THE ESTIMATION OF SOIL MOISTURE DEFICITS BY PENMAN'S AND THORNTHWAITE'S METHOD IN MID CANTERBURY

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ABSTRACT

A comparison was made between soil moisture deficits measured gravimetrically and soil moisture deficits obtained by estimating daily losses by evapotranspiration combined with daily rainfall to determine estimated soil moisture deficits. The methods used to estimate evapotranspiration were Thornthwaite's method and a number of modifications of Penman's method. Comparisons were based on (1) estimates of irrigation water requirements, and (2) use of the methods for irrigation timing.

The results showed that the original Penman method, based on the calculation of open-water evaporation, was the best of the Penman modifications, and that the Thornthwaite method was slightly superior to this.

INTRODUCTION

Penman's method of calculating potential evapotranspiration (*PE*) from climatic data was first published in 1948 (Penman, 1948). Since then Penman and other authors have introduced modifications to the method. It was decided to investigate the effect of such modifications in calculating *PE* on the estimation of irrigation water requirements and irrigation scheduling in Mid Canterbury.

The original Penman formula calculated evaporation from an open water surface and used a factor, *f*, which varied from month to month and place to place, to convert this open-water evaporation to *PE*. This method has been used under New Zealand conditions (Fitzgerald and Rickard, 1960) and derived values of the *f* factor for Mid Canterbury are given in Table 1, together with the values found by Penman to hold for south-east England.

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TABLE 1 — Values of Penman's factor (*f*).

Month	*Winchmore*	*South-east England (corresponding months)*
September	0.6	0.7
October	0.6	0.7
November	0.6	0.8
December	0.7	0.8
January	0.7	0.8
February	0.8	0.7
March	0.7	0.7

In the Penman equation the evaporating power of the air is given by:

$$E_a = 0.35\ (a + U/100)\ (e_a - e_d)$$

where U is the wind speed at a height of 2 metres in miles per day;
e_a is the saturation vapour pressure at mean air temperature;
e_d is the mean vapour pressure of the atmosphere derived from relative humidity observations.

Originally, Penman used a value of 0.5 for a, but later (Penman, 1956) he altered this to 1.0. Thus, variations can be introduced to the method by choice of two a factors.

Penman (1963) described a modification of his original method by which *PE* was calculated directly, using an albedo of 0.25 for a short green crop. There is little agreement in the literature, however, on values to use for this coefficient: "flat ground, grass covered 0.25–0.33" (Sutton, 1953); "meadows 0.15–0.25" (Budyko, 1958); "short grass, meteorological enclosure 0.25–0.27" (Monteith and Szeicz, 1961); "grass, green 0.16–0.27" (Van Wijk, 1963). Investigation by Chudnowskii (1953) showed that under summer cereals the albedo of sprinkler-irrigated crops was generally higher than that of non-irrigated crops.

In view of this, Fitzgerald and Cossens (1966), in their investigation of the applicability of Penman's method to irrigated areas of Central Otago, used a value of 0.30 for the albedo. Use of either 0.25 or 0.30 introduces more possible modifications of the Penman method.

Penman's original and modified methods used values of the wind speed recorded at a height of 2 metres. In New Zealand, wind speeds are measured at a height of 5.8 metres. Wind speeds at this height were used in an investigation of Penman's method in Mid Canterbury (Fitzgerald and Rickard, 1960). The relationship between wind speeds at two heights was determined from the power law profile (Sutton, 1953):

$$U = U_1 \, (Z/Z_1)^p$$

where U_1 is the wind speed at height Z_1;
U is the wind speed at height Z;
p is a constant.

The value of the exponent p was taken to be 0.25 as recommended by Lang *et al.* (1973). Thus, further modifications can be introduced into the method by converting wind speeds to those which would be expected at 2 metres.

