JOURNAL OF HYDROLOGY (N.Z.) Vol. 15, No. 1, 1976

ALTERNATIVE MEASURES OF RIVER CHANNEL SHAPE AND THEIR SIGNIFICANCE

G. Pickup*

ABSTRACT

The width/depth ratio is not always the best measure of river channel shape, because it is measured only at the bankfull stage. It is therefore insensitive to some types of shape difference, and as a result may not reflect the effect of variations in sediment characteristics. Two alternative measures are proposed: bed width, and the exponent x in the equation $W_i = cD_i^x$, where W_i is width at stage D_i. In Cumberland Basin stream channels, bed width increases with mean size of the bed material, while x decreases, reflecting shape adjustment to increasing bed-load transportation.

INTRODUCTION

One of the most widely used measures of river channel shape is the width/depth ratio, or its inverse, the form ratio, which was presented by Gilbert (1914). The width/depth ratio was popularized by Schumm (1960, 1961a, 1961b) in a series of papers describing the relationship between channel shape and perimeter sediment for stable and unstable channels.

A major reason for the popularity of the width/depth ratio is that it has been found to correlate satisfactorily with perimeter sediment characteristics under a wide range of conditions (Schumm, 1960). Consequently, there has been little incentive to develop alternative measures except in the case of complex channels containing benches for which Riley (1972) has developed the bench index. Recent work from Australia, however, indicates that the width/depth ratio may not always be the most suitable shape index (Pickup, 1974; S. J. Riley, pers. comm.; R. F. Warner, pers. comm.). This paper describes some alternatives to it.

---

* Department of Geography, University of Papua New Guinea.

PROBLEMS OF THE WIDTH/DEPTH RATIO

There are three main problems in using the width/depth ratio:

(1) it measures shape only at the bankfull stage;
(2) it may not reflect some types of difference in channel shape;
(3) as a result of (2), it may be insensitive to the effect of changes in perimeter sediment.

The first problem, the fact that width/depth ratio is tied to the bankfull stage, arises from difficulties of defining bankfull, especially in channels which are slightly incised, as the following example shows. Two commonly used definitions of bankfull are:

(1) the stage occupied by the 1.58-year flood on the annual series (Dury *et al.*, 1963);
(2) the stage at which the width/depth ratio is at a minimum (Wolman, 1955; Harvey, 1969).

Fig. 1 shows two channel cross sections, A and B, whose shape is identical below the stage reached by the 1.58-year flood ($Q_{1.5}$). In channel A, which is not incised, the two definitions of bankfull give the same stage. Channel B, however, is slightly incised by the criterion of the first definition and has a smaller width/depth ratio at banktop – the point where the ratio is at a minimum – than at

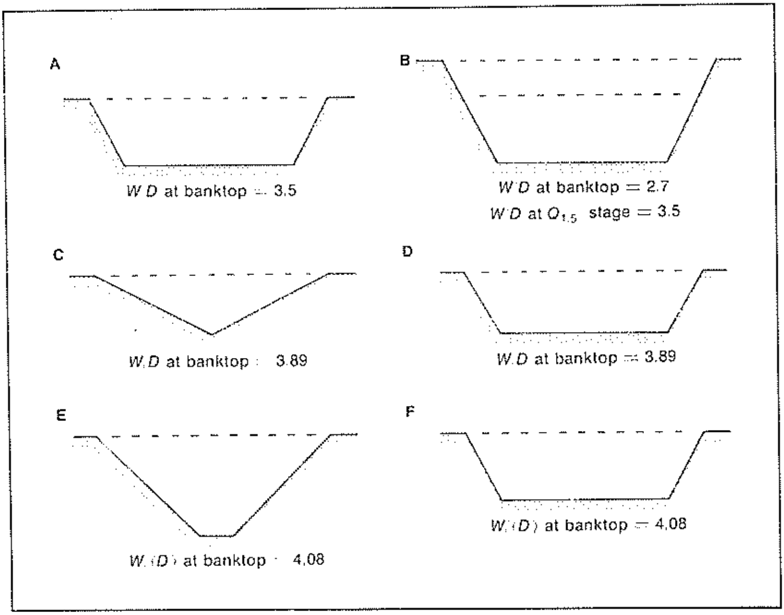

FIG. 1 — Width/depth and width/mean-depth ratios for different channel shapes.

the 1.58-year flood stage. Bankfull stage is usually defined in terms of discharge frequency, so if a stage–discharge curve were not available for channel B, the incision could pass unnoticed. As a result, two channel cross sections with the same effective shape at bankfull would be assigned different width/depth ratios.

