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Towards estimating areal reduction factors for design rainfalls in New Zealand

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Abstract
Areal reduction factors (ARFs) are used to convert rainfalls measured at a point to equivalent rainfalls over a specified area for particular rainfall durations and return periods. Currently in New Zealand, ARF values derived overseas are employed in hydrological design and planning. To obtain values based on New Zealand rainfall data ARFs are estimated empirically by the United States Weather Bureau (USWB), the United Kingdom Flood Studies Report and Bell's methods using recorded rainfalls at a number of raingauges. The only locations found to have a sufficient density of raingauges and period of common record were Christchurch and Auckland. ARF values are calculated as a function of area, rainfall duration and, with Bell's method, return period. Results from the USWB and Bell's methods are the most realistic and consistent and are in accord with many international findings. ARF values are greater in Christchurch than in Auckland, owing perhaps to differences in topography and the nature of storm rainfalls. Bell's method is recommended for use as a basis in design in Christchurch and Auckland and perhaps elsewhere, mainly because it accommodates return period. Considerably more data from more locations is required to make substantive progress in empirical estimation of ARF values nationally.

Keywords
areal reduction factor; areal rainfall; design rainfall; spatial rainfall distribution; design floods; flood estimation.

Introduction
In hydrological design and planning it is usually necessary to prescribe a catchment design rainfall event consisting of a set of rainfall depths or intensities varying in space (i.e., areal variation) and time reflecting natural conditions within a storm. However, data from raingauges applies at a point and is normally uniform over a small surrounding area only. To convert point rainfalls to areal rainfalls an areal reduction factor (ARF) is employed, which denotes the ratio between areal average rainfall and point rainfall for a given area, duration and return period; it may vary depending on catchment climate and characteristics as well as the record period and methods used in its derivation (Svensson and Jones, 2010).

At present, areal rainfall cannot be measured so both indirect theoretical and empirical approaches have been developed (Srikanthan, 1995; Svensson and Jones, 2010; Pietersen *et al.*, 2015). Theoretical methods assume that rainfall processes are stationary and isotropic and have large data requirements. They are best suited for application to short duration rainfalls over small areas (Svensson and Jones, 2010).

In this nationwide exploratory study where appropriate rainfall data are sparse, we employ an empirical approach. Two empirical methods are used internationally: storm-centred or geographically centred (fixed area). Broadly speaking, fixed-area methods estimate ARF values by averaging rainfall data over an area whereas storm-centred ARF values are the result of calculating ARF values for each of a large sample of storms and averaging them (Oliveria *et al.*, 2008). The storm-centred approach has not seen widespread application mainly due to lack of information about storms with large return periods, problems with including multi-centred storms and limitation to specific types of storm events (Asquith and Famiglietti, 2000). The approach is generally used in calculating catchment probable maximum precipitation (Srikanthan, 1995).

In New Zealand, Tomlinson (1978) examined seven storms with extreme rainfalls, but was unable to calculate ARF values owing to very large variations in rainfall depth-area characteristics. Subsequently, Tomlinson (1980) recommended the use, with caution, of ARF values given in Natural Environment Research Council (NERC, 1975), which were derived by a fixed-area method. These values are commonly employed in New Zealand without verification, in the absence of alternatives.

Given these findings we adopt a fixed-area approach and apply three standard methods for estimating ARF values. Due mainly to limitations regarding record length and raingauge density, investigations are confined to areas in Christchurch and Auckland. We examined other areas in Northland, Hawke's Bay, and Southland, but were unable to obtain consistent results at these locations.

The purpose of the paper is to examine and compare the results of applying some empirical ARF calculation methods in New Zealand. The aim is to make progress towards determining ARF values for use in predicting catchment design rainfalls.

Theory

Three standard methods are employed internationally to derive ARF values empirically using recorded rainfall depths at a number of raingauges (Srikanthan, 1995). Two specify ARF as a function of area (A) and rainfall duration (D); the other includes return period (T) as well. A brief description of the methods and their behaviour follows (Srikanthan, 1995; Pietersen *et al.*, 2015).

