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Evidence of trend in return levels for daily rainfall in New Zealand

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Abstract

Annual maxima of daily rainfall over time are modeled for sixteen locations spread throughout New Zealand. The generalized extreme value distribution is fitted to each data set to describe both the mean and the variability of annual maximum daily rainfall for each location and to predict their behavior in the coming decades. Only in a few cases did we find evidence of any trend. We also give estimates of 2, 10 and 100-year return levels for daily rainfall and describe how they vary with the latitude of the locations.

Introduction

The T-year return level of a variable is the level exceeded on average only once in T years. Thus, for example, the 2-year return level is the median of the distribution of the variable recorded annually. The variable studied here is the annual maximum of daily rainfall. We estimate its 2, 10 and 100-year return levels for sixteen locations.

Estimation of return levels is usually based on the three extreme value distributions—Gumbel, Fréchet and negative Weibull—suggested by Fisher and Tippett (1928) for behavior of maxima of large samples. In this paper we use the generalized extreme value distribution. It was developed by Jenkinson (1955) by combining the three extreme value distributions; see also Hosking *et al.* (1985), Galambos (1987) and Davison and Smith (1990).

Little has been published on return levels for New Zealand rainfall. Work related to the Gumbel distribution for New Zealand rainfall and storms includes Tomlinson (1980), Revfeim and Hughes (1983), Thompson (1987), McKerchar and Pearson (1989) and Revfeim (1982, 1983, 1991, 1992, 1998).

The generalized extreme value distribution has been applied to rainfall and flood data in New Zealand by Pearson (1991), Griffiths and Pearson (1993), Withers and Silby (1994) and Pearson and Henderson (1998), and to overseas data by Jenkinson (1955), Schaeffer (1990), Buishand (1991) and others. Farago and Katz (1990) review applications of extreme value theory to climate data.

We also explore the data for trends. Rainfall data are notoriously difficult to analyze for trends. Various attempts to look for trends in rainfall data have been made with dubious success. For New Zealand rainfall, these include the use of regression, moving averages and other filters (de Lisle, 1961; Withers and Pearson, 1991; Bradford, 1992; Withers and Silby, 1994). Withers and Pearson (1991), using cusum techniques, found some suggestion of jumps in rainfall. Among other work on New Zealand rainfall we mention May and Kidson (1971), Trenberth (1977), Coulter and Hessell (1980), Renwick (1983), Thompson (1984) and Revfeim and Withers (1990). Turning to overseas rainfall, we also have the use of filters—Karl (1980) and World Meteorological Organization (1988, 1990), modeling rainfall as sums of sinusoids—Tabony (1981), Kane (1988), Kane and Trivedi (1988) and Kane and Teixeira (1991) and use of empirical and other methods—Barnett (1985) and Folland *et al.* (1991).

None of these model the variability as well as the mean of rainfall extremes, although simulated changes in variability, along with a discussion of rainfall predictions using global circulation models, may be found in Gordon *et al.* (1992) and Sections 6.4.3, 6.4.4 and 6.5.6 of Houghton *et al.* (1996). The novelty of this paper is that it provides a method for modeling both the mean and the variability of rainfall extremes.

Rainfall data sets

The data consists of annual maxima of daily rainfall (9am-9am) in mm, which is variable forty one of the New Zealand climate database (Penney, 1997; see also Fouhy *et al.*, 1992). The data sets cover sixteen locations spread from Auckland in the North Island to Invercargill at the bottom of the South Island.

The locations of the sixteen stations are shown in Figure 1 (opposite). The station names, station numbers and years of data are listed in Table 1.

Of these series, Auckland at Albert Park, Rotorua at Whakarewarewa and Gisborne at Patutahi have been discontinued.