Using all these modifications, 12 variants of the Penman equation can be used. These are:

(1) 1948 method with wind speed at 5.8 metres and $a = 1.0$.
(2) 1948 method with wind speed at 5.8 metres and $a = 0.5$.
(3) 1948 method with wind speed at 2.0 metres and $a = 1.0$.
(4) 1948 method with wind speed at 2.0 metres and $a = 0.5$.
(5) 1963 method with albedo of 0.25, wind speed at 5.8 metres and $a = 1.0$.
(6) 1963 method with albedo of 0.25, wind speed at 5.8 metres and $a = 0.5$.
(7) 1963 method with albedo of 0.25, wind speed at 2.0 metres and $a = 1.0$.
(8) 1963 method with albedo of 0.25, wind speed at 2.0 metres and $a = 0.5$.
(9) 1963 method with albedo of 0.30, wind speed at 5.8 metres and $a = 1.0$.
(10) 1963 method with albedo of 0.30, wind speed at 5.8 metres and $a = 0.5$.
(11) 1963 method with albedo of 0.30, wind speed at 2.0 metres and $a = 1.0$.
(12) 1963 method with albedo of 0.30, wind speed at 2.0 metres and $a = 0.5$.

Each of these modifications of the Penman method is designed to calculate potential evapotranspiration, which was then converted to actual evapotranspiration (*AE*) as described by Rickard and Fitzgerald (1969). Estimates of potential evapotranspiration were also made by Thornthwaite's (1948) method, and these were also converted to *AE*.

The methods used to determine the usefulness of the estimates were based on their application to irrigation practice. It is important that any method used should produce a reliable estimate of irrigation water requirements. Too high an estimate could result in an unduly high investment in an irrigation system. Too low an estimate could leave the system vulnerable to drought. The other use of these methods is to determine when irrigation should be applied.

METHODS

A Fortran programme was written for an ICL-190A digital computer to calculate the 12 modifications of Penman's method. The Thornthwaite estimates were available from an earlier Algol programme written for an Elliot 503 digital computer. Climate data used were recorded at the Winchmore Irrigation Research Station's meteorological station adjacent to the trial area in which gravimetric soil moisture measurements were made. Mean monthly values of the climate parameters were used to calculate a daily value of *PE* by the Penman method, while daily values were used for the Thornthwaite method. This value was then multiplied by a factor which depended on the daily soil moisture deficit to give the *AE* for each day; details of the method used are given by Rickard and Fitzgerald (1969).

To determine soil moisture deficits it was assumed that the soil moisture deficit was zero at the start of calculations – 1 September. Several years of soil sampling at Winchmore indicate that this is a reasonable assumption. The deficits were accumulated day by day in a bookkeeping-like procedure in which daily *AE* increased the deficit and rainfall reduced the deficit. These calculations were used to produce estimated deficits which could be compared with gravimetrically determined deficit on the days of irrigation of a pasture experiment at the Winchmore Irrigation Research Station (Rickard, 1972). This experiment was carried out on the Lismore silt loam, which has 60 mm available moisture-holding capacity in the top 300 mm of soil.

The treatments used in this experiment were:

(1) Irrigated when 100 percent of the available soil moisture was depleted.

(2) Irrigated when 75 percent of the available soil moisture was depleted.

(3) Irrigated when 50 percent of the available soil moisture was depleted.

(4) Irrigated at intervals of not less than three weeks.

Irrigation was applied by the border strip method, and each irrigation restored the top 300 mm to field capacity. Soil moisture deficits were measured, before irrigation, by the gravimetric method.

For the purpose of comparing measured and calculated deficits, two seasons, 1964/65 and 1967/68, during which a large number of irrigations had been applied to the trial area, were selected. These gave a total of 42 irrigations for which deficits could be compared.

RESULTS

Estimate of Water Requirements

A comparison between the various modifications can be based on the estimates of seasonal water requirements. These are calculated by summing the deficits on the irrigation days, and a comparison can then be made with the measured water requirements.

For example, if the calculated deficits at each irrigation over a particular period were 30, 35, 32, 40 and 32 mm, then the water requirement would be 169 mm. The results of this comparison are given in Table. 2.

A reasonable criterion to adopt is that estimated water requirements be within ±10 percent of the measured requirements. When all treatments are combined, all the modifications with an albedo of 0.05 give reasonable estimates of water requirements. Only one of the modifications with an albedo of 0.25 meets the standard; three of the modifications with an albedo of 0.30 also meet the standard.

