PERSISTENCE OF DAILY RAINFALL AT SOME NEW ZEALAND STATIONS

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ABSTRACT

An analysis was carried out of the frequency of runs of various types of rainfall days at Wellington, Milford Sound, Alexandra, and Invercargill. In all seven sets of runs examined the results indicated that the probability of the event occurring increased appreciably after it had occurred once. The frequencies all fitted the simple Markov chain model reasonably well. The highest persistence found was for days with at least 0.01 in. of rain at Milford Sound in winter, and the lowest for days with at least 0.01 in. of rain at Alexandra in spring and summer. All the sets of runs showed a maximum of persistence in autumn or winter. The minimum in spring at some stations was considered to be associated with the generally rapid movement of weather systems in spring in the New Zealand area.

INTRODUCTION

Intensity and persistence of rainfall are two factors of importance in causing erosion. In New Zealand, rainfall intensities have been studied by Robertson (1963).

Persistence may be studied by tabulating the frequencies of sequences of days with not less than a certain amount of rain. The frequencies may be compared with results produced by mathematical models.

Studies of this type have been made for about 50 years. A new approach was first suggested by Brooks and Carruthers (1953). It was based on the statistical technique of the Markov chain, although not at the time referred to by this name. Later workers further developed the method; among these were Lawrence (1950), Gabriel and Neumann (1952), Caskey (1963), Weiss (1964) and Feyerherm and Bark (1965).

The application of various models to some New Zealand rainfall data is illustrated below. The aim is to find which model best describes the data. Rainfall figures from Milford Sound, Invercargill, Wellington and Alexandra are used.

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MODELS USED TO DESCRIBE RUNS OF EVENTS

Let P be the probability of occurrence of an event. If the event occurs n_1 times in N days then P can be taken as n_1/N.

Random Model

Suppose the probability of occurrence P of an event on any day is quite independent of its occurrence on any other day. The probability of the non-occurrence of the event is $(1-P)$ and the probability of a day without the event followed by a day with the event is $(1-P)P$.

The number of runs of at least 1 day $= N(1-P)P$.

The number of runs of at least 2 days $= N(1-P)P^2$.

By subtraction it follows that the number of runs of just 1 day $= N(1-P)P - N(1-P)P^2 = N(1-P)^2P$.

Similarly the number of runs of just n days $= N(1-P)^2P^n$.

This may be called a random model.

Simple Persistence Model

Suppose that the occurrence of an event on any day affects its occurrence on the following day. Let the probability of occurrence of the event if it has already occurred on the previous day be p_1. Let the total number of runs be d_1.

The number of runs of at least 1 day $= d_1$.

The number of runs of at least 2 days $= d_1p_1$.

By subtraction the number of runs of just 1 day $= d_1(1-p_1)$.
Similarly the number of runs of just m days $= d_1(1-p_1)p_1^{m-1}$.

In order to calculate p_1 we use the fact that:

$$n_1 = d_1(1-p_1)(1+2p_1+3p_1^2+ \ldots \text{ to } \infty)$$
$$= d_1(1-p_1) \times \frac{1}{(1-p_1)^2} = \frac{d_1}{1-p_1}$$

so that $p_1 = 1 - \frac{d_1}{n_1}$.

Note also that the mean length of run is given by $\frac{n_1}{d_1} = \frac{1}{1-p_1}$.

The above derivation follows mainly Brooks and Carruthers (1953, pp. 310–12) and Weiss (1964).

More complicated models may be postulated in which the occurrence of an event 2, 3, 4 . . . times in succession affects its occurrence on the following day.

MEASURES OF PERSISTENCE

Various statistics have been suggested to give the degree of persistence. Brooks and Carruthers (1953, p. 314) define the "Persistence Ratio" (R) as "the ratio of the mean length of runs of occurrences in a series of observations to the mean length in a random series in which the general probability is the same". Thus:

$$R=\frac{1}{1-p_1}\Big/\frac{1}{1-P}=\frac{1-P}{1-p_1}$$

Another useful parameter is r_B which is defined as

$$r_B=1-\frac{1}{R^2}=1-\left\{\frac{1-p_1}{1-P}\right\}^2$$

r_B normally varies from 0 to +1 and is analogous to the serial correlation coefficient (or autocorrelation) for a continuous variable.