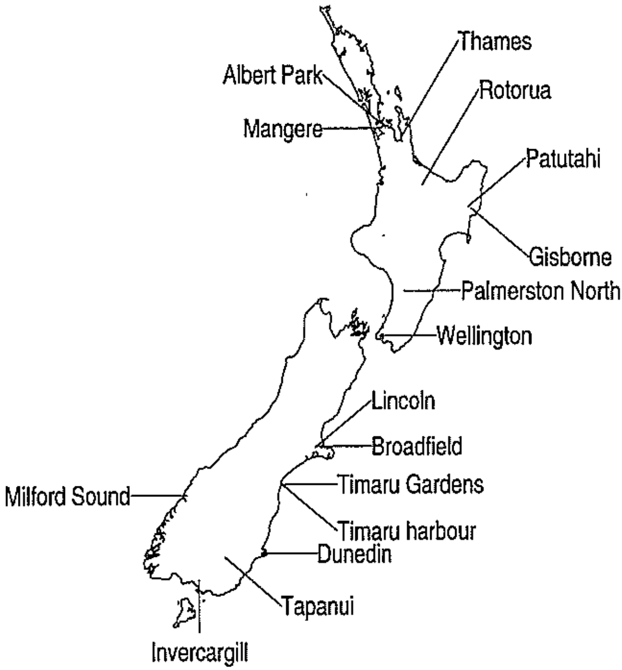

Figure 1 – Locations of the climate stations

Table 1 – Climate station data sets used

Station Name	Number	Years
Auckland, Albert Park	A64871	1863-1992
Mangere	C64971	1962-97
Thames	B75152	1958-97
Rotorua, Whakarewarewa	B86124	1899-1981
Gisborne, Patutahi	D87691	1893-1957
Gisborne Airport	D87692	1938-91
Palmerston North, BHS	E05362	1929-97
Wellington, Kelburn	E14272	1928-98
Lincoln	H32641	1881-1987
Lincoln, Broadfield	H32642	1973-98
Timaru Gardens	H41421	1884-1984
Timaru Harbour	H41422	1943-97
Milford Sound	F47691	1930-98
Dunedin, Bot. Gardens	I50852	1913-97
Tapanui	I59921	1898-1996
Invercargill Airport	I68433	1943-98

Methods

The method that we use allows both the mean and the variability of rainfall extremes to change with time. It has had some success with temperature data (see Withers and Nadarajah, 2000).

In terms of the standardized time variable

$$x_N = N/(n+1),$$

the model chosen for the annual maximum rainfall, Y_N , for years N=1, ..., n is

$$Y_N = \mu_N(\alpha) + \sigma_N(\beta)\varepsilon_N, \tag{1}$$

where either

$$\mu_N(\alpha) = \sum_{i=0}^{m} \alpha_i x_N^i \quad \text{and} \quad \alpha_N(\beta) = \exp\left(\sum_{i=0}^{m} \beta_i x_N^i\right),$$

giving a model denoted L(m), or

$$\mu_N(\alpha) = \exp\left(\sum_{i=0}^{m} \alpha_i x_N^i\right) \quad \text{and} \quad \alpha_N(\beta) = \exp\left(\sum_{i=0}^{m} \beta_i x_N^i\right),$$

giving a model denoted E(m). Here, $\mu_N(\alpha)$, the mean of the process, and $\sigma_N(\beta)$, the variability or scale of the process, have been parameterized in such a way as to model a variety of trend behavior. The case $m = 1$ corresponds to linear trends and $m = 2$ to quadratic trends. The case $m = 0$ corresponds to no trend. The models L(0) and E(0) are equivalent in this case.

