Journal of Hydrology (N.Z.) Vol. 21, No. 1, 1982

A LINEAR MODEL OF STORM RUNOFF FROM SOME URBAN CATCHMENTS IN NEW ZEALAND

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ABSTRACT

Rainfall and runoff from gauged urban catchments are used to design hydrographs for ungauged New Zealand urban catchments from a given rainfall hyetograph and known catchment physiography. The linear model of a catchment, used for simplicity, is characterized by the instantaneous unit hydrograph (IUH). Harmonic analysis is used to find the IUHs for 159 rainfall-runoff events on seven New Zealand urban catchments. No single IUH is found to characterize the response of any catchment for all events. It is found that the quickflow hydrograph may be satisfactorily generated from a known hyetograph using an IUH with the same shape as the gamma distribution and having a peak and time to peak derived from those of the harmonic IUH. This peak-fitting gamma-IUH is described by two parameters estimated from the peak discharge and time to peak. The gamma-IUH discharge and time to peak can be regionalized by equations that include catchment physiography and rainfall. The multiple regression analyses also strongly suggest that only a single parameter, the time lag of each event, is required to find the gamma IUH for that event.

INTRODUCTION

Current practice for design of urban stormwater systems in New Zealand revolves around the use of the Rational Method (e.g. Anderson, 1977) and TM61 (Ministry of Works and Development (MWD), 1975) to estimate peak flow given design rainfall characteristics. Anderson presented a procedure with coefficients relating the proportion of rainfall contributing to runoff based largely on Australian and American data. Waugh (1978) used factors developed overseas to adjust peak flow frequencies from some rural catchments in Northland for application to urban conditions.

It is often important to know not only peak flow, but also the volume of water that is likely to need pumping, impounding, or piping. The hydrograph may also be required for the design of flood storage and pumps. From instantaneous unit hydrographs (IUHs) derived from 56 storms on 7 North Island catchments, this paper provides the background for deriving flood hydrographs for either gauged or ungauged urban catchments.

Unit hydrograph techniques have been criticised because they represent a linear black-box approach to complex non-linear rainfall-runoff processes (e.g. Henderson, 1966, p. 394 and Fleming, 1975, p. 165). In spite of these obvious shortcomings, IUHs are used in this study for the following reasons:

1. a conceptual approach to rainfall-runoff modelling is not feasible;
2. under suitable restrictions IUHs frequently give results as good as those of more complex models (restrictions on the model presented here are discussed later);
3. unit hydrograph methods have been successfully regionalized in a number of countries (e.g. Natural Environment Research Council (NERC), 1975);
4. the restrictions of linearity and time invariance, which give unit hydrograph theory both its power and its limitations, can be relaxed somewhat to account for observed variations in the derived unit hydrographs from storm to storm on the same catchment.

BACKGROUND AND REVIEW OF PREVIOUS WORK

If a catchment is modelled as a linear system, Dooge (1973) has shown that the linear system may be represented by the equation

$$Q(t) = \int_{0}^{t} h(t-\tau) \, . \, i(\tau) \, d\tau \qquad (1)$$

where $Q(t)$=the output (quickflow),
$i(t)$ =the input (rainfall),
τ =a dummy variable for time, t,
and $h(t)$ =the kernel function (IUH).

Physically, the IUH represents the catchment output from an instantaneous unit input. The linear theory implies that a single IUH should be able to reproduce the catchment streamflow response to any given rainfall pattern.

To regionalize the linear catchment model the IUH must be related to catchment physiography. In a major study of flood hydrology (NERC, 1975), one-hour unit hydrographs were derived for 1631 storms using matrix inversion with smoothing. Unit hydrographs thus derived were approximated by a triangular construction. The time to peak of the unit hydrograph, t_p, was not sensitive to flood peak or rainfall intensity, and could be completely defined by an equation involving catchment slope, degree of urbanization, main stream length, and a climatic index of flood runoff potential of the catchment. NERC also found that the unit hydrograph peak, u_p, and width at $u_p/2$ could be completely described in terms of t_p. The time base of the unit hydrograph was constrained by the requirement that the area under the unit hydrograph be unity. Hence, the entire unit hydrograph was described in terms of only one parameter, t_p.

Rao, Delleur, and Sarma (1972) studied linear rainfall-runoff models for eight urban and five rural catchments of less than 52 km^2 using the Fourier transform method, Clark's (1945) method, Nash's (1959) method, and a single linear reservoir approach in which the IUH is taken to be of the form:

$$h(t) = K_r^{-1}\exp(-t/K_r), \qquad (2)$$

where K_r is a storage delay constant. The Fourier transform solution of (1) reproduced the storms best; however, attempts to characterize the peak, time to peak, and shape of this IUH were unsuccessful. The time lag and IUH derived for the catchments were found to vary form storm to storm, changing with different average storm intensities. The Nash method uses an IUH based on the two-parameter gamma probability density function,

$$h(t) = (t/k)^{n-1}\exp(-t/k)/k\Gamma(n),\ t \geqslant 0 \qquad (3)$$

where k, n are the two gamma distribution parameters and Γ is the gamma function. Good results were obtained using the gamma-IUH to reproduce storm runoff from corresponding rainfall for catchments greater than 13 km^2, while the single linear reservoir (2) best represented catchments smaller than 13 km^2. Rao *et al.* successfully developed regression equations for the parameters K_r, n, k in terms of catchment area, degree of urbanization, and volume and duration of rainfall after loss separation.

The present study follows Rao *et al's* (1972) study in the attempt to derive an IUH shape for use in synthesizing design flood hydrographs.

CATCHMENTS USED IN STUDY

Seven catchments (Table 1, Fig. 1) were used that represent a range of urban development from zero to 65%. All catchments had reliable runoff data in computer compatible form. The rain gauge used for all of the Rotorua catchments is at a distance of 5-7 km from the City Catchments, as there is no closer site for which continuous rainfall data are readily available in computer storage.