US Weather Bureau (USWB) method

Using data from raingauge networks covering areas from 200 to 1000 km^2, the US Weather Bureau (1957) employed the expression:

$$ARF = N \sum_{j=1}^{N} \sum_{i=1}^{N} w_i P_{dij} \,/\, \sum_{j=1}^{n} \sum_{i=1}^{N} P_{ij} \qquad (1)$$

in which N is the number of raingauges, w is a Thiessen weighting factor for gauge i, P_{dij} is the point rainfall for gauge i on the day the annual maximum areal rainfall occurs in year j, and P_{ij} is the annual maximum point rainfall for the chosen duration for year j at gauge i. The ARF for each event of specified duration is calculated using Thiessen weights and the largest of these in each year of record is selected. The sum of the resulting annual series multiplied by the number of gauges constitutes the numerator in Equation 1. The denominator in Equation 1 is the sum of the largest point rainfalls at each gauge in each year over all gauges and all years. In deriving this method, the average the number of raingauges employed was six and the data period ranged from seven to 15 years. Leclerc and Shaake (1972) expressed the ARF values derived by the USWB as:

$$ARF = 1 - exp(-1.1\, D^{0.25}) + exp(-1.1\, D^{0.25} - 0.0039 A) \qquad (2)$$

where D is in hours and A is in km^2.

UK Flood Studies Report (UKFSR) method

In the UKFSR method, areal rainfall for each raingauge is computed to identify the time the maximum occurs and the P_{dij} values are abstracted (NERC, 1975). Also, the maximum point values, $P_{ij,}$ are identified. The mean of the ratio of P_{dij}/P_{ij} over all gauges and all years of record is determined and *ARF* is defined as

$$ARF = (1/nN)\sum_{i=1}^{N}\sum_{j=1}^{n}(P_{dij}/P_{ij}) \qquad (3)$$

Generally for a given area the ARF values produced by this method are greater than those given by Equation 2, but this difference decreases with increasing duration and area.

Bell's method

The annual maximum areal rainfall and the annual maximum point rainfall series at each raingauge are ranked (Bell, 1976), then the Thiessen weighted mean of point rainfalls of the same rank is computed to give an annual series of weighted maximum point rainfalls. The ARF of rank *m* is defined as the ratio of the areal rainfall of rank *m* to the weighted average point rainfall of rank *m,* that is:

$$ARF_m = \sum_{i=1}^{N} w_i P_{dij} / \sum_{i=1}^{N} w_i P_i \qquad (4)$$

If ARF values are assumed to be independent of rank or return period, *T*, then the mean of ARF_m over all ranks gives an estimate dependent only on *A* and *D*. Bell's method is the only empirical ARF estimation method that takes return period into account.

As noted above, the true value of an ARF cannot be measured at present so in practice one must choose the method that gives the most consistent results and is the most appropriate for use in a particular instance.

Rainfall data

Rainfall data for this study came from automatic raingauges operated by the Meteorological Service of New Zealand, regional and city councils and the National Institute of Water and Atmospheric Research Ltd. Locations in New Zealand were sought where there was a sufficient density of raingauges having enough simultaneous record length to be able to apply the three methods of ARF estimation. The requirements regarding density and record length are quite stringent and were satisfied only by locations in Auckland and Christchurch (Table 1).

Time series of annual maxima for the various durations exhibited no evidence of trend, periodicity persistence or shifts according to the Mann-Whitney and Wald-Wolfowitz statistical tests and visual inspection (see also Withers and Pearson, 1991; Griffiths *et al.*, 2014). This means that records can be assumed to be stationary and composed of independent values.

Table 1 – Summary of records used

Location	Number of gauges	Period of record	Length of record
Christchurch	18	2005-2016	12 yrs
Auckland	11	2002-2012	11 yrs

Analysis

The USWB, UKFSR and Bell's methods were applied to calculate ARF values at Christchurch and Auckland. Automatic raingauge sites at these locations are shown in Figure 1. In applying the three methods, circular areas of 10, 20, 50, 100 and 500 km^2 were specified along with durations of 1, 6, 12, 24 and 48 hours, and for Bell's method return periods of up to 11 or 12 years. The areal rainfall for each raingauge for a given area, duration and return period (where applicable) was determined using the Thiessen polygon method (Smith, 1993). Results for each location are presented separately and then compared.