The noise or residuals, $\{\varepsilon_1,...,\varepsilon_n\}$, are assumed to be independent, with the generalized extreme value distribution

$$F_\xi(x) = \Pr(\varepsilon_N \le x) = \exp\left\{-(1+\xi x)^{-1/\xi}\right\} \text{ for } 1+\xi x > 0 \tag{2}$$

where ξ is referred to as the shape parameter. The constraint $1 + \xi x > 0$ implies that for $\xi < 0$, maxima are bounded above by $\mu_N - \sigma_N/\xi$, and for $\xi > 0$, maxima are bounded below by $\mu_N - \sigma_N/\xi$. The particular case of (2) for $\xi = 0$,

$$F_0(x) = \exp\{-\exp - (-x)\} \text{ for } -\infty < x < \infty$$

is known as the EV1 or Gumbel distribution, while the cases $\xi > 0$ and $\xi < 0$ are known as the EV2 or Fréchet, and the EV3 or negative Weibull distributions, respectively. For maximum rainfall and floods, usually $\xi > 0$ (EV2), although, as noted, earlier work tended to fit the Gumbel model, $\xi = 0$.

Results and discussion

Fitting of models

Fitting the model, (1), means obtaining estimates of the unknown vectors α, β and the unknown shape parameter ξ. We used the maximum likelihood estimates (see Appendix for details) denoted (α, β, ξ).

For each of the sixteen data sets we fitted the following models by the method of maximum likelihood: L(0) with $\xi = 0$ – denoted below by 'constant-Gumbel', L(0), L(1), L(2), E(1) and E(2). We then applied the likelihood ratio test (see Appendix for details) to discriminate at the 95% level between the models constant-Gumbel, L(0), L(1) and L(2) in a step-down manner. This was repeated for the models constant-Gumbel, E(0), E(1) and E(2).

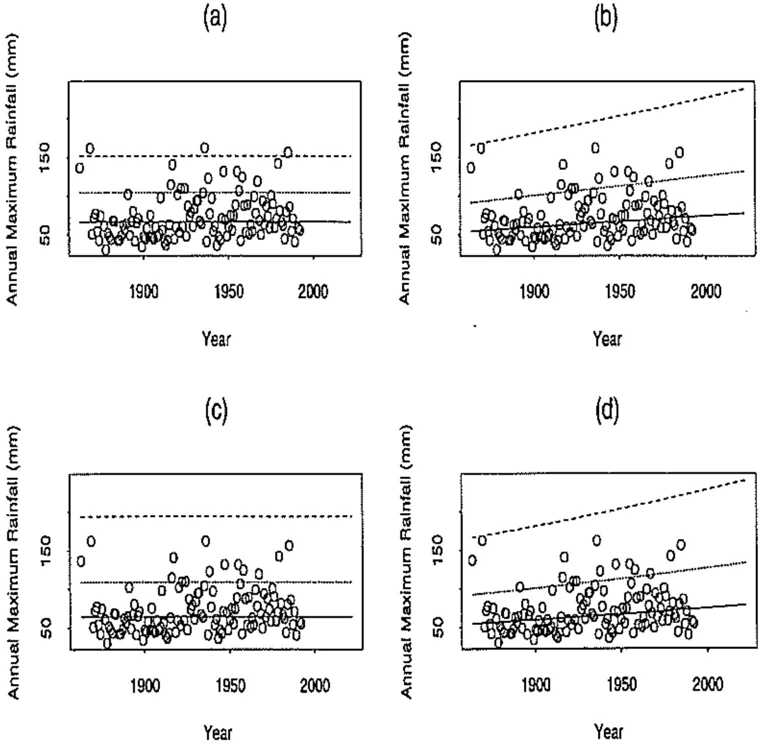

Figure 2 – Annual maximum rainfall for Auckland at Albert Park. Models: (a) constant-Gumbel; (b) L(1); (c) L(0); (d) E(1). Statistical significance: for L(0) versus L(1) $p = .02$; for L(0) versus E(1) $p = .02$. Line types: ———— for 2-year rainfall; .... for 10-year rainfall; - - - - - - for 100-year rainfall

Higher order polynomials are often better at describing a data set, but their projections into the future tend to fluctuate too wildly, and in the case of variability, to either shrink too quickly or expand too quickly. This was often found to be the case with fitting the cubic models, L(3) and E(3), to the data, thus those results were discarded.