IUH DERIVATION

Wairau Creek, the best documented and best instrumented of the catchments, was chosen for a pilot study to develop methods for deriving IUHs from observed rainfall and runoff. Eight events were chosen from the short record (Table 1) to allow comparisons between the IUHs. Peak flows ranged from 0.27 to 12.5 m^3/s. IUHs for the eight events were derived from rainfall-runoff records using the harmonic analysis method described by O'Donnell (1960, 1966).

Rainfall Losses

Rainfall losses due to interception, infiltration, and storage are calculated as a percentage of total rainfall rather than as a loss rate (NERC, 1975, pp. 393, 411). Rainfall losses are distributed by multiplying rainfall intensity recorded over each sampling interval by a separation factor. This factor is the ratio of the total volume of quickflow to the total volume of recorded rainfall.

TABLE 1—Catchments studied and coincident periods of rainfall-runoff records.

Catchment Name	Catchment Characteristics				Automatic Raingauge		Coincidental Period of Rainfall and Runoff Record
	Area km^2	Average slope x 10^3	Channel Length, km	Fraction Urbanised	Name	Site No.	
Wairau Creek	11.4	9.6	5.2	0.23	Lockies	647716	April 1972-September 1972
Rewarewa River	1.34	25.4	1.7	0.16	Boyds	649620	April 1972-June 1976
Manukau River	0.30	48.0	0.8	0.00	Hawthorne Hedge	649910	June 1969-June 1978
Fenton Street Drain	0.029	5.3	0.5	0.65	8 Mile Road	861218	July 1975-May 1978
Te Ngae Drain	0.92	8.1	2.1	0.17	8 Mile Road	861218	February 1976-July 1978
Pomare Stream	0.81	7.9	1.2	0.21	8 Mile Road	861218	November 1975-June 1978
Porirua Stream	41.0	23.7	13.3	0.05	Reservoir	141812	April 1969-May 1969 and August 1972-January 1979

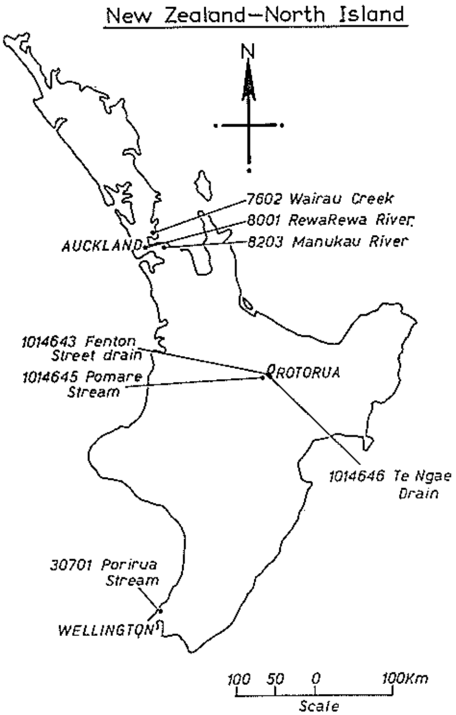

FIG. 1—Location of catchments studied.

Baseflow Separation

The method of baseflow separation used here is adapted from Rao *et al.* (1972). An alternative baseflow separation technique consisting of fitting straight lines to the recession limb of the hydrograph plotted on semi-logarithmic paper (Linsley *et al.*, 1975) can be related to physical processes and was also tested here. A wide variation in the time and flow rate at which the change in recession slope occurred was found on the semi-logarithmic plot of the Wairau Creek events. In addition, the change in slope was quite subtle so that finding the point at which it occurred could not be automated for computer use, and no further attempt was made to use this technique.

Data Sampling Interval

For the eight storms examined, convolution of rainfall data with the derived harmonic-IUH exactly reproduced the runoff hydrograph. However, in common with other numerical solutions of (1), the harmonic-IUH often exhibited marked oscillations, mainly in the tail ordinates (Fig. 2a). Spurious negative ordinates indicate computational instability and cannot be justified if the IUH is taken to represent the catchment time-area diagram (Boneh and Golan, 1979). Consequently the IUH oscillations are

