ESTIMATION OF ROUGHNESS COEFFICIENT FROM VELOCITY MEASUREMENTS IN OPEN CHANNELS

A.J. Raudkivi

Civil Engineering Department,
University of Auckland

It is frequently necessary to estimate river discharge by slope-area measurements. The accuracy of the estimate depends on the value of the roughness coefficient used in the formula for the calculation of the mean velocity. These empirical formulae are of the form

$$V = K\, R^a\, S_e^{\,b} \qquad (1)$$

where V is the mean velocity,

R is the hydraulic radius,

S_e is the slope of the energy gradient,

K, a, b are coefficients.

The estimation of the roughness coefficient is probably one of the most difficult tasks in any method of discharge calculation. Useful guidance may be obtained from a consideration of the logarithmic velocity distribution law. For a flow over a rough boundary the theory yields (Rouse, 1946:195)

$$v/v_* = 5.75 \log (y/k) + 8.5 \qquad (2)$$

where v is the velocity at distance y from the bed in the direction of mean flow,

$v_* = (to/p)^{\frac{1}{2}}$ is the shear velocity

to is the boundary shear stress

p is the density of the water

k is the mean equivalent roughness height.

Integrating equation (2) for unit width and dividing by the area of the unit strip yields

$$V/v_* = 5.75 \log (yo/k) + 6 \qquad (3)$$

where yo is the mean depth.

Deducting equation (3) from equation (2) gives

$$(v - V)/v_* = 5.75 \log (y/yo) + 2.5 \qquad (4)$$

Equation (4) can be transcribed in terms of the Darcy-Weisbach friction factor f, using the relation quoted by Rouse (1946:203),

$$\sqrt{to/_p} = \sqrt{f/_8}$$

as

$$(v - V)/(Vf^{\frac{1}{2}}) = 2 \log (y/yo) + 0.88 \qquad (5)$$

Observational results agree well with these formulae under laboratory conditions. In the field the velocity distribution can be modified by the secondary currents, but the agreement, usually, is still reasonable.

Since $t = g\,pR\,S_e$ where g is the acceleration of gravity, v_* in equation (2) can be replaced by $(g\,R\,S_e)^{\frac{1}{2}}$. Using the approximations that v = V at y = 0.4 yo (equivalent to the mean in vertical at 0.6 D measured from the surface) and R = yo yields

$$V = 32.6\ (yo\ S_e)^{\frac{1}{2}}\ \log\ (13.2\ yo/k) \qquad (6)$$

Comparing this with the Chezy formula

$$V = C\ (yo\ S)^{\frac{1}{2}} \qquad (7)$$

yields

$$C = 32.6\ \log\ (yo/k) + 36.5 \qquad (8)$$

This relationship is shown in Figure 1 in graphical form.

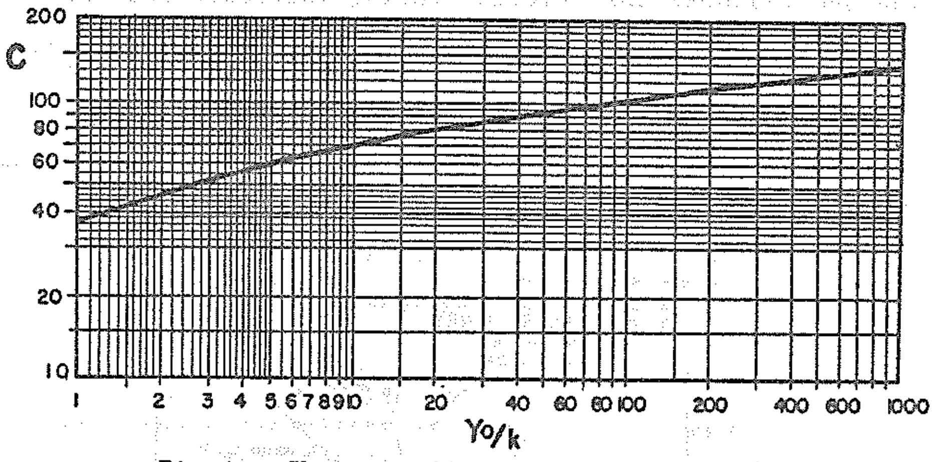


Fig.1 - Chezy coefficient C versus yo/k (yo being mean depth, k is mean equivalent roughness height)

At about yo/k = 30 the slope is approximately $\tan^{-1}(1/6)$ and in this region

$$C \propto (yo/k)^{1/6} = K(yo/k)^{1/6}$$

which upon substitution into the Chezy formula (8) yields the Manning formula.