The second problem associated with use of the width/depth ratio is that it may not reflect some differences in channel shape. An example of this is illustrated by cross sections C and D of Fig. 1. Both have the same width/depth ratio, but C is triangular in shape while D is trapezoidal. This problem may be overcome by using the width/mean-depth ratio:

$$W/\langle D\rangle = W/(A/W) = W^2/A \qquad (1)$$

where W is the width, $\langle D\rangle$ is the mean depth, and A is the cross-sectional area. There are, however, situations where channels with quite different shapes have the same width/mean-depth ratio, as cross sections E and F of Fig. 1 illustrate.

The third problem, that the width/depth ratio may be insensitive to differences in channel perimeter sediment characteristics, was noted during a study of stream channel characteristics in the Cumberland Basin of eastern New South Wales (Pickup, 1974). Width/depth ratios, measured at 47 sites, were correlated with Schumm's silt-clay index M, but as Fig. 2 shows, no significant correlation is apparent. Similar poor results were obtained when width/mean-depth ratio was substituted as the dependent variable and when bed- and bank-material mean size, silt content and clay content were used as the independent variable.

ALTERNATIVE MEASURES OF CHANNEL SHAPE

The poor results obtained using width/depth ratios made it necessary to develop alternative measures of channel shape without the problems described in the previous section. Two indices were finally selected – the bed width W_b, and the width/depth exponent x. They were derived in the following manner.

After a channel cross section was surveyed, it was plotted up and measurements of width were obtained at various stages above the bed. These stages were at intervals of 0.15 m or 0.3 m depending on channel depth. Using least-squares regression, a relationship was then derived with the form:

$$W_i = cD_i^x \qquad (2)$$

where W_i is the width at stage D_i. The fit was usually very good and in no case was less than 90 percent of the variance explained.

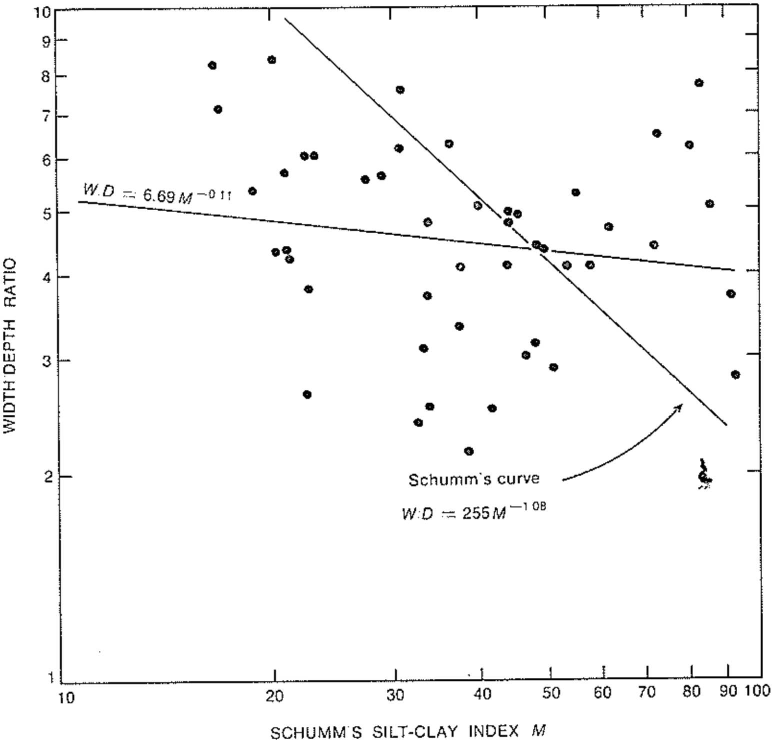

FIG. 2 — The relationship between width/depth ratio and Schumm's silt–clay index M, for Cumberland Basin stream channels.

Equation (2) is similar in form to and can be functionally related to the power relationships derived by Leopold and Maddock (1953) to describe at-a-station hydraulic geometry.

The value of width/depth exponent was obtained directly from equation (2). Bed width W_b was arbitrarily defined as the width of the channel 0.3 m above the lowest point of the bed and was calculated as:

$$W_b = c\,0.3^x \qquad (3)$$

This measure seems to correspond quite closely with the section of channel perimeter occupied by loose bed material. It may therefore be used as a measure of the width of the 'live bed', or the section of perimeter over which bed-load transport occurs. It should be noted, however, that while 0.3 m may be the appropriate level at which to measure bed width in the Cumberland Basin, this may not be the case in other areas, especially where there are sand-bed streams with dunes. It is therefore necessary to exercise some care in selecting the appropriate stage at which to measure W_b. Thus, use of W_b has its limitations and it would be rather difficult to apply it to mountain streams with very irregular beds.