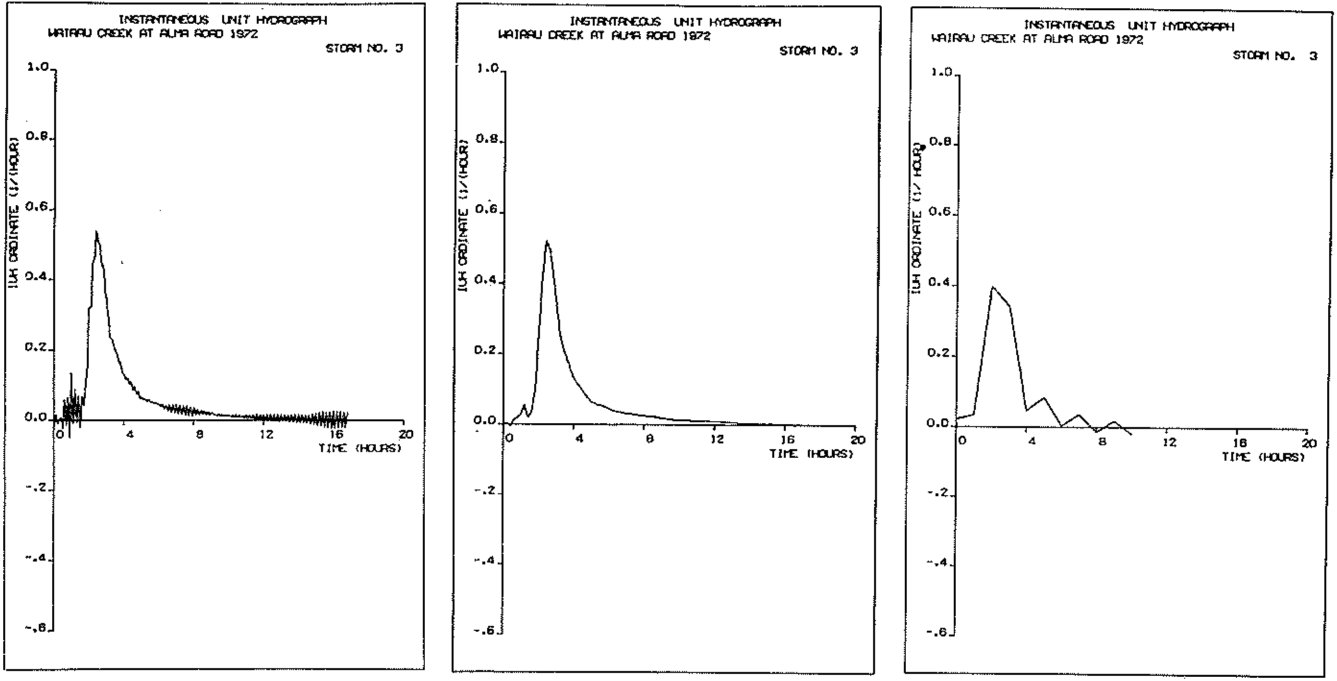

FIG. 2—Effect of sampling time interval on IUH oscillations. (a) 0.1 hour sampling interval; (b) 0.2 hour sampling interval; (c) 1.0 hour sampling interval.

customarily smoothed out before the IUH is considered to satisfactorily explain the linear response of the catchment. IUH oscillations may be removed by mathematical filtering (Rao and Delleur, 1974) or by adjustment of the sampling interval for rainfall and runoff ordinates (Kavvas and Schulz, 1973). The effect on the IUH of changing the sampling time interval was examined for some of the Wairau Creek events. The IUHs derived for Wairau Creek event number 3 when the sampling time interval is 0.1, 0.2, and 1.0 hour respectively show that the sampling interval of 0.2 hour, for this event, smooths out most of the IUH oscillations (Fig. 2). Unfortunately, the sampling time interval that produces a smooth IUH for one event does not have a similar effect when used with other events on the same catchment (Hossain *et al.*, 1978). However, it was found that if a suitable sampling interval for one event was used for other events, the IUH peak was generally free of oscillations and the shape of the IUH tail could be discerned through the remaining oscillations. Thus an approximately suitable sampling time interval appropriate to each catchment was used for every event on that catchment; a sampling interval of 0.2 hour was used for the Wairau Creek events.

Harmonic-IUH Shape and the Gamma-IUH

All the IUHs generated from the eight Wairau Creek events appeared to have a shape similar to that of the two-parameter gamma probability density function (3). The area of the Wairau Ck catchment is 11.4 km^2; from Rao *et al.* (1972) one would expect the IUH shape to be similar to that of a single linear reservoir. However, there was little suggestion of this. It was therefore decided to fit an IUH based on the gamma probability density function (3) to each Wairau Ck event, and then to determine how well this gamma-IUH reproduced the observed quickflow hydrograph, by convolution of the gamma-IUH with the excess precipitation hyetograph.

Goodness-of-Fit and Fitting the Gamma-IUH

Percentage peak error (E_p) was chosen to measure the precision of peak flow reproduction, as defined by:

$$E_p = (Q_{r,max} - Q_p)\, 100/Q_p \tag{4}$$

where $Q_{r,max}$ = reproduced hydrograph peak,
Q_p = observed hydrograph peak.

Several measures of assessment of the reproduction of the entire hydrograph, based manily on statistical correlation theory, were evaluated for each Wairau Ck event. These included the linear correlation coefficient (Rao *et al.*, 1972), a special correlation coefficient and the integral square error (March and Eagleson, 1965), and the model efficiency (Nash and Sutcliffe, 1970). For Wairau Ck data, model efficiency seemed to be the most sensitive at detecting observed discrepancies in hydrograph reproduction and was consequently used as an objective measure of general hydrograph reproduction. Model efficiency (M_e) is defined in an analogous manner to the statistical coefficient of determination, as

$$M_e = 1 - \left[\sum_{j=1}^{N} (Q(jd) - Q_r(jd))^2 \Big/ \sum_{j=1}^{N} (Q(jd) - \overline{Q})^2 \right] \tag{5}$$

where $\overline{Q} = \frac{1}{N} \sum_{j=1}^{N} Q(jd)$, the mean value of Q(jd),

d = data sampling interval
Q(jd) = observed quickflow hydrograph
$Q_r(jd)$ = reproduced quickflow hydrograph
N = number of quickflow hydrograph ordinates.

A perfect fit is represented by $M_e = 1$ and a poor fit by large negative values of M_e.

The quantities E_p and M_e were used as goodness-of-fit indices to assess the relative merits of two different methods of fitting a gamma-IUH to rainfall-runoff data. Fitting requires estimating the gamma distribution parameters k, n (see equation 3). The two methods tested were the method of moments (Nash, 1959) and a peak fitting method; the latter was found to yield consistently better results for the Wairau Ck data (Broome, 1980). In the peak fitting method the actual IUH is determined from storm data; the peak (h_p) and the time to peak (t_p) of the gamma-IUH and the actual IUH are then constrained to be equal. This requires that the gamma-IUH parameters k, n be given by

$$k = t_p/(n-1) \tag{6}$$

$$h_p t_p = (n-1)^n e^{-(n-1)}/\Gamma(n) \tag{7}$$

These equations may be solved by an iterative method (Croley, 1980).

Multiple-peaked Events

Although the gamma-IUH is reasonably successful at reproducing the response of the major peaks of multiple peaked events, it is less successful at reproducing the response of the secondary peaks (Fig. 3). Spurious oscillations in the actual (harmonic) IUHs are evident in multiple-peaked events (Fig. 4); in addition, there is often systematic variation of the harmonic-IUH from the usual single-peaked IUH shape. Generally, multiple-peaked IUHs are explained in terms of differing response mechanisms for different sections of the catchment (NERC, 1975). This is not the case for the single-peaked Wairau Creek events. Another explanation of the phenomenon of multiple-peaked IUHs is that the rainfall loss or baseflow separation techniques fail to adequately recognise that the first period of rainfall causes a change in the state of the catchment. Because of these difficulties, only single-peaked hydrograph events were used in this study.