The main results of the fits are shown in Figures 2-4. The three stations illustrated go from north to south with the best models on the right. In each set of four plots, the top two refer to the first two models listed and the

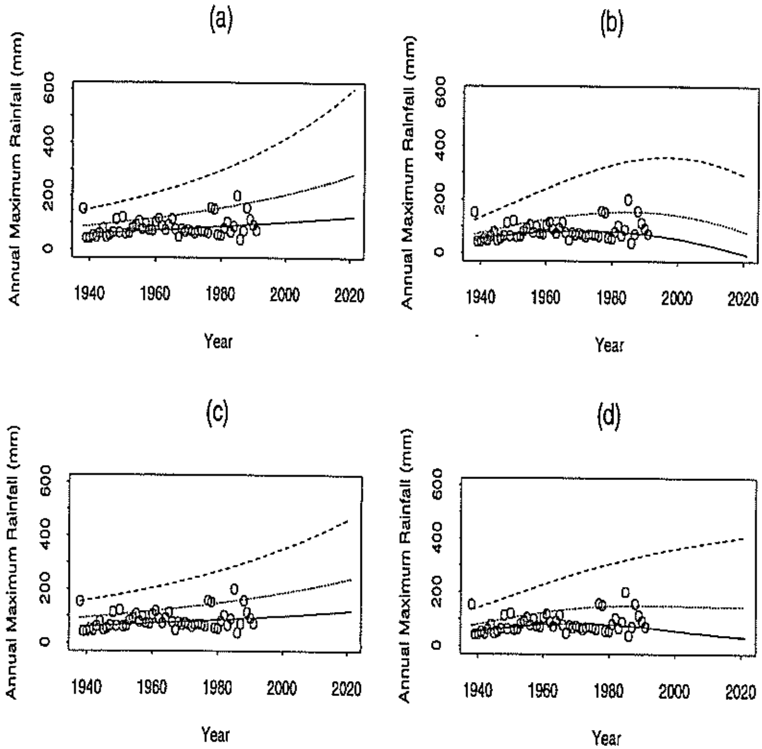

Figure 3 – Annual maximum rainfall for Gisborne Airport. Models: (a) L(1); (b) L(2); (c) E(1); (d) E(2). Statistical significance: for L(1) versus L(2) $p = .02$; for E(1) versus E(2) $p = .02$.
Line types: ———— for 2-year rainfall; .... for 10-year rainfall; - - - - - - for 100-year rainfall