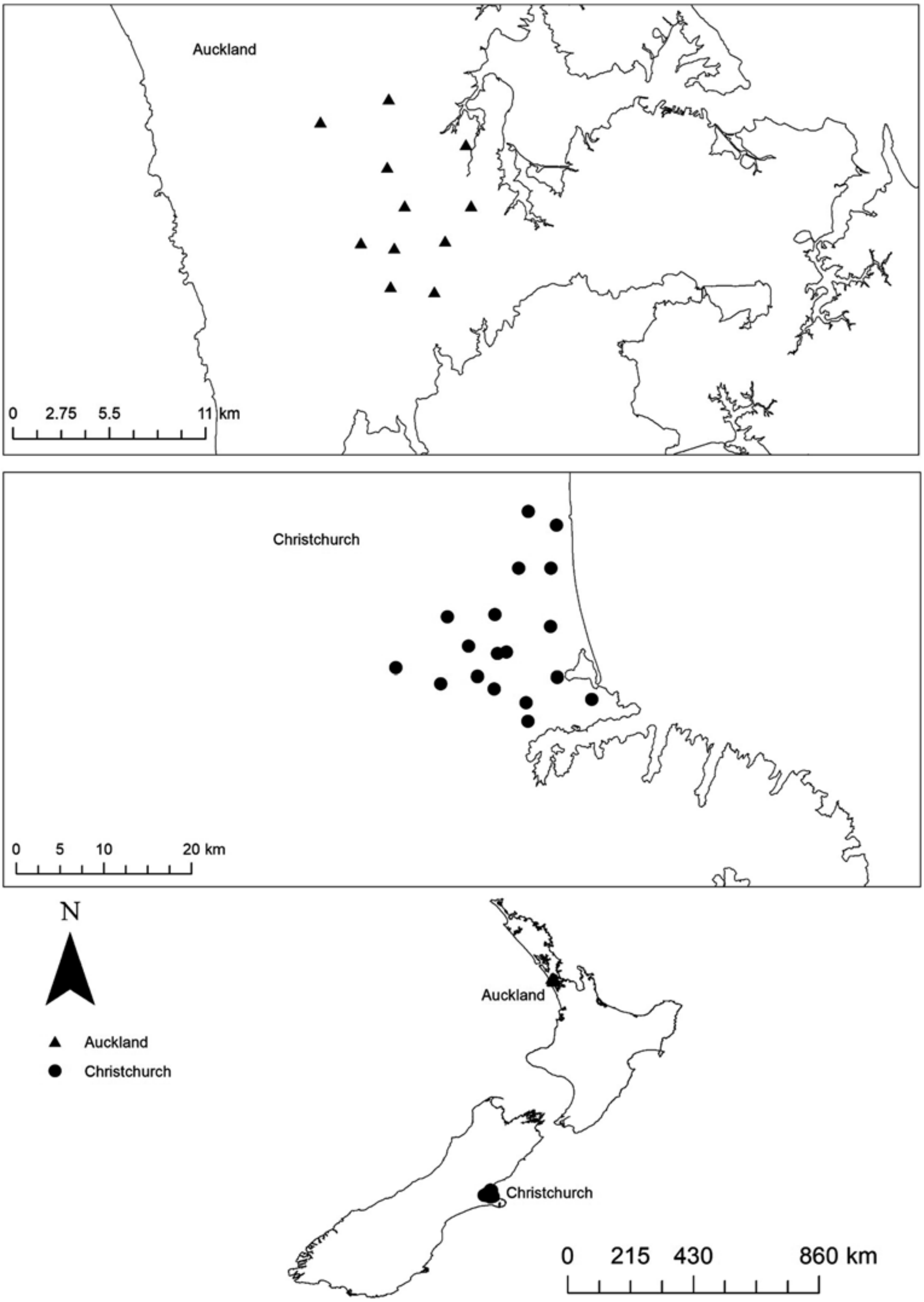

Figure 1 – Locations of raingauges used in Auckland and Christchurch

Table 2 – Areal reduction factors estimated by USWB, UKFSR and Bell's methods for various rainfall durations and areas in Christchurch

Duration (hrs)	Method	Area (km^2)				
		10	20	50	100	500
1	USWB	0.903	0.908	0.915	0.785	0.653
	UKFSR	0.927	0.927	0.907	0.791	0.695
	Bell's	0.944	0.939	0.937	0.836	0.702
6	USWB	0.976	0.978	0.980	0.950	0.841
	UKFSR	0.968	0.974	0.974	0.950	0.858
	Bell's	0.982	0.980	0.979	0.951	0.843
12	USWB	0.991	0.993	0.994	0.973	0.899
	UKFSR	0.992	0.992	0.992	0.954	0.897
	Bell's	0.989	0.989	0.990	0.965	0.900
24	USWB	0.985	0.984	0.985	0.998	0.952
	UKFSR	0.968	0.970	0.970	0.984	0.946
	Bell's	0.981	0.978	0.977	0.992	0.951
48	USWB	0.981	0.974	0.982	0.988	0.936
	UKFSR	0.977	0.973	0.971	0.982	0.923
	Bell's	0.974	0.970	0.968	0.987	0.938

Christchurch

ARF values for the specified areas and durations determined using the three methods are listed in Table 2. For a given area, ARF values generally increase as duration increases and for a given duration ARF values decrease with increasing area. Comparatively, the differences between the ARFs estimated by the three calculation methods are less than about 10% (Table 2).

