AN APPLICATION OF TIME-SERIES ANALYSIS TO GROUNDWATER INVESTIGATION AND MANAGEMENT IN CENTRAL CANTERBURY, NEW ZEALAND

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ABSTRACT

The dynamic behaviour of a semi-confined aquifer in the Central Canterbury Plains area of New Zealand has been described by simple single-input single-output linear transfer function models, relating monthly instantaneous water level at a monitoring bore to monthly totals of rainfall recharge estimated from a daily water balance model. Model identification and parameter estimation were done by the Simple Refined Instrumental Variable (SRIV) algorithm using MICROCAPTAIN software. First-order and second-order transfer function models fitted the 23 years data with R^2 values of 0.88 and 0.90 respectively. Noise was modelled as a second-order autoregressive process and incorporated into a forecast equation. A second-order model had the lowest one-step-ahead forecast error variance of 0.0041 of total variance. Split-sample calibration demonstrated the superior predictive ability of the first-order model. Simple regression equations at zero time lag provided good water-level estimates at other bores from short records concurrent with the monitoring bore. The simulation and forecast models were implemented using spreadsheet software.

INTRODUCTION

The Canterbury Regional Council has a statutory responsibility to monitor and analyse the water resources of the region, and to apply this knowledge when considering water right applications. A typical example of this process is groundwater resource allocation in the Greendale area of the Canterbury Plains (Fig. 1). During the 1970s and early 1980s a large number of bores were sunk here to abstract water for irrigation. In one particular aquifer over a 30 km^2 area there are 22 bores, with water rights authorising peak abstraction of 86,000 m^3/day.

The natural piezometric levels of this semi-confined aquifer have a recorded range of 25 m over several years in response to natural recharge. Periods of natural low water level reduce yields in some bores, accentuating interference between bores due to pumping drawdown. This can cause misunderstanding between bore users who cannot easily separate the slowly varying, natural water levels from the shorter term effects of pumping.

Quantification of the processes causing natural water level variation has two benefits: increased understanding of the resource by both users and the statutory