$$V \propto yo^{2/3}\ S^{\frac{1}{2}}$$

In other words, the Manning formula is applicable in the neighbourhood of yo/k = 30. In the region of, say yo/k= 10, C is proportional to $(yo/k)^{\frac{1}{4}}$, which gives

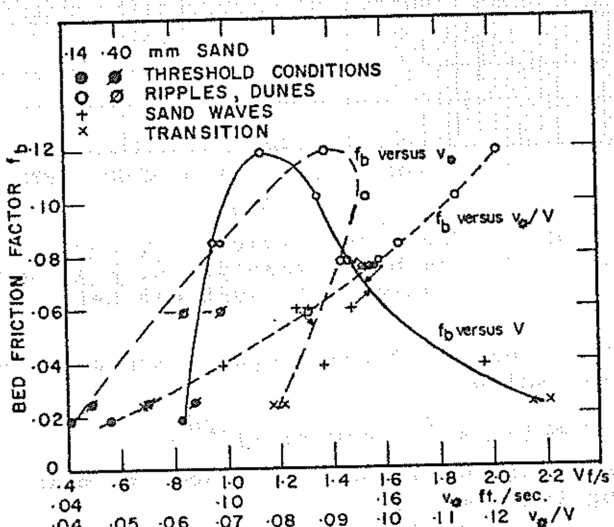
$$V \propto yo^{\frac{3}{4}}\ S^{\frac{1}{2}}$$

Thus depending upon the ratio of (yo/k) one may derive a wide variety of formulae of the same kind but with different exponents.

This treatment so far assumes that the boundary is rough but fixed and has the mean roughness height k. It is important to realise that k is not the measure of sand or shingle size but implies that the same fixed and constant cross section having a uniform roughness formed by the particles of diameter k would yield, with a given gradient, the same velocity and discharge as the actual channel. The k measures the combined effect of particle size and surface waviness. Flow over a rippled sandbed could show a k value about a hundred times the particle diameter.

For example, when laboratory results obtained from a straight flume are plotted in terms of the bed friction factor f_b, versus shear velocity v_* or mean velocity V, then it can be seen (Figure 2) that with the formation of the bed features the friction factor increases and subsequently decreases as dunes give place to sandwaves. The friction factor is nearly as low for the transition flat bed as for the flat bed at the threshold conditions of sand movement. The anti dune range has not been investigated in these experiments.

For the peak value of f_b and 0.40 mm sand the k value is approximately 2 inches or 130 times the particle diameter.



BED FRICTION FACTOR VERSUS v_*, V AND v_*/V FOR MEAN SAND SIZE d OF 0·14 AND 0·40mm. DISCHARGE CONSTANT FOR 0·14mm SAND AND VARIABLE FOR 0·40mm SAND.

FIG. 2

The data on Figure 2 were first plotted as f_b versus $(v_{*b}/V)^2$, and by definition of f, viz. : $f = 8(v_*/V)^2$ should lie on a parabola.

The experiment with 0.40 mm sand was run at approximately constant mean depth and constant temperature. Hence if f_b is plotted against V or v_{*b} this is analogous to plotting f versus Reynolds number.

The experiment with 0.14 mm sand was run at constant discharge and variable depth, nevertheless the trend is the same.

Vertical velocity traverses, obtained by measurement, should plot approximately on a straight line on log-natural paper. The intercept with the y-axis at zero velocity is $y' = k/33$ or $k/30$. The Darcy-Weisbach friction factor f can be obtained with the aid of equation (5) (see Rouse, 1946:199, ex.37). Introducing this f value into the resistance equation for the two-dimensional case

$$\frac{1}{\sqrt{f}} = 2 \log (yo/k) + 2.12$$

yields yo/k, and the corresponding value of the Chézy coefficient C can be read from Figure 1.

From measurements at various river stages a plot of Chézy C or Manning N versus river stage could be established for a station and would assist in selection of the roughness values on the river. However, it must be kept in mind that on tortuous rivers the bend losses may be the primary cause of head loss and may overshadow the friction loss.

If vertical velocity traverses are taken in conjunction with bed load sampling it is useful to know the value of mean bed shear stress at this point. From

$$v_* = v/(5.75 \log y/y')$$

and the above mentioned velocity plot, this local shear stress is readily obtained.

REFERENCE

Rouse,H. 1946: Elementary Mechanics of Fluids. John Wiley & Sons, New York, 376 pp.