Linearity, Invariance, and Scaling; a Non-Dimensional IUH?

The six single-peaked events on the Wairau Ck catchment exhibit wide variations in the peak and time to peak of the IUHs (Fig. 5), suggesting that it would be unwise to characterize catchment response in terms of a single average IUH. Dependence of the IUH on storm characteristics such as peak rainfall intensity, total rainfall volume, and storm duration may explain some of the variation within a single catchment; hence,

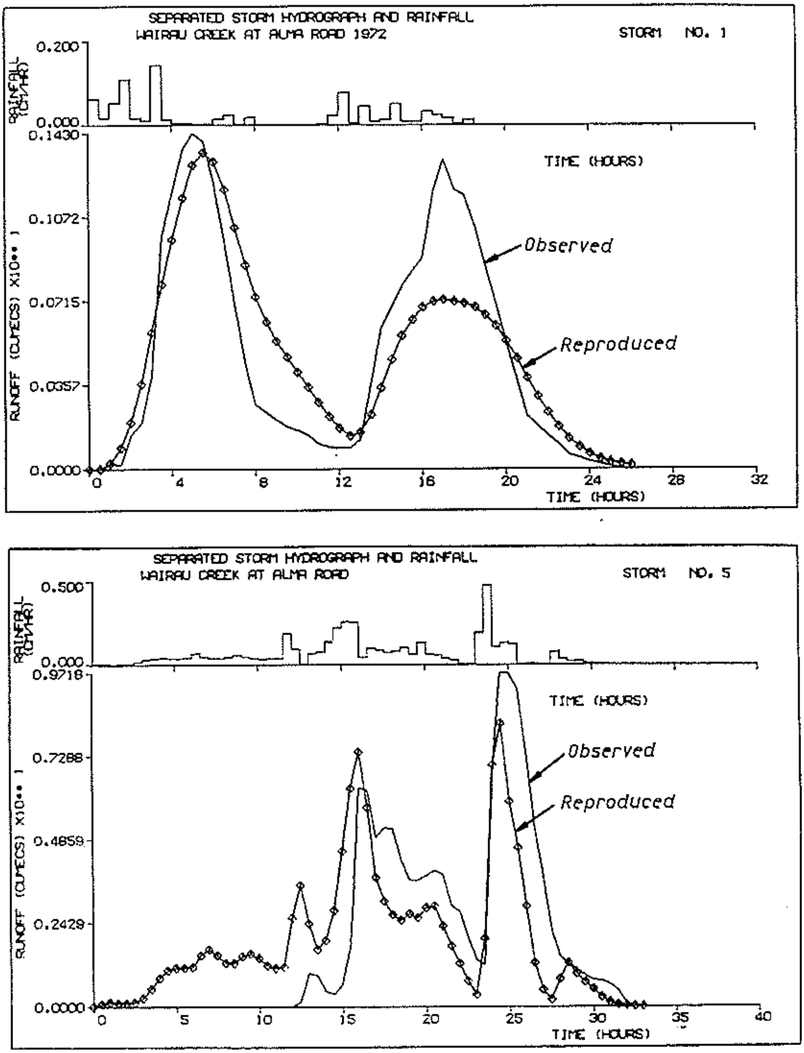

FIG. 3—Gamma-IUH reproduction of multi-peaked storms. (a) Wairau Creek event number 1; (b) Wairau Creek event number 5.

storm characteristics as well as catchment characteristics were included in the regression studies.

In addition, nondimensionalizing the IUH with a suitable time scale might reduce variation between IUHs, in effect reducing the different IUHs to a single "universal" curve. Non-dimensionalization may be performed because both axes of IUH plots are expressed only in units of

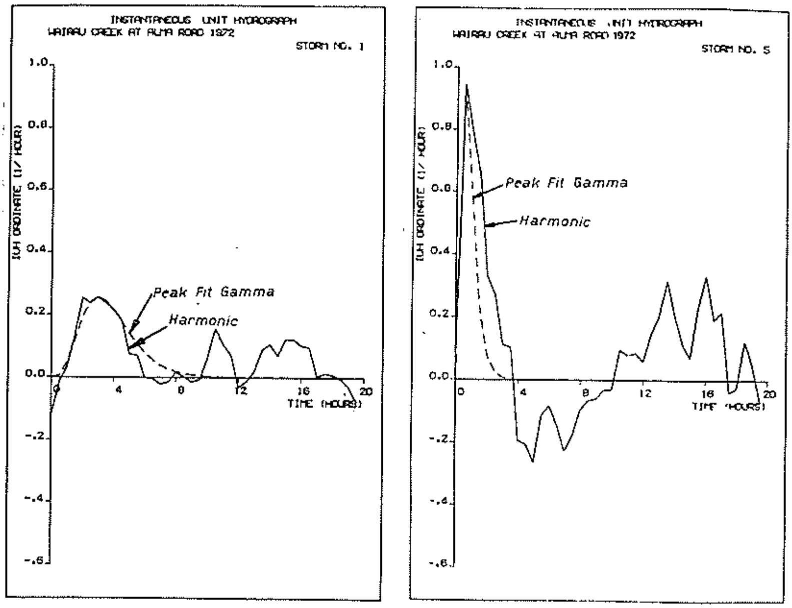

FIG. 4—Harmonic- and gamma-IUHs for multi-peaked events. (a) Wairau Creek event number 1. (b) Wairau Creek event number 5.

time. Examination of the defining equation (1) for the IUH ($h(t)$) reveals that if the rainfall ($i(t)$) and quickflow ($Q(t)$) are both expressed in units of length/time, then $h(t)$ must have dimensions $(\text{time})^{-1}$. Hence, a single time parameter which characterizes an event may be used to make the plot of the IUH dimensionless for that event. This is done by multiplying the ordinate axis and dividing the abscissa axis by the time parameter. Two quantities commonly employed in this scaling are the time to peak, t_p, and the time lag between the centroids of the rainfall hyetograph and the quickflow hydrograph, t_l. This scaling does not necessarily reduce all the IUHs to a single shape; moreover, the time scales t_p, t_l generally vary from event to event. However, it is worthwhile to consider at this point some implications of a possible "universal" scaling.