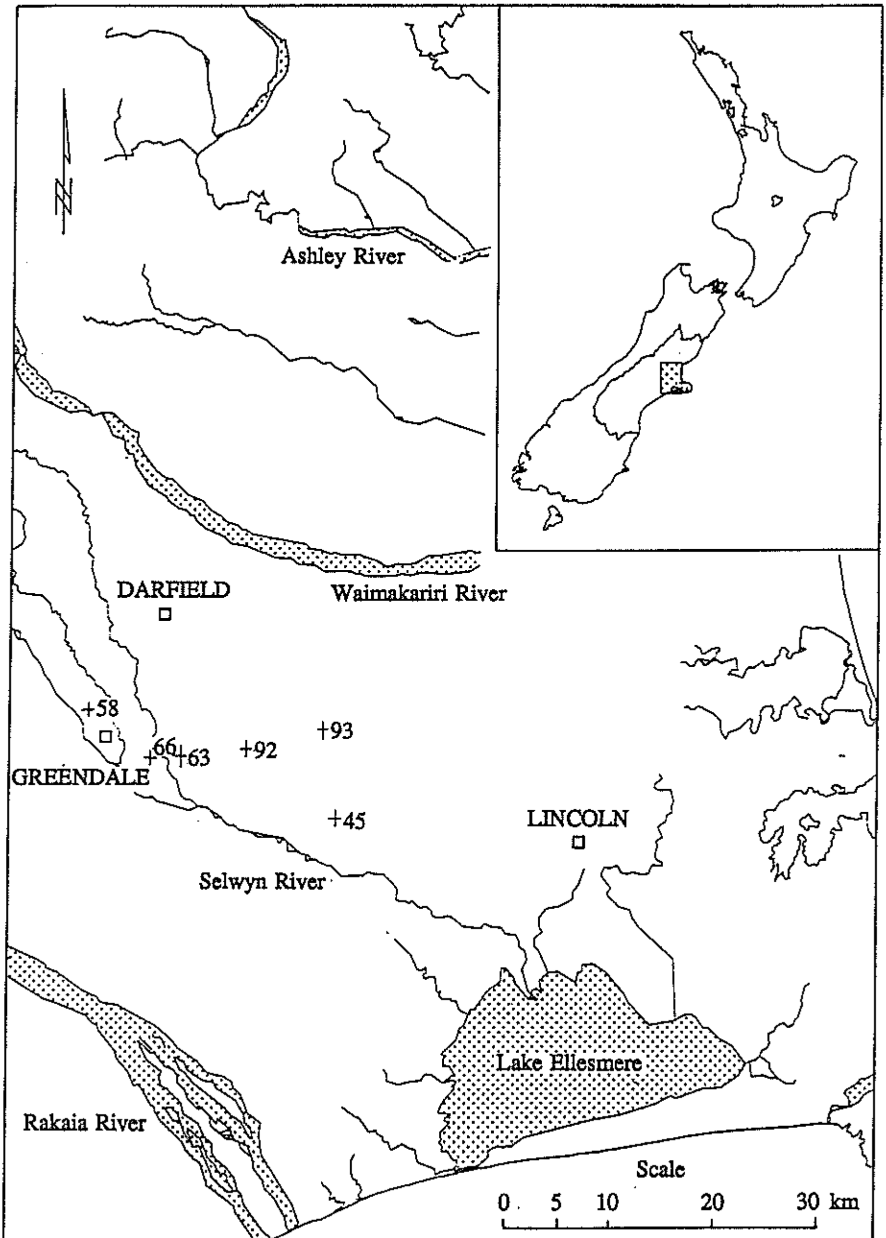

FIG. 1—The Central Canterbury Plains — the location of monitoring bores used for analysis are marked "+ nn".

authority, and accurate forecasting of natural water levels in order to aid farm management decisions when water availability is restricted.

The problem reduces to estimation of rainfall recharge, as the presumed major recharge source, and derivation of the dynamic relationship between recharge and available records of piezometric levels for the aquifer. Sharma (1986) has reviewed methods of measuring and predicting natural groundwater recharge. The water balance method is suitable for this investigation in terms of cost, data availability and climate.

A relationship between bore water levels and estimates of rainfall recharge from a water balance model initially was apparent from plots of the data. However, the relationship could not be adequately quantified using correlation techniques available in standard statistical software. This prompted consideration of more sophisticated time-series analysis techniques available as commercial software.

When time-series analysis is used to quantify the relationship between a time-varying input to and output from a physical system (called the transfer function problem), it is sometimes described as a "black box" approach. Such an approach can lead to erroneous conclusions if devoid of physical knowledge, and is usually not acceptable to resource managers. However, Box and Jenkins (1970) have described the direct relationship between the dynamic behaviour of physical systems and their structural identification and parameter estimation by means of time-series analysis.

The approach taken for the Greendale study was to describe the aquifer in terms of a simple physical model, express the model as a simple linear equation, and use time-series analysis to determine the equation coefficients and to test the predictive capability. A similar approach has been applied, for example, to modelling geothermal systems (Fradkin *et al,* 1981), and solute transport in streams (Dietrich and Jakeman, 1989).

HYDROGEOLOGY OF THE CANTERBURY PLAINS AQUIFERS AT GREENDALE

The Canterbury Plains formed as a series of coalescing alluvial gravel fans and glacial outwash during the colder climatic fluctuations of the last 2 million years. During the warmer interglacial periods these gravel deposits were resorted, leaving differentiated layers of greater and lesser permeability.

At Greendale, groundwater flow is predominantly from the northwest to the southeast through the permeable aquifers, with some vertical downward flow between layers. Piezometric gradients are approximately 4×10^{-3}, which is slightly flatter than the gradient of the land surface. Transmissivities, measured by pumping test, range from 0.003 to 0.23 m^2/s. Assuming an average hydraulic conductivity of 2×10^{-3} m/s and an effective porosity of 0.1 gives an average lateral groundwater velocity of 8×10^{-5} m/s or 6.9 m/day.