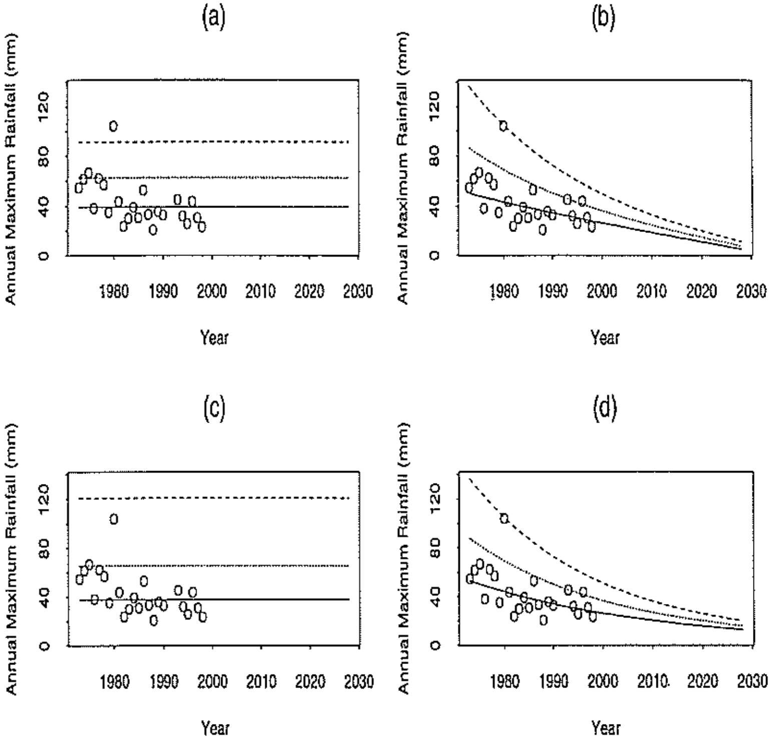

Figure 4 – Annual maximum rainfall for Lincoln at Broadfield. Models: (a) constant-Gumbel; (b) L(1); (c) L(0); (d) E(1). Statistical significance: for L(0) versus L(1) $p = .01$; for L(0) versus E(1) $p = .01$.
Line types: ————— for 2-year rainfall; .... for 10-year rainfall; - - - - - - for 100-year rainfall

bottom two to the last two models listed. For those data sets, where L(0) or the constant-Gumbel is the best model, no figure is given. For those data sets, where L(1) or E(1) is the best model, we plot the return levels for Y_N for the four models constant-Gumbel, L(0), L(1) and E(1). For those data sets, where L(2) or E(2) is the best model, we plot the return levels for the four models L(1), E(1), L(2) and E(2). The captions also include *p*-values of the likelihood ratio test (see Appendix) of one model versus another. In each figure the 2-year (median), 10-year and 100-year return levels of daily rainfall are plotted, i.e.

$$\mu_N(\hat{\alpha}) + \sigma_N(\hat{\beta})F_\xi^{-1}(q) \quad \text{for} \quad q = .5, .9, .99$$

The median ($q = .5$) is the solid line and the other return levels are the dotted and dashed lines. Note that F_ξ^{-1} (exp(-1)) = 0 so that $\mu_N(\alpha)$ is the exp(-1)≈37% quantile.

Return levels

Traditionally return levels were calculated on Gumbel paper, that is using the constant-Gumbel model. This model however gave the best fit only for some of the sixteen stations. For Milford Sound, Dunedin and Tapanui we found that the model, L(0), gave the best fit, but with $\xi \neq 0$. The corresponding return levels for these three stations were significantly higher than what would have been obtained using the constant-Gumbel model. This shows that the traditional method using Gumbel paper can seriously underestimate return levels, as shown by Pearson and Henderson (1998).

With regard to change of return levels with time, there was no evidence of significant trend except for Albert Park, Broadfield and Gisborne Airport. For Albert Park and Broadfield the linear models, L(1) and E(1), gave the best fits (Figs. 2 and 3). For Gisborne Airport the quadratic models, L(2) and E(2), gave the best fits (Fig. 4). The slopes of the linear model, L(1), for these three stations were 1.4 mm/decade for Albert Park, 5.6 mm/decade for Gisborne Airport, and -7.6 mm/decade for Broadfield. Variability was also increasing for the North Island series (Albert Park and Gisborne Airport) and decreasing for the South Island series (Broadfield). Trees growing up