Let T be the time scale of interest (e.g., $T=t_p$ or $T=t_l$). Denoting the non-dimensional flow and time by $h^*=hT$ and $t^*=t/T$, respectively, the expression for the non-dimensional gamma-IUH is

$$h^*(t^*) = (T/k)^n (t^*)^{n-1} e^{-t^*(T/k)}/\Gamma(n) \qquad (8)$$

The peak fitting equations are

$$t_p^* = (n-1)/(T/k) \qquad (9)$$

$$h_p^* t_p^* = (n-1)^n e^{-(n-1)}/\Gamma(n) = h_p t_p \qquad (10)$$

where $h_p^* = h^*(t_p^*)$ and $t_p^* = t_p/T$. In order for all the IUHs to collapse to a single shape it is sufficient to require that all their peaks coincide, i.e., that both h_p^* and t_p^* be constants. This is true because the gamma-IUH has only two parameters, so that by fixing h_p^*, t_p^*, the entire shape is fixed. t_p^* can only be a constant if t_p is proportional to T for each event; this is automatically satisfied if we choose $T = t_p$ (so that $t_p^* = 1$ identically), or $T = t_l$ and $t_l = ct_p$, where c is some constant. Then if $h_p^* t_p^*$ is also a constant, all of the non-dimensional IUH peaks will coincide and a single non-dimensional gamma-IUH shape will exist. Referring to (10), $h_p^* t_p^*$ will be constant only if the parameter n is constant from event to event. To summarise, we can expect to find a universal non-dimensional gamma-IUH for all catchments, with a time scale T, only if we choose T such that t_p/T is the same from event to event and catchment to catchment; in addition $h_p t_p$ and n must be constant for different events and different catchments. This scaling was tested on the complete data using the multiple regression analyses discussed in the next section.

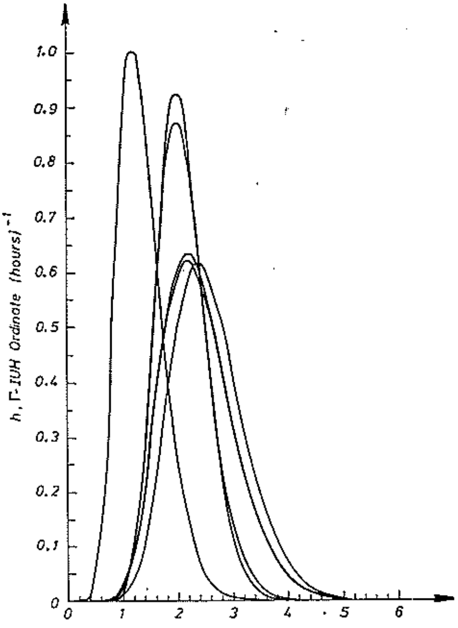

FIG. 5—Gamma-IUHs of the six single-peaked Wairau Creek events.

RESULTS FROM MULTIPLE REGRESSION

Storm Parameters

Following the findings of Rao et al. (1972), Heerdegen (1974), and Hossain *et al.* (1978), rainfall duration and volume of rainfall after separation of losses were incorporated in the regression study (see definitions, Table 2). Rainfall peak intensity after loss separation was also included

TABLE 2—Definition of catchment and storm parameters.

Name	Units	Definition
Catchment Area (A)	km^2	The area contributing to the runoff at the gauging point.
Main Stream Length (L)	km	The length of the longest stream channel in the catchment.
Catchment Slope (S)	m/km	The difference in elevation between the end of the main stream and the gauging point (in meters) divided by the mainstream length.
Degree of Urbanisation (U)	—	An estimate of the proportion of the catchment that was impervious to rainfall infiltration.
Rainfall Duration (T_r)	hour	Total time prior to the peak flow during which rain fell.
Excess Rainfall Volume (V_e)	cm	Total volume of rain that fell before the peak flow after separation of losses.
Peak Rainfall Intensity (i_p)	cm/hour	Peak rate at which rain fell after the separation of losses.
Time Lag (t_l)	hour	Time between the centroid of rainfall after separation of losses and the centroid of the quick-flow hydrograph.

because the rainfall peak intensity and runoff peak flow appeared, in many cases, to be separated by an interval of time equal to the time to peak of the IUH. The final storm parameter included was the time lag, the difference in time between the centroid of the hyetograph and the centroid of the quickflow hydrograph; studies by Nash (1960), Askew (1968), Diskin (1973), and Pilgrim (1976) discuss the importance of the time lag.

Catchment Physiography

The catchment physiographic parameters defined in Table 2 were extracted as follows

1. Catchment area: As listed by Ministry of Works and Development (1978).
2. Main stream length: Scaled from available catchment maps. These maps were from a variety of sources and were to varying scales.
3. Catchment slope: Because the maps available showed contour intervals ranging from 1.5 m to 100 ft. only the simple definition of slope of Table 2 was warranted.
4. Degree of urbanization: Ideally this parameter should be measured

from large-scale aerial photographs as the percentage catchment area covered by buildings, roads, footpaths, or other impervious surfaces. Unfortunately, aerial photographs were available only for the Rewarewa catchment. Detailed maps that showed streets and buildings to scale were available for two of the Rotorua catchments. The only type of detailed map that was available and covered all of the catchments and showed urbanized areas was the NZMS 271 series (Dept. Lands and Surveys; scale varies, but typically 1:20,000). These maps differentiate between residential areas, schools, parks and reserves, and industrial areas, but do not show roadways to scale. Therefore, the detailed aerial photographs and maps that were available were used to derive factors for the degree of imperviousness likely to be found for these different types of urban development. Roadways were assessed as part of the residential area. Factors derived in this way by Broome (1980) were: light industrial 0.5, residential 0.3, schools 0.2, parks and reserves 0.03. These factors were appropriate for only five of the seven catchments. The Manukau R. catchment is a farm and was taken to have an insignificant impervious proportion. The Fenton Street Drain catchment is wholly developed and largely covered with commercial properties. A 1:4800 scale map was used to assess the proportion of the area of the Fenton Street Drain catchment that is covered by buildings and streets.

