is lost.

frequencies periodically coincide to produce apparent cyclic changes in amplitude, in the same way as an out-of-tune instrument interacts with a reference instrument. In music both the waves are continuous and the beating produces a wave with frequency $(f_1+f_2)/2$, whose amplitude is modulated by a wave of frequency $\left|f_1 - f_2\right|$. When one wave is discrete, such as a digital recorder, it is possible that information on the wave will be lost and only the $\left|f_1 - f_2\right|$ frequency wave will be visible (Fig. 1). "Aliasing" occurs when only this lower frequency cycle due to the interaction of the recorder interval and seiche is recorded (i.e. information on the continuous wave is lost). Figure 1 shows the beating that occurs when two waves (either continuous, as in music, or discrete as in a digital recorder) interact.

The frequency of the beating wave, or alias cycle for two waves of similar frequency is given by the beats formula (Halliday and Resnick, 1988):

$$f_{alias} = \left|f_{sampled} - f_{actual}\right| . \qquad (1)$$

Converting this into periods gives

$$T_{alias} = \frac{T_{actual} \cdot T_{sampled}}{\left|T_{actual} - T_{sampled}\right|} . \qquad (2)$$

It is evident that the closer the sampling period is to the actual seiche period the longer the spurious cycle will be.

To avoid aliasing in lake level records, the recording interval should be set to less than half the seiche period. This criterion is based on the Nyquist sampling rate (Brigham, 1974) which states that a spacing of

$$T \leq \frac{1}{2f} \qquad (3)$$

will accurately sample the period of a periodic function. A sampling rate near half the natural period is more likely to record a reduced amplitude.

As part of an investigation into long-term yields from hydroelectric lakes, Thompson and Ibbitt (1978) examined lake seiche and aliasing. They aimed to smooth lake level records by removing the effects of seiching in order to calculate accurate outflows over medium to long periods. Seiche oscillations commonly persist only a few days and therefore did not significantly affect their calculations. They concluded that the standard punch tape interval of 15-minutes was satisfactory for most New Zealand lakes.

Most lake level recorders in the country are now telemetered. This almost instantaneous availability of data has led to its use for real-time assessment of lake inflows and for inflow forecasts. To calculate inflows into a lake at the resolution of several hours requires accurate water-level data, since the short calculation time step excludes the use of long-term smoothing to remove seiche effects. The period of the seiche on New Zealand lakes is sufficiently less than the 3-6 hour time step commonly used in inflow calculation; averaging will thus not cause calculation inaccuracies, while still smoothing the effect of seiche in the record. However, the aliasing cycle period can be of similar length to the calculation time step and this can cause errors in the calculation.

Figure 2 shows Lake Coleridge levels recorded in May 1994 using a fifteen-minute interval. An approximately seven-hour cycle can be seen on this section of record. Figure 3A shows a section of five-minute data for Lake Coleridge recorded in January 1995. The bold line shows the result of sampling at fifteen-minute intervals, and the resulting alias cycle is evident. Using equation 2 and a seven-hour alias cycle to back-calculate the period of the seiche cycle gives a period of either 15.55 or 14.48 minutes. The 15-minute recording interval therefore violates the inequality of equation 3 and hence the loss of information on the seiche cycle. The 5-minute recording interval, however, is less than half the seiche cycle and results in no loss of information on the seiche period. The lower plot (3B) shows the effect of taking three-hour averages of both the 5-minute and 15-minute data. Averaging the 15-minute data in this way clearly does not accurately represent lake surface behaviour. Figure 4 shows a period of lake level record from the Moose Lodge recorder on the southern end of Lake Ohau. At the time this was recorded (September 1991) the recorder interval was 15-minutes; it clearly shows an approximately 2-hour oscillation in the record.

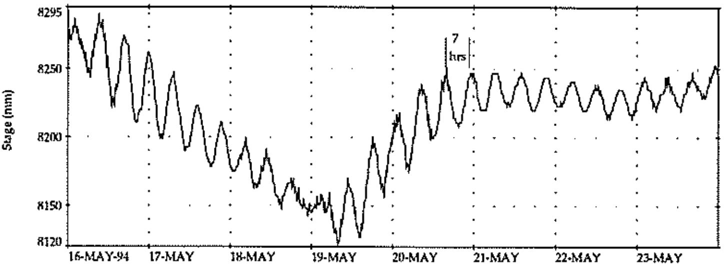

Figure 2 - Fifteen-minute Lake Coleridge level record showing a spurious cycle of approximately seven hours.