MILFORD SOUND EXAMPLE

To illustrate the testing of the two models, let us use the frequencies of runs of days at Milford Sound with at least 0.50 in. of rain. This station, with an average annual rainfall of 253 in., is the wettest in New Zealand for which we have a considerable number of years of record.

The driest season at Milford Sound is the winter (June, July and August), which receives only 19% of the total annual rainfall.

The result of an analysis of runs is shown in Table 1.

For the purposes of testing the two models, all the months have been combined. It should be noted, however, that:

(a) Runs of more than 8 days commenced only during September–December and in April and May. Four of the eight commenced in December.

(b) The three longest runs in the period 1935–64 were:
   13 days, 16–28 April 1953: total 30.86 in.
   13 days, 2 December 1963–9 January 1964: total 25.61 in.
   11 days, 14–24 December 1957: total 31.49 in.

For these Milford Sound runs P=0.300, but the random model (A) has far more runs of 1 day than the observed number, and fewer runs of 5 days and more, than those observed. $p_1=0.495$, and Model B, the simple Markov chain, is a fairly satisfactory fit; that is, the chance of recording at least half an inch of rain on a day following one on which this event has already occurred, is considerably greater than the over-all chance.

TABLE 1 — Milford Sound. No. of runs of days with at least 0.50 in. of rain (1935–64), and calculated values with three different models

No. of days	No. of Runs Observed	*Calculated Values* Model A (Random)		Model B (Markov Chain)		Modified (p_7=0.629)
1	841	$N(1-P)^2P$	1630	$d_1(1-p_1)$	840	—
2	413	$N(1-P)^2P^2$	542	$d_1(1-p_1)p_1$	416	—
3	215	$N(1-P)^2P^3$	180	$d_1(1-p_1)p_1^2$	206	—
4	98		60		102	—
5	47		20		50	—
6	22		6.6		25	—
7	7		2.2		12.4	9.6
8	8		0.7		6.1	6.1
9	4		0.2		3.1	3.7
10	4		0.1		1.5	2.4
11	1		—		0.7	—
12	0		—		0.37	—
13	2		—		0.18	—
14	0		—		0.09	—
11+	3		0.04		1.5	4.1

Total number of days in period: 10958 (N).

Total number of days with at least 0.50 in. of rain: 3297 (n_1).

Total number of runs: 1662 (d_1).

Over-all probability of day with at least 0.50 in. of rain:

$$\frac{3297}{10958}=0.300\ (P).$$

$$p_1=1-\frac{d_1}{n_1}=0.495$$

Persistence ratio $$R=\frac{1-P}{1-p_1}=1.39$$

$$r_B=1-\frac{1}{R^2}=0.48$$

Seasonal persistence ratios R and over-all probabilities P

	R	P
Summer	1.31	0.309
Autumn	1.36	0.287
Winter	1.55	0.269
Spring	1.36	0.332

NOTE: 1. In this and subsequent tables in this paper, season and month refer to the date of commencement of runs of any kind.

2. "11+" means 11 days or more.

Further Adjustment to Model B

A χ^2 test indicates that the fit of the calculated values in Model B is still not entirely satisfactory. Difficulties of a somewhat similar nature have occurred with tests of the Markov chain model made elsewhere. It may be seen from above that in this model the ratio: Number of runs of at least (m+1) days: Number of runs of at least m days, should be constant and of value p_1.

For the observed data these ratios are:

At least 2 days: at least 1 day	0.495
At least 3 days: at least 2 days	0.495
At least 4 days: at least 3 days	0.475
At least 5 days: at least 4 days	0.494
At least 6 days: at least 5 days	0.505
At least 7 days: at least 6 days	0.545
At least 8 days: at least 7 days	0.728
At least 9 days: at least 8 days	0.843
At least 10 days: at least 9 days	0.635

As there is a discontinuity between 7:6 and 8:7, a calculation was made of the value of what we shall call p_7, the probability that the event will occur after it has already occurred 7 times. The value of p_7 thus obtained was 0.629, considerably higher than p_1. In the last column the amended values for the calculated runs of 7 days and more are given. The agreement is now satisfactory.