Choice of Storms

159 events were selected for analysis from rainfall-runoff data from the seven catchments. For each event, the methods described earlier were used to derive an IUH by harmonic analysis, fit a gamma-IUH, and check goodness-of-fit for reproduced quickflow hydrographs using the gamma-IUH. Overall goodness-of-fit for each event was classified according to Figure 6, which is based on Beable (1976) but with a more stringent requirement for peak error. Plotted points in Figure 6

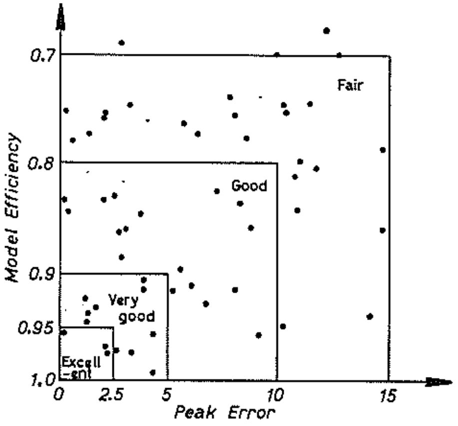

FIG. 6—Goodness of fit classification and the selected events.

correspond to the 56 events which were subsequently used in the regression study; for 54 of these events, gamma-IUHs produced "fair" or better fits. The other two catchments were used even though they do not strictly adhere to these goodness-of-fit requirements. One of these events was used because it was the second largest in ten years, and there was a lack of suitable large events available for analysis. The other event was used because very few events from the non-urbanized Manukau R. catchment proved suitable for analysis and this event very nearly fitted within the goodness-of-fit criteria.

Reasons for poor fit for the remaining 103 events include gross timing errors, such as runoff being recorded prior to the associated rainfall, or runoff recorded as lagging the rainfall by an unreasonable time; poor selection of data sampling time interval and resulting poor model efficiency; and occurrence of a single-peaked rainfall hyetograph associated with a multiple-peaked runoff hydrograph. As is usual in this type of study, problems often arise in matching the event hyetograph and hydrograph. Relative timing is a critical factor in the analysis, especially for small catchments with time lags of the order of six minutes or less. For some of the events on the Rotorua catchments, hyetographs recorded at the 7 km distant rain gauge clearly could not have produced the hydrograph recorded by the runoff gauge.

Regression Analyses

The analyses used multiplicative regression of the form

$y = cx_1^{g_1} x_2^{g_2} x_3^{g_3} \ldots$, where y is the dependent variable, the x_i are the independent variables, and c, g_i are regression constants. Transformation of the variables to logarithms was used, and a stepwise procedure employed so that independent parameters were added one at a time on the basis of having the greatest F-test statistic.

Linear dependence between the independent variables was checked and an expected strong correlation was found between catchment area and main stream length (Table 3). As a result, stream length was not used in the regression study. There was also a moderate degree of correlation between area and urbanization, and between catchment slope and urbanization (Table 3); this correlation led to some unexpected results in the initial regression equations.

Dependent variables investigated by multiple regression analysis included the IUH peak ordinate (h_p), the IUH time to peak (t_p), and the two gamma-IUH parameters k, n; independent variables were catchment area (A), catchment slope (S), urbanization (U) as $(1+U)$, rainfall duration (T_r), and rainfall peak intensity (i_p); the remaining variables listed in Table 3 did not add significantly (5% level) to the regression equations, or were not used for reasons of linear dependence, as explained above. h_p was the only variable to which S and U added significantly to the relationship at the 5% level. Initial attempts to determine the dependence of h_p on U resulted in a negative correlation between h_p and U, i.e., a negative exponent for the term $(1+U)$ in the equation for h_p. This surprising result implied that increases in urbanization would act to reduce

TABLE 3—Correlation coefficients for the inter-relationship of the parameters

	h_p	t_p	k	n	T_r	V_e	i_p	t_l	A	L	S	1+U
h_p	1.000											
t_p	—0.812	1.000										
k	—0.954	0.614	1.000									
n	0.511	0.059	—0.740	1.000								
T_r	—0.721	0.632	0.682	—0.362	1.000							
V_e	—0.198	0.049	0.253	—0.302	0.576	1.000						
i_p	0.228	—0.359	—0.151	—0.083	—0.003	0.524	1.000					
t_l	—0.946	0.844	0.873	—0.385	0.791	0.248	—0.224	1.000				
A	—0.823	0.775	0.728	—0.257	0.564	—0.022	—0.304	0.820	1.000			
L	—0.806	0.738	0.725	—0.281	0.521	—0.051	—0.265	0.793	0.983	1.000		
S	—0.605	0.520	0.570	—0.305	0.584	0.463	—0.083	0.602	0.495	0.445	1.000	
1+U	0.670	—0.643	—0.589	0.208	—0.568	—0.213	0.177	—0.671	—0.751	—0.683	—0.811	1.000

Note: All parameters have been defined earlier in the text.

flood peaks. Further investigation showed that S and U only had a significant effect when both were together in the relationship. It was concluded that the negative correlation between h_p and U was spurious and both S and U were dropped from the relationships. If a wide range of catchments, including rural catchments of varying degrees of slope, were to be examined it is thought a more meaningful relationship between h_p, S and U may be possible.