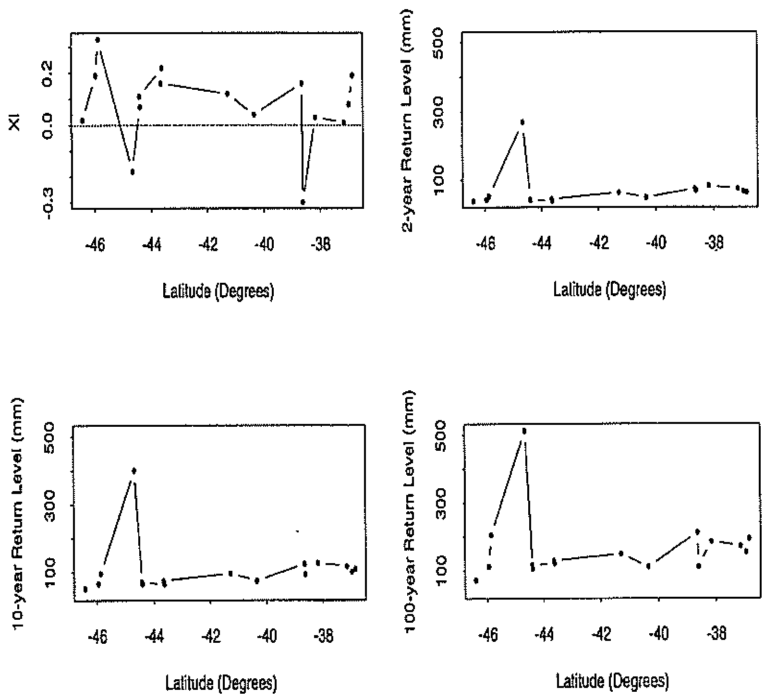

Figure 5 – Estimates of the shape parameter, Xi, and the return levels (2, 10 and 100 years) versus latitude for the model L(0)

around Albert Park and urbanization during the period of records (1863-1992) may both have contributed to the trend for that station.

Figure 5 plots the estimates of the shape parameter, ξ (denoted Xi), and the 2, 10 and 100-year return levels for daily rainfall against latitude (in degrees) for the model, L(0), for the sixteen stations, running from south on the left to north on the right. The third and fourth points from the left (Dunedin at -45.9 and Milford Sound at -44.7) behave quite differently from the rest. Without them, there appears to be some evidence of a mild increase in the return levels going from south to north.

Appendix

Here we comment on maximum likelihood estimation and likelihood ratio tests.

Let $\theta = (\alpha, \beta, \xi)$. The likelihood of θ, denoted $L_n(\theta)$, is defined as the joint density of $(Y_1,...,Y_n)$ evaluated at the realized observations, i.e.

$$L_n(\theta) = f(y_1, ..., y_n; \theta).$$

Thus, using equations (1) and (2), we can write

$$L_n(\theta) = \exp\left\{-\sum_{N=1}^{n}\left(1+\xi\frac{y_N-\mu_N(\alpha)}{\sigma_N(\beta)}\right)^{-1/\xi}\right\}\prod_{N=1}^{n}\frac{1}{\sigma_N(\beta)}\left(1+\xi\frac{y_N-\mu_N(\alpha)}{\sigma_N(\beta)}\right)^{-(1/\xi+1)}.$$

The principle of maximum likelihood estimation is to estimate θ by the value that maximizes $L_n(\theta)$. There are many numerical algorithms that maximize functions of several variables: we used the E04JAF routine from the NAG Fortran library to perform the maximization.

For the likelihood ratio test, suppose we have $\theta = (\theta_1, \theta_2)$, where θ_2 is a vector of length k, and that we wish to test the hypothesis that $\theta_2 = \mathbf{0}$. Let

$$l_n = \sup_{\theta_1} L_n(\theta_1, \mathbf{0}) \,/\, \sup_{\theta} L_n(\theta_1, \theta_2).$$

The likelihood ratio test of level approximately 1-α (usually, 0.95) is to accept the hypothesis if $-2 \log l_n \leq \chi^2_k (1-\alpha)$, where $\chi^2_k (1-\alpha)$ is the $1-\alpha$ quantile of the χ^2_k distribution. For testing L(m) given L(m + 1), or for testing E(m) given E(m + 1), $k = 2$ so that l_n is approximately uniformly distributed on (0,1), and hence the p-value, a measure of the level of significance of the test, is identical to l_n. The test accepts the hypothesis that L(m + 1) is not a significant improvement on L(m) or that E(m + 1) is not a significant improvement on E(m) if the respective p-value is greater than α.

For further details on maximum likelihood estimation and likelihood ratio tests, see Wald (1943) and Cox and Hinkley (1974).

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