For Bell's method the results for Christchurch may be approximately expressed in the form of Equation 2 as:

$$ARF = 1 - exp(-1.24\,D^{0.192}) + exp(-1.24\,D^{0.192} - 0.005A) \quad (5)$$

where D is in hours and A is in km^2.

Agreement among the three calculation methods improves a little for larger areas and longer durations. This result is consistent with previous work by Griffiths and Pearson (1993) and Griffiths *et al.* (2009) using storm and flood data for the Avon River catchment in Christchurch, which yielded ARF values similar to those predicted by NERC (1975) as presented in Faulkner (1999). For a given area and duration, ARF values generally increase with decreasing return period and are more or less independent of duration for $D \geq 12$ hr and $A \leq 100$ km^2 (Table 3). Tables 2 and 3 display calculated values; inconsistencies occur owing mainly to the positioning of raingauges and anomalies in the time series of recorded rainfalls. The inconsistencies are greater in Table 3 and in a more detailed study with longer simultaneous records might largely be removed by fitting extreme value distributions to rainfall versus return period relations at each raingauge for each duration (see, for example, Svensson and Jones, 2010).

The above trends may be expressed by the simple power law model:

$$ARF = f(A^{-m} D^{n} T^{r}) \quad (6)$$

in which m, n and r are positive exponents, and are consistent, along with the magnitudes of the ARF values, with many international findings (Pietersen *et al.*, 2015).

Table 3 – Areal reduction factors estimated by Bell's method for various rainfall durations, return periods and areas in Christchurch

Duration (hrs)	Return period (yrs)	Area (km^2)				
		10	20	50	100	500
1	12	0.809	0.830	0.848	0.616	0.445
	6	0.894	0.879	0.866	0.758	0.535
	4	1.000	1.000	1.000	0.852	0.627
	2	0.947	0.938	0.931	0.823	0.794
6	12	0.965	0.960	0.955	0.982	0.753
	6	1.000	1.000	1.000	0.959	0.803
	4	0.988	0.986	0.985	0.933	0.823
	2	0.983	0.980	0.977	0.931	0.863
12	12	1.000	1.000	1.000	0.999	0.924
	6	1.000	1.000	1.000	0.901	0.832
	4	1.000	1.000	1.000	0.951	0.816
	2	1.000	1.000	1.000	0.998	0.970
24	12	1.000	1.000	1.000	0.986	0.846
	6	1.000	1.000	1.000	1.000	0.987
	4	0.972	0.967	0.963	0.980	1.000
	2	1.000	1.000	1.000	0.987	0.960
48	12	1.000	1.000	1.000	0.986	0.846
	6	1.000	1.000	1.000	1.000	0.987
	4	0.964	0.959	0.953	0.990	1.000
	2	1.000	1.000	1.000	0.990	0.960

Auckland

Trends in the ARF values with A and D follow Equation 6 in general and results from the USWB and Bell's methods are in good agreement, with Bell's values greater than the USWB values except for D = 1 hr. The result for Bell's method may be expressed approximately as:

$$ARF = 1 - exp(-0.597 D^{0.229}) + exp(0.597 D^{0.229} - 0.121 A) \quad (7)$$

where D is in hours and A is in km^2.

UKFSR values approach the others as duration increases but are still less, except for some of the long duration and large area values (Table 4). With return period included, ARF values again follow Equation 6 in general (Table 5).

Comparison

ARF values predicted by Bell's method are mostly about 20% greater in Christchurch than Auckland except for D = 1 hr, for which differences are much greater. With the USWB method differences average about 30% except (again) for D = 1 hr with Christchurch once again having larger values. The same pattern is exhibited with UKFSR values, which are generally less than those predicted by the other two methods. With return period, Christchurch ARF values again exceed Auckland values but the latter do not exhibit independence for larger areas and longer durations as they do in Christchurch.