To test the significance of this effect, the record was divided into two 15-year periods, and these were tested separately. In the first period, 1935–49, there was no effect of this nature, but in the period 1950–64 the effect was just as marked as in the whole period shown above.

Seasonal Values

No attempt has been made to investigate the agreement for separate seasons, but values of P, p_1 and the persistence ratio have been calculated separately for the four seasons, indicating appreciably higher persistence in winter, the driest season, than in the other seasons. Note, however, that P is lowest in winter.

Throughout this paper the seasons are defined as: summer — Dec., Jan., Feb.; autumn — Mar., Apr., May; winter — June, July, Aug.; spring — Sept., Oct., Nov.

OTHER EXAMPLES OF PERSISTENCE

Raindays* at Milford Sound

These were tabulated over the same period (1935–64), the longest runs being:

30 days, 21 July–19 August 1946.
25 days, 10 March–3 April 1964.
21 days, 29 October–18 November 1951.

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* A rainday is a day when at least 0.01 in. of rain is recorded.

TABLE 2 — Frequencies of runs of raindays (day with at least 0.01 in. of rain) at Milford Sound, Invercargill and Alexandra

Days	(a) *Milford Sound* (1935–64) *Obs.*	*Calc. Markov* (p_1 *const.*)	*Calc. Modified Markov* (p_2 *const.*)	(b) *Invercargill* (1911–65) *Obs.*	*Calc. Markov*	*Calc. Modified Markov* (p_6 .. p_1)	(c) *Alexandra* (1929–65) *Obs.*	*Calc. Markov*
1	390	456	405	1155	1093	—	1149	1186
2	364	326	337	808	753	—	547	508
3	246	232	242	455	519	—	222	217
4	190	166	174	326	357	—	96	93
5	102	119	125	210	246	—	36	40
6	84	85	90	152	170	154	19	17
7	49	60	64	113	117	112	4	7
8	46	43	46	87	81	81	1	—
9	38	31	33	58	56	59		
10	22	22	24	34	38	42	—	—
11, 12	29	27	29	59	44	53	(8 days+	5.4)
13, 14	20	14	15	31	21	28	—	—
15–17	9	9.1	10	15	13	15	—	—
18–20	2	3.4	3.7	7	4.2	6.2	—	—
21–25	2	1.5	1.8	4	—	—	—	—
26+	1	0.4	0.4	(21 days+	2.3	3.9)	—	—
Total runs d_1	1594	—	—	3514	—	—	2074	—
Total raindays n_1	5567	—	—	11308	—	—	3628	—
Total days N	10958	—	—	20089	—	—	13514	—
P	0.511	—	—	0.563	—	—	0.267	—
p_1	—	0.714	—	—	0.689	—	—	0.428
p_2	—	—	0.718	—	—	—	—	—
p_6	—	—	—	—	—	0.725	—	—
Persist. ratio	—	1.72	1.74	—	1.41	1.59	—	1.28

SEASONAL VALUES

	Milford Sound *R*	*P*	Invercargill *R*	*P*	Alexandra *R*	*P*
Summer	1.58	0.51	1.42	0.53	1.24	0.28
Autumn	1.72	0.51	1.38	0.58	1.37	0.26
Winter	1.87	0.47	1.53	0.57	1.31	0.25
Spring	1.69	0.59	1.31	0.58	1.24	0.27

The results of the analysis and of the calculations are shown in Table 2 (a). It is seen that if we assume that p_1 is constant the agreement is reasonably satisfactory apart from the number of runs of 1 day, for which calculated values are too high. Following Brooks and Carruthers (1953, p. 312), we may assume instead that p_2 is constant, that is, that the probability of a rainday is constant after there have already been **two** raindays. From the last column of Table 2 (a) it may be seen that the value of p_2 does not differ appreciably from that of p_1, but the agreement is much better.