Values of W_b were found to correlate with mean bed-material size and drainage area in the Cumberland Basin, yielding the equation:

$$\log W_b = 0.316 - 0.183 d_m + 0.097 \log A_d \qquad (4)$$
$$r = 0.783 (P < 0.001)$$

where W_b is the bed width in metres, d_m is the mean bed-material size in φ units*, and A_d is the drainage area in km^2. W_b was not significantly correlated with Schumm's silt-clay index M, or with measurements of bank sediment characteristics including mean size, percentage silt-clay and percentage clay. The width/depth exponent x was also not significantly correlated with these sediment variables, but when correlated with W_b and d_m yielded the relationships:

$$\log W_b = 0.953 - 0.775x \qquad (5)$$
$$r = -0.824\ (P < 0.001)$$

$$x = 0.529 - 0.071 d_m \qquad (6)$$
$$r = -0.554\ (P < 0.001)$$

Equations (4) and (6) indicate that W_b and x depend on the size of the bed material. As a channel with coarse bed material is likely to carry most of its load as bed load, then large values of W_b and small values of x are associated with bed-load channels. Small values of W_b and large values of x, on the other hand, are associated with finer bed material and probably reflect the increasing importance of suspended-load transport. Equation (5) indicates how close the association between W_b and x is, although there is some spurious correlation here since the two variables are already related by equation (3).

The reason that the width/depth ratio does not reflect changes in channel shape associated with differences in sediment load results from the characteristics of the bank material. In the Cumberland Basin, channels are cut into very cohesive red and yellow podsolic soils which are highly compacted and contain a high proportion of clay. As a result, channel widening in response to changes in sediment load is severely restricted by the resistance of the banks to erosion (Pickup, 1974). Changes in channel shape therefore involve little variation in width. On the other hand, when Schumm (1960) derived a relationship between width/depth ratio and perimeter

---

* Phi units are based on a log scale and calculated from the relationship: $\varphi = -\log_2 d$, where d is the diameter in mm.

sediment, his sample included sand-bed streams with friable banks which allow extensive channel widening.

The changes in shape which do occur in Cumberland Basin streams may be explained in the following manner. The rate of bed-load transport in a channel depends on the total shear stress at the bed (see, for instance, Colby, 1963). Shear at the banks does not contribute to bed-load transport. The efficient bed-load channel will therefore tend to adjust its cross section to maximize total bed shear (which partly depends on the width of the bed) and at the same time minimize head losses due to bank friction. This adjustment may be achieved by reducing the area of bank surface and increasing the area of bed. Thus, the greater the proportion of bed load carried, the greater the total bed shear required to transport it, and the wider the bed.

The small value of the width/depth exponent associated with wide beds and coarse bed material reflects the same adjustment. A high value of x indicates a rapid increase of width with depth, a large bank area, and relatively high bank-friction head losses. A small value of x indicates only a slow increase in width with depth, a small bank area and therefore relatively low bank-friction head losses. It follows that the most efficient bed-load channel cross section is wide with steep banks – indeed, almost a rectangle.

While a rectangle may be an efficient shape for bed-load transport it has poor hydraulic characteristics. The most efficient channel shape for flow is a semicircle, which encloses the maximum area for a given perimeter and therefore has the largest hydraulic radius. Channels which carry most of their sediment as wash load will tend to develop towards a semicircular cross section, within the limits imposed by the resistance of the bank material, since the transport of the sediment load does not require high total bed shear. Such a cross section has a very small bed width, but a large increase of width with depth and therefore a high value of x.

An example may illustrate these principles more clearly. For a given value of W_b, equation (3) may be used to calculate c and equation (5) used to calculate x, the two coefficients of equation (2). This equation may be used to calculate channel width as depth increases, and assuming that the channel is symmetrical, the cross section for a given bed width may then be plotted. Cross sections have been determined using this method for bed widths of 2, 4, 6 and 8 m and are shown in Fig. 3. This figure also shows the relationships between:

(1) hydraulic radius and channel area for each cross section;

(2) the proportion of the total perimeter made up by the bed and the channel area for each cross section.