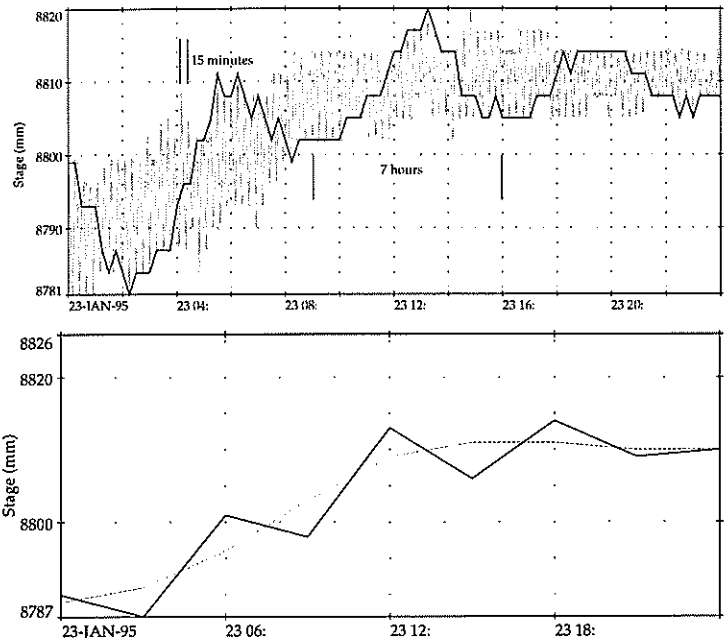

Figure 3 - A) A record of Lake Coleridge levels recorded five-minute intervals is shown by the light dotted line; diamond marks are the actual data points. Aliasing occurs if a fifteen-minute recorder interval (bold line) is used. B) Three-hour averages of both the five- and fifteen-minute records shown in part A; the light dotted line is the five-minute record and the bold line the fifteen-minute record.

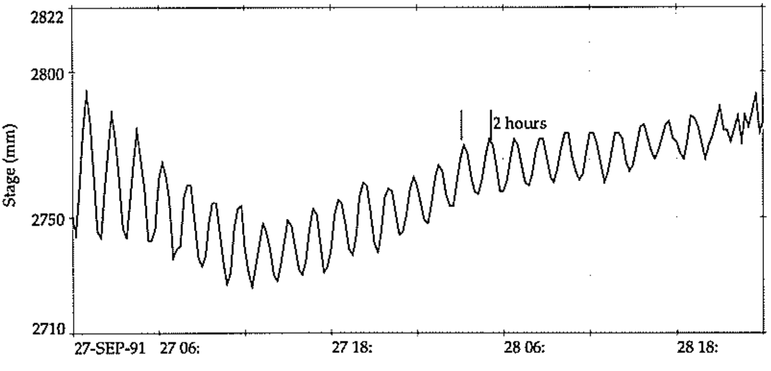

**Figure 4** - A hydrograph of Lake Ohau levels at Moose Lodge showing the 2-hour alias cycle that occurs when the recording interval is 15 minutes.

## A possible solution: moving averages

A number of recorders are used for both control (e.g. hydropower generation and resource consent monitoring) and data collection, and they are checked at regular frequent intervals in case an alarm needs to be raised. It may not be desirable to record the data at the same frequency as the recorder is interrogated for storage considerations. Can a moving average of the finely sampled data be used to smooth the data and allow a longer period between filed data without resulting in an aliased record?

There is a well developed theory for moving averages in the literature (e.g. Burroughs, 1992). Goring and Bell (1996) discuss the limitations of using moving averages to remove semi-diurnal tides from sea level records. By converting the equation for a symmetric moving average into frequency space it is possible to define a relationship between H, the ratio of amplitude of the surface oscillation in the recorded series and the amplitude of the oscillation in the averaged series, with respect to the period of the moving average. Figure 5 shows this relationship. The important features to note are: 1) the oscillation is filtered out completely only when the period of the moving average is an integer multiple of the oscillation period; 2) the longer the moving-average period, the smaller the maximum amplitude of the oscillation remaining in the filtered series; 3) the phase of the filtered series may be 180° out from the original series.

A 24-minute moving average was applied to an 18-minute oscillation. This oscillation has a maximum amplitude of 20 mm and mean of 100 mm; the calculated values are recorded at the end of each averaging period. This is to allow a moving-average value to be recorded at the current time, not 12 minutes later as with a symmetric moving-average. Figure 6A shows the resulting time series. The 18-minute oscillation remains, although the amplitude has been reduced to 4.13 mm and the cycle is inverted, as predicted by the relationship shown in Figure 5. A 24-minute moving average of an 18-minute oscillation gives a t/T value of 1.3, which corresponds to $H = -0.21$. (i.e. the cycle will be inverted and the amplitude will be 21% of the original). Although the theory is developed for a symmetric moving average, the predictions still hold when the results are filed at the end of the averaging period.