Raindays at Invercargill

Invercargill (43 in.) has only about a sixth the annual rainfall of Milford Sound but has a few more raindays (204 per annum). An analysis was carried out of runs of raindays there, with the result shown in Table 2 (b).

Using the simple Markov chain model with p_1 constant, we find reasonably satisfactory agreement until the higher runs are reached, when the calculated values are seen to be too low. This discrepancy is the same as was found with the simple Markov chain model for days with at least 0.50 in. of rain at Milford Sound. By the use of the same technique, comparing ratios: Number of runs of at least (n+1 days) : Number of runs of at least n days, for different values of n, a discontinuity was found. This resulted in the separate calculation of p_6, the probability of a rainday after six successive raindays. The values as thus calculated from runs of 6 days or longer agree better with the observed values. The significance of the effect was tested in the same way as for the similar effect at Milford Sound for days with at least 0.50 in. of rain, by dividing the record into two periods. In the first period, 1911–48, there was an effect, but in the second period, 1949–65, there was no effect.

The seasonal variation of persistence shows a maximum in the same season as at Milford Sound, namely in winter.

Raindays at Alexandra

Alexandra has a precipitation regime which is in sharp contrast to those of both Milford Sound and Invercargill. The annual rainfall of 13 in. is the lowest in the country, and the average annual number of raindays (102) is just half the number at Invercargill. The analysis of runs of raindays is shown in Table 2 (c), together with the calculated values using the Markov chain model. The agreement is reasonably good.

Seasonal calculations show that persistence is highest in autumn. They also indicate that in summer and autumn there are no runs of raindays higher than six. Summer is the season of highest rainfall at Alexandra, with 34% of the total; while winter, the season of lowest rainfall, receives only 16%.

TABLE 3(a) — Frequencies of runs of raindays by season at Wellington (1862–1943) and calculated values with persistence ratios

No. of days per run	Summer Obs.	Summer Calc.	Autumn Obs.	Autumn Calc.	Winter Obs.	Winter Calc.	Spring Obs.	Spring Calc.
1	7.2	7.3	6.1	6.1	4.8	5.2	6.5	6.7
2	4.0	3.8	4.0	3.7	3.8	3.6	4.5	4.0
3	2.0	2.0	2.2	2.3	2.5	2.4	2.4	2.4
4	1.0	1.0	1.3	1.4	1.8	1.7	1.3	1.4
5	0.5	0.5	0.7	0.8	1.2	1.1	0.8	0.9
6, 7	0.3	0.4	0.7	0.8	1.4	1.31	0.8	0.8
8, 9	0.1	0.11	0.4	0.3	0.6	0.61	0.3	0.3
10 or more	0.02	0.04	0.2	0.2	0.4	0.5	0.2	0.17
Total No. of runs (d_1)	15.1		15.6		16.5		16.8	
No. of raindays per season (n_1)	31.5		40.0		52.1		41.9	
Total days N	90.25		92		92		91	
P	0.345		0.436		0.566		0.462 (yr 0.452)	
p_1	0.518		0.618		0.683		0.598	
Persist. ratio	1.36		1.45		1.36		1.34 (yr 1.38)	

TABLE 3(b) — Frequencies of runs of days with at least 0.10 in. of rain at Wellington (1862–1965) by season with calculated values and persistence ratios

No. of days per run	Summer Obs.	Summer Calc.	Autumn Obs.	Autumn Calc.	Winter Obs.	Winter Calc.	Spring Obs.	Spring Calc.
1	669	697	762	760	775	794	802	842
2	296	259	319	328	411	396	381	345
3	98	96	149	142	207	198	154	141
4	29	36	58	61	112	99	68	58
5, 6	15	18	43	37	68	74	19	12
7, 8	1	1.8	6	7	15	18	2	(7+6.8)
9, 10	—	(9+1.1)	—	(9+1.5)	4	(9+6.1)	—	—
d_1	1108		1337		1592		1426	
n_1	1761		2352		3175		2410	
N	9233		9476		9476		9282	
P	0.191		0.245		0.326		0.260 (yr 0.253)	
p_1	0.371		0.432		0.499		0.409	
Persist. ratio	1.29		1.33		1.35		(1.25–) (yr 1.30)	