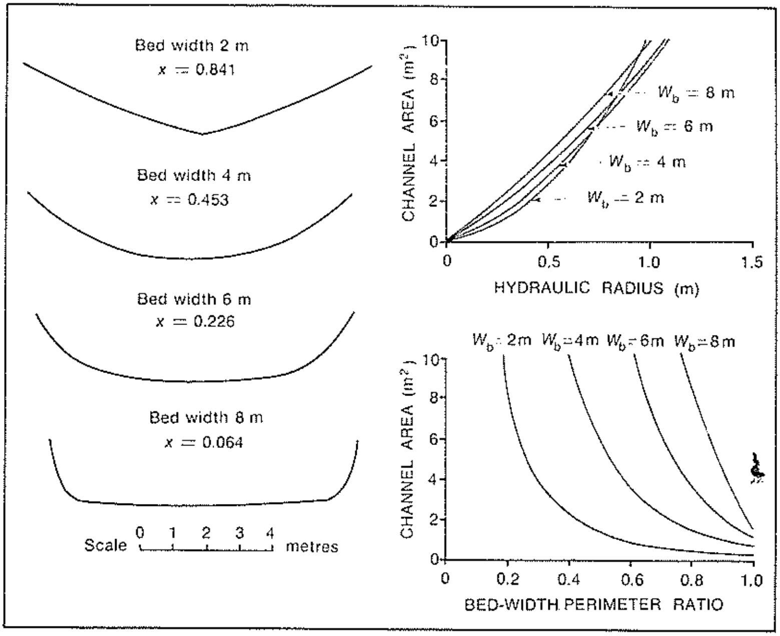

FIG. 3 — Channel shapes for different W_b and x values, and the associated relationships between channel area and hydraulic radius and between channel area and the ratio of bed width to total perimeter length.

The first of these relationships, between hydraulic radius and area, illustrates how efficient the channel shape associated with each bed width is in the conveyance of flow. The second relationship, between the bed-width/total-perimeter-length ratio and channel area illustrates the suitability of each channel cross section for the transport of bed load, the idea being that a greater proportion of total energy is dissipated at the bed in those channels where the bed forms a large proportion of the total perimeter.

The channels with a small bed width and a high value of x are clearly the most efficient, having larger hydraulic radii for most values of area, but the bed forms a rapidly decreasing proportion of the total perimeter – suggesting that head losses would be increasingly due to bank friction. The channels with larger bed widths and small values of x, on the other hand, have a low hydraulic efficiency, but most of the perimeter is made up by the bed, so that even at high stages, bank-friction head losses are relatively low. Such channels are therefore best suited to bed-load transport.

CONCLUSION

Two alternative measures of channel shape have been proposed for situations where the width/depth ratio is a poor indicator of channel form and is insensitive to the effects of variations in channel sediment type. These measures are bed width and the width/depth exponent, and have the advantage that they are not measured at the bankfull stage and are therefore not affected by channel incision. Neither of the two variables seem to be significantly correlated with measures of bank sediment characteristics but both are related to the mean size of the channel bed material. Bed width tends to be larger and values of the width exponent are small in channels with coarse bed material, while small bed widths and large values of the exponent are found where bed material is fine. This reflects channel shape adjustment to sediment load characteristics.

ACKNOWLEDGMENTS

The author wishes to thank Drs R. F. Warner and S. J. Riley for their helpful comments on an earlier draft of this paper. Financial support was provided by the University of Sydney and the Department of Education and Science. Secretarial and drafting services were provided by the University of Papua New Guinea.

REFERENCES

Colby, B. R. 1963: Fluvial sediments – a summary of source, transportation, deposition and measurement. *U.S. Geological Survey Bulletin 1181-A.*

Dury, G. H.; Hails, J. R.; Robbie, H. B. 1963: Bankfull discharge and the magnitude–frequency series. *Australian Journal of Science 26*: 123–124.

Gilbert, G. K. 1914: The transport of debris by running water. *U.S. Geological Survey Professional Paper 86.*

Harvey, A. M. 1969: Channel capacity and adjustment of streams to hydrologic regime. *Journal of Hydrology (Netherlands) 8*: 82–98.

Pickup, G. 1974: Channel adjustment to changed hydrologic regime in the Cumberland Basin, New South Wales. Unpublished Ph.D. thesis, University of Sydney.

Leopold, L. B.; Maddock, T. 1953: The hydraulic geometry of stream channels and some physiographic implications. *U.S. Geological Survey Professional Paper 252.*

Riley, S. J. 1972: A comparison of morphometric measures of bankfull. *Journal of Hydrology (Netherlands) 17*: 23–31.

Schumm, S. A. 1960: The shape of alluvial channels in relation to sediment type. *U.S. Geological Survey Professional Paper 352-B.*

Schumm, S. A. 1961a: Dimensions of some stable alluvial channels. *U.S. Geological Survey Professional Paper 424-B.* pp. 26–27.

Schumm, S. A. 1961b: The effect of sediment characteristics on erosion and deposition in ephemeral stream channels. *U.S. Geological Survey Professional Paper 352-C.*

Wolman, M. G. 1955: The natural channel of Brandywine Creek, Pennsylvania. *U.S. Geological Survey Professional Paper 271.*