The expressions best justified by the data studied are:

$$h_p \text{ (hours}^{-1}) = 1.339\, A^{-0.310}\, T_r^{-0.445}\ (R^2 = 0.774,\ SE = 0.574) \qquad (11)$$

$$t_p \text{ (hours)} = 0.474\, A^{0.228}\, T_r^{0.336}\, i_p^{-0.202}\ (R^2 = 0.691,\ SE = 0.576) \qquad (12)$$

$$k \text{ (hours)} = 0.139\, A^{0.333}\, T_r^{0.613}\ (R^2 = 0.639,\ SE = 0.945) \qquad (13)$$

where R is the correlation coefficient and SE the standard error of the estimate of the natural logarithm of the dependent variable. Units of the independent variables are: catchment area (A), km^2; rainfall duration (T_r), hours; peak rainfall intensity (i_p), cm/hour. The correlation for the second gamma distribution parameter, n, was not significant (R^2=.131) and will be discussed below. Because of the natural logarithmic transform, confidence intervals may be estimated as exp($\pm z_c$ x standard error), where z_c is the standard normal variate determined by the desired confidence level. For example, for the 68% confidence interval on h_c, $z_c = 1.00$ and $0.56\ h_p < h_p < 1.78\ h_p$. Confidence intervals for the other variables are of a similar size; it can be seen that the regression equations give only a rough estimate for the dependent variables.

The lack of correlation for the gamma distribution parameter n raises the possibility that n may be a constant. In line with earlier discussion about a nondimensional IUH, this implies that if a time scale T can be found such that T is proportional to t_p, then a single nondimensional gamma-IUH exists for all events and for all catchments. The conditions for a single dimensionless gamma-IUH are automatically satisfied if one takes $T = t_p$ and uses the average value of n from all storms as

$$n = 5.57 \qquad (14)$$

From (10), this implies

$$h_p t_p = h_p^* t_p^* = 0.84 \qquad (15)$$

Multiple regression was also used to check the possibility that $T = t_l$, the centroid to centroid time lag. Regression showed the following relations between t_l, t_p, and h_p:

$$h_p = 1.358\, t_l^{-0.960}\ (R^2 = 0.895,\ SE = 0.387) \qquad (16)$$

$$t_p = 0.578\, t_l^{0.729}\ (R^2 = 0.712,\ SE = 0.546) \qquad (17)$$

$$t_l = 2.160\, t_p^{0.977}\ (R^2 = 0.712,\ SE = 0.632) \qquad (18)$$

Results of both regressions of t_l with t_p are included to illustrate that there is a large difference depending on which parameter is the dependent variable. All time scales have units of hours.

Clearly, the exponents relating t_l with h_p and t_l with t_p are remarkably close to minus one and one, respectively. From the earlier discussion, these are precisely the exponents that might be expected if a dimensionless

IUH was to apply to the catchments studied. Because the dimensionless IUH has constant values of h_pt_l and t_p/t_l, the average values of these factors were calculated for the data examined. These average values were

$$h_pt_l = 1.501 \tag{19}$$

$$t_p/t_l = 0.558 \tag{20}$$

In addition from (7) with $n = 5.57$ we have

$$k = 0.122\ t_l \tag{21}$$

Finally, time lag may be found from catchment and storm characteristics

$$t_l = 0.998\ A^{0{\cdot}275}\ T_r^{0{\cdot}560}\ (R^2 = 0.831,\ SE = 0.488) \tag{22}$$

where t_l, T_r are in hours and A in km^2.

AN EXAMPLE OF THE USE OF THE METHOD

For purposes of illustration, the method described in this paper was used to generate flood hydrographs for the Mt Pleasant catchment in Christchurch. This catchment is located in the Port Hills and is completely developed with residential housing. Catchment characteristics were extracted from a 1:1000 scale map with 2-metre contours, supplied by the Christchurch Drainage Board. The catchment has an area of 74.3 hectares, a main stream length of 1.21 km, and a difference in elevation between the highest and lowest points in the catchment of 174 m. Storm runoff hydrographs, assuming no baseflow and no rainfall losses, were generated for the 5 year and 20 year storms, using two different rainfall patterns and two different rainfall durations, giving a total of eight separate cases. The storm patterns were (a) a constant rainfall intensity, and (b) the *Flood Studies Report* (NERC, 1975, vol. 2, Table II.6.3) 75% winter recommended design storm profile, which distributes the total rainfall over the total storm duration T_r as

Time t/T_r	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
% Storm Rainfall	25	20	15	12	8	6	5	4	3	2

Storm durations were chosen so as to be equal to (a) the time of concentration (T_c) of the catchments, and (b) the catchment time lag (t_l). Time of concentration (T_c) was estimated from an empirical formula attributed to the U.S. Soil Conservation Service and listed in the Ministry of Works' *Technical Memorandum No. 61* (MWD, 1975), as:

$$T_c(hr) = (0.87L^3/H)^{0.385} \tag{23}$$

where L is maximum flow length (km) and H is the difference in elevation (m) between the highest and lowest points in the catchment. Substituting the values of L, H for Mount Pleasant gives $T_c = 0.16$ hours, or just under 10 minutes. Time lag (t_l) is a function of the runoff hydrograph as well as the rainfall hyetograph, and is not known in advance. However, because we are considering the case in which storm duration is equal to basin time lag, for the purpose of this example an estimate of t_l was obtained from the regression equation (22) by equating $T_r = t_l$ and solving for $t_l = 0.82$ hr, or just under 50 minutes. Values of storm rainfall were obtained from data in Carver and Greenland (1977). The gamma-IUHs for each of the eight cases were derived by specifying h_p, t_p using the regression equations (11) and (12), and then determining the gamma distribution parameters k, n by the peak fitting method specified earlier (see equations (6) and (7)). A computer program was then used to carry out the convolution between rainfall (i(t)) and the gamma-IUH (h(t)) specified by equation (1). (This convolution could easily be performed on a programmable calculator.) The results are summarized in Table 4; the storm runoff hydrograph for the case of the 5-year, .82 hour storm, using the Flood Studies Report 75% winter storm profile, is illustrated in Figure 7. Runoff hydrographs for the other seven cases have a similar shape, but with different peak flows and times to peak.