Raindays at Wellington

Wellington is about average for New Zealand as regards both annual rainfall (48 in. at the present site, Kelburn) and raindays (159). The frequency of runs of raindays of different length per season have already been given by Seelye (1944), and in Table 3 a) these are shown together with the calculated values using the Markov chain model. The agreement is reasonably satisfactory, with a maximum of persistence in the autumn.

It should be noted that these figures are in a different form from those quoted in the other tables, as they give frequency per season instead of total frequency.

Days with at least 0.10 in. of Rain at Wellington

In this case the data extend over a period of a little more than a century, and as in (4), this includes several sites. It is not considered, however, that this is likely to affect the analysis.

The observed and calculated values are listed in Table 3 (b). The agreement is reasonably satisfactory except in spring, the season of lowest persistence. From 5 days upward the calculated values in spring are much too high. It appears that in spring there is virtually no persistence above about 3–4 days. For this reason the value for spring has been bracketed with a negative sign.

Dry Days at Wellington

In Table 4, runs of days with no appreciable rain at Wellington are listed for the four seasons. "No appreciable rain" means not enough rain to register as 0.01 in. Calculations have again been made by the simple Markov model, and the fit is reasonably satisfactory in all seasons. However, in summer the model underestimates the small number of runs of more than 25 days; and in spring it overestimates the small number of runs of more than 16 days. The persistence ratios are almost identical with those obtained for raindays at Wellington over a somewhat shorter period; there is a maximum of persistence in the autumn.

DISCUSSION

The Markov chain is evidently a satisfactory model for these daily precipitation data. Of the few slight modifications required for the frequency of long runs, two are very similar: for days with at least half an inch of rain at Milford Sound p_7, six days after p_1, is higher, 0.629 against 0.495; and for Invercargill raindays p_6, five days after p_1, is also somewhat higher, 0.725 against 0.689. In the period 1950–64 the model required this modification for days with 0.50 in. of rain at Milford Sound, but in the almost identical period 1949–65 for raindays at Invercargill the model did not require this modification. The significance of this effect is therefore doubtful.

TABLE 4 — Observed and calculated runs of dry days (days with no appreciable rain) at Wellington February 1862–January 1963*

Days	Summer Obs.	Summer Calc.	Autumn Obs.	Autumn Calc.	Winter Obs.	Winter Calc.	Spring Obs.	Spring Calc.
1	383	385	512	473	700	668	617	568
2	302	287	324	330	359	395	337	376
3	214	214	197	230	254	234	244	249
4	156	160	151	160	118	139	152	165
5	130	119	110	112	84	82	95	109
6	90	89	89	78	44	48	79	73
7, 8	100	115	101	92	50	46	85	80
9, 10	73	64	44	44	16	16	47	35
11, 12	30	36	25	22	10	6	20	16
13, 14	17	20	9	10	1	2	5	7
15, 16	15	11	6	5	2	0.9	2	3
17–20	10	10	4	3.7	—	(17+1.4)	1	1.9
21–25	3	3.3	1	1.0	—	—	—	(21+2.0)
26–30	2	0.8	—	(26+0.2)	—	—	—	—
31–34	2	(31+0.2)	—	—	—	—	—	—
Total runs (d_1)	1517		1573		1638		1684	
Dry days (n)	5980		5194		4012		5002	
N	9024		9200		9200		9100	
P	0.663		0.565		0.436		0.550 (yr 0.552)	
p_1	0.746		0.697		0.592		0.663	
Persist. ratio	1.33		1.45		1.38		1.35 (yr 1.38)	

* Note: Omitting one year (1866) in which there were no daily observations, the period is exactly 100 years.