The differences in ARF magnitude between the two locations probably arise from differences in topography (with Christchurch flat and Auckland largely

Table 4 – Areal reduction factors estimated by USWB, UKFSR and Bell's methods for various rainfall durations and areas in Auckland

Duration (hrs)	Method	Area (km^2)				
		10	**20**	**50**	**100**	**500**
1	USWB	0.654	0.654	0.447	0.360	0.307
	UKFSR	0.500	0.500	0.364	0.287	0.224
	Bell's	0.591	0.591	0.492	0.352	0.306
6	USWB	0.833	0.834	0.591	0.593	0.602
	UKFSR	0.758	0.758	0.573	0.581	0.572
	Bell's	0.896	0.896	0.667	0.642	0.655
12	USWB	0.822	0.822	0.673	0.719	0.726
	UKFSR	0.748	0.748	0.690	0.762	0.686
	Bell's	0.874	0.874	0.747	0.759	0.746
24	USWB	0.825	0.825	0.712	0.758	0.763
	UKFSR	0.789	0.789	0.731	0.773	0.760
	Bell's	0.863	0.863	0.770	0.791	0.787
48	USWB	0.815	0.815	0.734	0.762	0.777
	UKFSR	0.776	0.776	0.773	0.808	0.759
	Bell's	0.852	0.851	0.780	0.797	0.811

Table 5 – Areal reduction factors estimated by Bell's method for various rainfall durations, return periods and areas in Auckland

Duration (hrs)	Return period (yrs)	Area (km2)				
		10	**20**	**50**	**100**	**500**
1	11	0.668	0.668	0.311	0.272	0.322
	6	1.000	1.000	0.564	0.535	0.488
	4	0.812	0.812	0.632	0.538	0.422
	2	0.717	0.718	0.408	0.372	0.409
6	11	0.659	0.660	0.364	0.418	0.432
	6	1.000	1.000	0.761	0.614	0.603
	4	0.825	0.825	0.662	0.656	0.674
	2	0.816	0.816	0.624	0.665	0.696
12	11	0.660	0.660	0.413	0.538	0.601
	6	1.000	0.998	0.743	0.701	0.871
	4	0.750	0.750	0.817	0.825	0.740
	2	0.874	0.873	0.795	0.819	0.719
24	11	0.666	0.667	0.472	0.582	0.606
	6	0.860	0.859	0.796	0.809	0.886
	4	0.864	0.864	0.794	0.772	0.794
	2	0.831	0.831	0.878	0.865	0.789
48	11	0.667	0.668	0.500	0.592	0.598
	6	0.735	0.734	0.794	0.848	0.785
	4	0.815	0.814	0.791	0.819	0.872
	2	0.888	0.888	0.773	0.812	0.787

rolling country), storm type and rainfall amounts. Heavy rainfalls in Christchurch occur mainly with easterly or southerly quarter winds and are modest compared with Auckland. High intensity, short duration rainfall may occur in convective showers and thunderstorms in Christchurch, but are uncommon. In contrast, Auckland receives periods of moderate or heavy rain generally associated with the passage of a depression that may sometimes be of tropical origin and, more frequently than Christchurch, heavy short period rainfalls in convective showers and thunderstorms (see Podger *et al.*, 2015 for Australian experience with topography).

Overall, the USWB and Bell's methods give the most consistent and reasonable results and are a basis for estimating ARF values. We recommend the use of Bell's method largely because it accommodates return period.

Finally, it is important to note that with natural and human-induced changes in climate, ARF calculation may be affected. This study uses data from particular periods (2002-2012 and 2005-2016, see Table 1) of stable climate.

Future work

Results from this pilot study and earlier work suggest that further substantial progress in estimating ARF values throughout New Zealand using empirical methods must await the collection of longer records from a greater density of raingauges in many locations. Prediction of ARF values for higher return periods could perhaps be achieved by fitting statistical distributions to rainfall versus return period relations for each raingauge for each duration.

Conclusions

The USWB and Bell's methods for estimating ARFs as a function of area and duration supply the most realistic and consistent results for Christchurch and Auckland locations and are similar to many international findings. ARF values are greater in Christchurch than in Auckland, probably owing to differences in, at least, topography and the nature of storm rainfalls. Bell's method is recommended for use in design in Christchurch, Auckland and perhaps elsewhere as a basis for estimating ARF values, mainly because it accommodates return period. Longer records of common period from a greater density of raingauges in more locations are required to make significant progress in empirical calculation of ARF values nationally.

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