Water requirements estimated by Thornthwaite's method also provide an accurate estimate of requirements. However, as can be seen from Table 2 there are differences in accuracy of estimation when the water requirement estimates are divided into their individual irrigation treatments. In general, the Thornthwaite estimate is consistent over the irrigation treatments.

Use for Predicting Irrigation Dates

A comparison of practical importance can be made by assessing the number of days earlier, or later, that irrigation would have occurred if the date had been decided by calculation rather than by measurement. To do this, the difference between the calculated and measured deficit was divided by the average daily *PE* for the month. Use of potential rates rather than actual will tend in some cases to produce a lower number of days. However, all the calculated variants should be affected similarly. Table 3 gives the percentage of irrigation that would have fallen within 3 days of the actual applications.

TABLE 2 — Calculated water requirements as a percentage of measured requirements for the periods 1964/65 and 1967/68.

Factor *a*	*Height of wind measurement (m)*	*Albedo*	*Percentage available moisture depletion at irrigation*			*Irrigated at 3-weekly interval*	*All irrig. treatments combined*
			100	*75*	*50*		
1.0	5.8	0.05*	91	106	121	102	106
0.5	5.8	0.05*	88	101	113	96	100
1.0	2.0	0.05*	88	100	112	96	100
0.5	2.0	0.05*	85	93	103	89	93
1.0	5.8	0.25	96	118	145	121	121
0.5	5.8	0.25	94	113	134	113	115
1.0	2.0	0.25	93	112	133	112	114
0.5	2.0	0.25	91	106	121	102	106
1.0	5.8	0.30	94	114	136	115	116
0.5	5.8	0.30	91	108	125	105	108
1.0	2.0	0.30	91	108	124	105	108
0.5	2.0	0.30	87	101	110	94	99
Thornthwaite:			87	99	111	97	99
Measured water requirement (mm):			359	353	461	448	1621

* The albedo of 0.05 is for an open water surface, the results are modified by the factor f to give evapotranspiration from pasture.

TABLE 3 — Percentage of irrigation within three days.

Factor a	*Height of wind measurement (m)*	*Albedo*	*Percentage available moisture depletion at irrigation*			*Irrigated at 3-weekly interval*	*All irrig. treatments combined*
			100	*75*	*50*		
1.0	5.8	0.05*	83	100	88	100	93
0.5	5.8	0.05*	83	100	100	83	93
1.0	2.0	0.05*	83	100	88	83	90
0.5	2.0	0.05*	67	100	100	83	90
1.0	5.8	0.25	100	100	44	75	71
0.5	5.8	0.25	100	100	56	83	79
1.0	2.0	0.25	100	100	62	83	81
0.5	2.0	0.25	100	100	75	83	86
1.0	5.8	0.30	100	100	44	83	74
0.5	5.8	0.30	100	100	75	83	86
1.0	2.0	0.30	100	100	75	83	86
0.5	2.0	0.30	83	100	110	92	95
Thornthwaite:			83	100	93	100	95

* The albedo of 0.05 is for an open water surface, the results are modified by the factor f to give evapotranspiration from pasture.

If the criterion of the system that produces the highest number of irrigations within 3 days of the actual irrigation is chosen, then the best Penman estimates over all treatments are any with an albedo of 0.05 and the last of the group with an albedo of 0.30. The choice of height at which the wind was measured has no effect when an albedo of 0.05 is used but has some effect when an albedo of 0.25 is used. The height at which wind is measured has no clear influence when the albedo is 0.30. The Thornthwaite method is shown to be a good method for estimating when to irrigate. The effect of irrigation treatment on the percentage of irrigations within 3 days of the actual date can also be seen from Table 3.

CONCLUSIONS

As an estimate of irrigation requirements in general, the original Penman method and the method with an albedo of 0.30 would be the most suitable. If these variants had been used to determine the dates of irrigation, all the estimates with an albedo of 0.05 and one with an albedo of 0.30 would be suitable. The results show that on both criteria the Thornthwaite estimate gives equally satisfactory results.

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