Three major aquifers have been identified at Greendale:

(1) A shallow unconfined aquifer (0–25 m) in the riparian zone of the Selwyn River. The water level ranges from 0 to 6 m below ground level, fluctuating in response to recharge from the river.

(2) A middle aquifer (35–85 m) which is confined by an overlying aquitard through which vertical recharge flows. Outside the riparian zone this aquitard extends

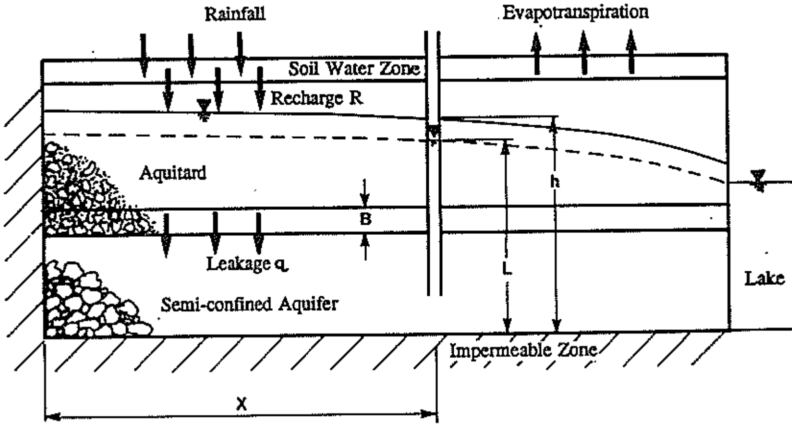

FIG. 2—A simple physical model of the Greendale aquifer.

above the water table. Piezometric water levels in the middle aquifer have a range of about 25 m.

(3) A deep aquifer (105–125 m), separated from the middle aquifer by an intervening aquitard zone. Piezometric water levels have a range of about 25 m.

The aquifers largely are recharged by rainfall, apart from some river recharge into the riparian shallow aquifer. Piezometric levels indicate vertical flow downwards between the aquifers. Outflow from the aquifers is predominantly by lateral flow to the southeast and by bore abstractions. Most irrigation bores in the Greendale area abstract water from the middle aquifer; the present study analyses bore data from this aquifer.

THE PHYSICAL MODEL

The aquifer system is illustrated by a vertical section along the principal direction of flow (Fig. 2). The three horizontal zones are:

(1) Zone of soil moisture storage which receives rainfall, loses moisture by evapotranspiration, and infiltrates its soil water surplus R to recharge the aquitard.

(2) An aquitard, with storativity P, in which the flow is only vertical. Hydraulic resistance occurs predominantly in a thin layer (thickness B hydraulic conductivity K) at the boundary with the semi-confined aquifer.

(3) A semi-confined aquifer (transmissivity T, storativity S) in which the flow is only horizontal. Vertical leakage q from the aquitard is the only input, and the downstream end of the aquifer has a constant piezometric head related to the level of Lake Ellesmere.

Soil moisture storage was simulated using a model developed by Heiler (1982) for water resource studies of the Canterbury Plains to estimate soil moisture surpluses. The surpluses, estimated on a daily time basis, were summed as monthly totals and used as recharge input data to the aquitard zone.

The aquifer system can be described by partial differential equations in the space variable x and time t. For a particular location, where bore levels are measured, the following discussion shows that the piezometric level L can be described by a linear ordinary differential equation in t.

Recharge R, leakage q, aquitard water level h, and aquifer piezometric head L are all functions of x and t. Some restrictions on the nature of these functions will be applied during the discussion.