SUMMARY AND CONCLUSIONS

A total of 159 storms on seven North Island catchments were studied to derive hydrographs for gauged and ungauged catchments. Using a synthetic IUH based on the two-parameter gamma probability density function, fitted such that the peaks of the gamma-IUH and actual IUHs coincide, it was found that storm quickflow hydrographs could be satisfactorily reproduced using the gamma-IUH. In the proposed design method, the gamma-IUH peak and time to peak must be specified, either from storm analysis (gauged catchment) or from the regression equations (11) and (12) given in this paper (ungauged catchment). The gamma-IUH parameters k, n can then be determined from the peak and peak time using (6) and (7). The design hydrograph is derived by convolution of the gamma-IUH with a design rainfall pattern.

Results of regression for the gamma distribution parameters k, n showed that n was independent of storm and catchment characteristics, indicating the possibility of deriving a single dimensionless gamma-IUH, with time scale equal to peak time or time lag. The conditions for such a dimensionless hydrograph are satisfied if one uses the relations for n, k,

TABLE 4—Example of storm hydrograph derivation for Mt. Pleasant catchment, Christchurch.

Storm Return Period (years)	Rainfall Duration T_r(hr)	Rainfall Timing Pattern	Gamma-IUH Time to Peak, t_p (hr)	Gamma-IUH Peak h_p (hr^{-1})	Peak Rainfall Intensity i_p (cm/hr)	Storm Peak Discharge (m^3/s)
20	.16	Constant	.16	3.3	8.6	8.6
20	.82	"	.34	1.6	2.8	5.0
5	.16	"	.18	3.3	4.8	4.8
5	.82	"	.37	1.6	1.7	3.1
20	.16	*Flood*	.082	3.3	21,6	8.4
20	.82	*Studies*	.18	1.6	6.9	5.2
5	.16	*Rept.*	.093	3.3	12.0	4.7
5	.82	Profile*	.19	1.6	4.3	3.3

* 75% winter recommended design storm profile (NERC, 1975, Vol. 2, Table II.6.3, and Vol. 1, Fig. 6.65).

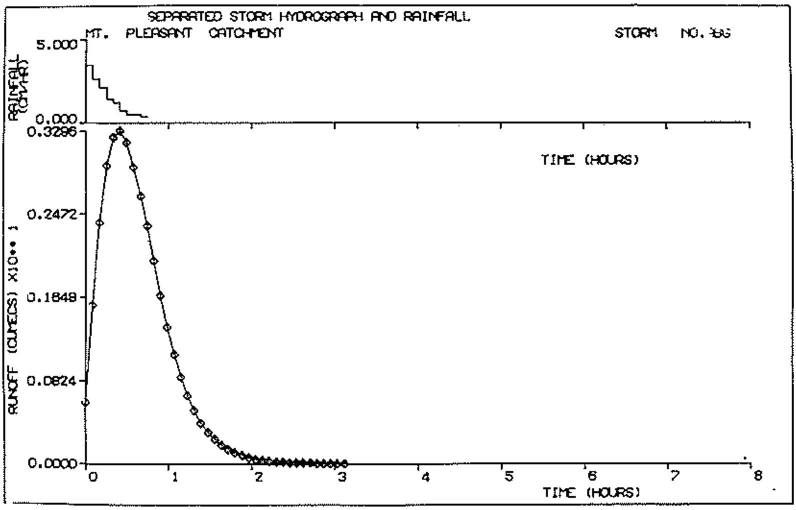

FIG. 7—Storm runoff hydrograph generated by gamma-IUH for 5 year, 0.82 hour rainfall, distributed according to Flood Studies Report (NERC, 1975) 75% winter recommended design storm profile.

h_pt_p, and t_p/t_l specified by (14), (21), (15) and (20), respectively, with t_l related to storm and catchment characteristics by (22). However, this method is subject to greater error than that of specifying h_p, t_p as described above. There is some conjecture as to the general applicability of a single parameter gamma-IUH. Croley (1980) has argued that a single parameter gamma hydrograph is not supported by the results of a large number of studies. Although Nash (1960) obtained very low coefficients of correlation of n with catchment physiographic characteristics, he felt there was sufficient statistical evidence for maintaining that such a relationship exists. On the other hand, NERC (1975), in an examination of over 1500 events on British catchments, concluded that the product of unit hydrograph peak with the unit hydrograph peak time was a constant (i.e. by implication that h_pt_p was constant). All that can be concluded from this study is that a single parameter gamma-IUH described the response of the catchments studied. Considering that the catchments used in this study were from very different hydrological zones, some further study of the general applicability of the relationship found is warranted.

A sensitivity study carried out on the Wairau Creek events showed that a change in IUH peak was reflected by an almost equal change in quickflow hydrograph peak. From the standard error estimate for h_p (see equation 11), it is seen that the IUH peak may be determined with 68% confidence limits of approximately —44% to +78%. Because the magnitude of the quickflow hydrograph peak depends almost directly on the magnitude of the IUH peak, the reproduced hydrograph peak is expected to vary from a 44% underestimate to a 78% overestimate, on average, in 68% of events.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the Water and Soil Division, MWD, and the NZ Meteorological Service for permission to use their data. Thanks are also due to the staff members of Water and Soil Division, MWD, to the Rotorua District Council, and the Christchurch Drainage Board, which supplied catchment information; and to Mr Ang Beng Ghee who carried out the calculations for the example.

The authors are particularly grateful for the assistance and advice of Dr M. E. Beable.

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