These moving-average values were then filed at 5- and 15-minute intervals. Figure 6B shows the moving-average data filed at five-minute intervals resulting in an 18-minute cycle with some amplitude beating. Figure 6C shows the moving average recorded at 15-minute intervals. This results in a 1.5 hour aliased cycle and loss of information about the 18-minute cycle. Therefore, like the raw data, moving-average data must also be filed at less that half the oscillation period.

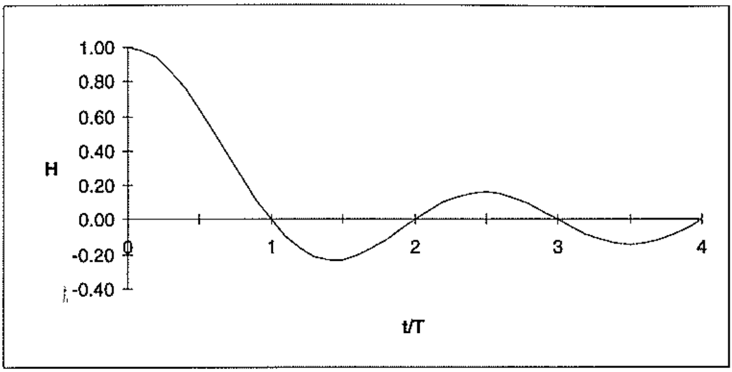

**Figure 5** - The relationship between H, the ratio of amplitude of the surface oscillation in the recorded series and the amplitude of the oscillation in the averaged series, with respect to the period of the moving average. A negative H value means that the cycle is the inverse of the original.

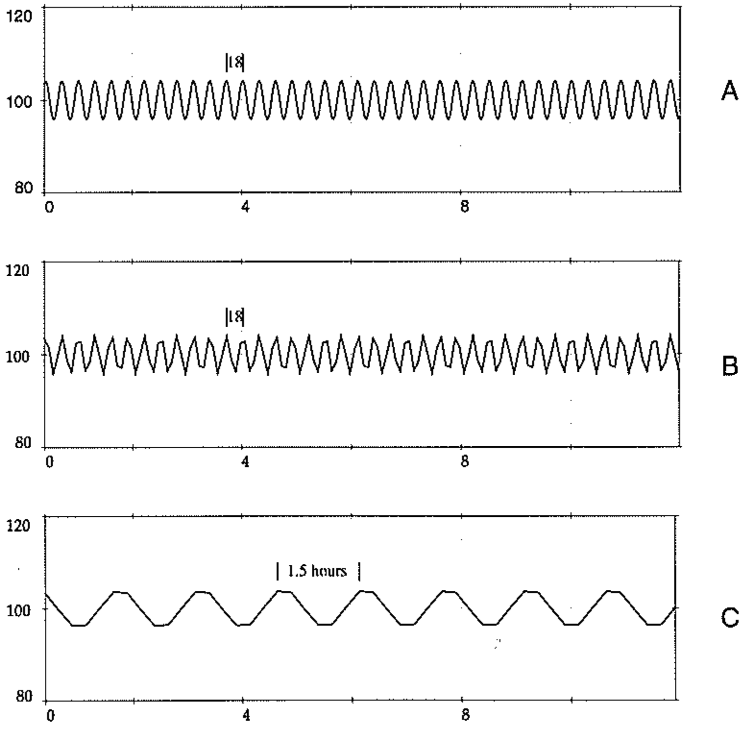

**Figure 6** - The effects of using a 24-minute moving average and of differing length filing intervals on the recording of an 18-minute sine wave with amplitude 20 mm and mean 100 mm. A) filed at the same rate as the incoming data (18-seconds), B) filed at 5-minute intervals, which is less than half the oscillation period allowing the 18-minute cycle to be reconstructed, and C) filed at 15-minute intervals (greater than half the period) resulting in loss of information on the 18-minute wave.

## Conclusion

When a digital recorder is used to measure lake level, the seiche cycle and the recording interval will interact with each other, and an alias cycle is likely to affect the calculation of an equilibrium water level. In order to avoid aliasing in the record, the recording interval should be less than half the period of the seiche. Seiche periods can be predicted by using the Merian formula or some other mathematical procedure.

Using a moving average on the data will reduce the amplitude recorded but will still result in an alias if recorded at more than half the seiche period. Unless a moving average is taken over a period that is an integer multiple of the seiche period it will result in a seiche oscillation of smaller amplitude remaining in the record. Inversion ($180^\circ$ phase shift) of the cycle may also occur. The effect of using a moving average over a given averaging interval can be calculated.

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