TABLE 5 — Summary of Seasonal and Annual Persistence Ratios (R) and Overall Probabilities (P)

Station and Data	Average annual rainfall (in.)	Summer		Autumn		Winter		Spring		Year		r_B
		R	P	R	P	R	P	R	P	R	P	
Milford Sound												
Days with at least 0.01 in.	253	1.58	0.52	1.72	0.51	1.87	0.47	1.69	0.59	1.74	0.51	0.67
Days with at least 0.50 in.	—	1.31	0.31	1.36	0.29	1.55	0.27	1.36	0.33	1.39	0.30	0.48
Wellington												
Days with at least 0.01 in.	48	1.36	0.34	1.45	0.44	1.36	0.57	1.34	0.46	1.38	0.45	0.47
Days with at least 0.10 in.	—	1.29	0.19	1.33	0.24	1.35	0.33	1.25	0.26	1.30	0.25	0.41
Dry days	—	1.33	0.66	1.45	0.56	1.38	0.44	1.35	0.55	1.38	0.55	0.47
Invercargill												
Days with at least 0.01 in.	43	1.42	0.53	1.38	0.58	1.53	0.57	1.31	0.58	1.41	0.56	0.50
Alexandra												
Days with at least 0.01 in.	13	1.24	0.28	1.37	0.26	1.31	0.25	1.24	0.27	1.28	0.27	0.39

Table 5 summarizes the seasonal and annual values of persistence and overall probaibility, and also r_B. The average annual rainfalls of each station are also given, for reference purposes.

It will be seen that there is some correlation between annual rainfall and persistence. Similarly the decrease of proability obtained by increasing the minimum amount of rain per day from 0.01 in. to 0.10 in. or 0.50 in. is associated with a decrease of persistence over the year, but this is not uniform throughout the seasons. For example, in Wellington the decrease in persistence in winter from 1.36 for runs of days of 0.01 in. to 1.35 for runs of days of 0.10 in. is virtually negligible.

An interesting feature of the persistence figures is the variation with seasons, which appears to bear no relation either to the overall The maxima are all in autumn and winter, while the minima are all in spring or summer. The winter maximum of persistence is particularly well-marked in both sets of Milford Sound data and also at Invercargill; while the spring minimum is particularly well-marked in runs of days with at least 0.10 in. at Wellington.
probability of rain (i.e. seasonal raindays) or to seasonal rainfall.

Over most of New Zealand the prevailing westerly winds near the surface are on the average at their strongest in spring and early summer. From an analysis over the Tasman Sea–New Zealand area during the three years 1941–43 Kerr (1944) referred to "a rapid succession of fronts and depressions" in spring and considered that this was to be expected from the known maximum of westerly flow near the surface. No later analyses are available for a longer period, but a minimum of persistence of daily rainfall in spring is to be expected if this is the season when weather systems move eastward most rapidly.

CONCLUSIONS

(1) For the data examined the probability of rain was dependent on whether there was rain on the previous day, and similarly for the probability of no rain.

(2) The observed runs fitted a simple Markov chain model reasonably well.

(3) The seasons of greatest persistence of daily rainfall at these stations were winter and autumn. The low persistence in spring appeared to be associated with the generally rapid movement of weather systems over New Zealand in this season.

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ERRATA

RESEARCH IN LOOSE BOUNDARY HYDRAULICS AT AUCKLAND UNIVERSITY by R. A. Callander, J. Hydrol. (N.Z.) Vol. 5 (2): 111–21

p. 115 2nd line from bottom. The comma after "eddy" should be after "water".

p. 117 Fig. 5: Section RS should be further to the left, so that it passes through the highest point on the steep lee face of the ripple. Δx is the distance travelled by the ripple in time Δt.

p. 117 2nd paragraph: The second sentence should read "In fact, the rate of change of bed load transport rate with respect to distance ($\frac{\Delta L}{\Delta x}$) is proportional to the slope of the ripple surface, if the feature travels at constant speed without changing its shape. The constant of proportionality equals the speed of migration."

p. 119 7th line from bottom: For "on" read "no".