The continuity equation for the aquitard is:

$$P \frac{\partial h}{\partial t} = R - q \qquad (1)$$

and for the semi-confined aquifer:

$$T \frac{\partial^2 L}{\partial x^2} + q = S \frac{\partial L}{\partial t} \qquad (2)$$

The leakage q from the aquitard into the aquifer is:

$$q = \frac{K}{B}(h-L) \qquad (3)$$

Substituting for q from (3) into (1) and (2):

$$P \frac{\partial h}{\partial t} = R - \frac{K}{B}(h-L) \qquad (4)$$

$$T \frac{\partial^2 L}{\partial x^2} + \frac{K}{B}(h-L) = S \frac{\partial L}{\partial t} \qquad (5)$$

Elimination of h from (4) and (5) yields:

$$\frac{PB}{K} \frac{\partial}{\partial t}\left\{S \frac{\partial L}{\partial t} - T \frac{\partial^2 L}{\partial x^2} + \frac{K}{B} L\right\} + S \frac{\partial L}{\partial t} - T \frac{\partial^2 L}{\partial x^2} = R \qquad (6)$$

Now assume the following:

(1) R = R(t), which means that the estimated recharge from soil moisture is the same at all locations.

(2) The solution L to equation (6) is of the form L = L(x)L(t), so that the separation-of-variables solution technique can be used.

When the above assumptions are applied to equation (6) the result is:

$$\frac{PSB}{K} \frac{d^2L(t)}{dt^2} + \left\{SL(x) - \frac{PTB}{K} \frac{d^2L(x)}{dx^2} + PL(x)\right\} \frac{dL(t)}{dt}$$

$$- T \frac{d^2L(x)}{dx^2} L(t) = R(t) \qquad (7)$$

For a particular location x, the terms in brackets are constant and equation (7) is a second-order linear differential equation which describes the response of the piezometric head L(t) of the semi-confined aquifer to recharge R(t) infiltrating from the soil mantle.

The validity of using the separation-of-variables method is not rigorously proven here. However, the aquifer system shown in Figure 2 is similar to unsteady flow in agricultural drainage. Van Schilfgaarde (1974) discusses solutions, including separation of variables, which lead to linear system response as a good approximation for unsteady water-table behaviour.

The use of estimated monthly recharge data (with no further optimisation of the soil moisture model) as input to the two-component aquifer system reduced the analysis to the dynamic behaviour of a simple linear system. It is the dynamic response which obscures the relationship between recharge inputs and water level changes.

The reduction of a nonlinear hydrological process to a simple linear dynamic system responding to inputs from a nonlinear data filter has been employed in several investigations, for example Jakeman *et al* (1990).

THE TRANSFER FUNCTION MODEL

Since the input and output data are measured at discrete time values, the differential equation (7) is more conveniently expressed as a second-order difference equation:

$$L_t = a_1 L_{t-1} + a_2 L_{t-2} + b_1 R_{t-1} + b_2 R_{t-2} \qquad (8)$$

The coefficients of equation (8) can be expressed directly in terms of the coefficients of equation (7) (Box and Jenkins, 1970), thus maintaining the linkage with physical concepts.

Equation (8) can be rewritten in the form:

$$(1 - a_1z^{-1} - a_2z^{-2})\, L_t = (b_1z^{-1} + b_2z^{-2})\, R_t \qquad (9)$$

by use of the shift operator z^{-1} where

$$z^{-m} L_t = L_{t-m}$$

The ratio of the two polynomials in z^{-1}

$$(b_1z^{-1} + b_2z^{-2}) \,/\, (1 - a_1z^{-2} - a_2z^{-1})$$

is the transfer function which relates the series L_t to R_t, or, in more general terms

$$L_t = \frac{B(z^{-1})}{A(z^{-1})} R_t \qquad (10)$$

where the numbers of terms in the z^{-1} polynomials A and B are to be determined by the model identification procedure. The conventional notation for model structure is (n, m, b) where:

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n} \tag{11a}$$

$$B(z^{-1}) = (b_0 + b_1 z^{-1} + \ldots b_m z^{-m})\, z^{-b} \tag{11b}$$

The common factor z^{-b} in the $B(z^{-1})$ polynomial is the time delay in the system, exclusive of dynamic effects. This is the time between an input event and the first effect on the output.

The values of L_t, calculated by applying the transfer function model (10) to the R_t series are related to the measured bore water levels H_t by:

$$H_t = L_t + v_t \tag{12}$$

The noise series v_t is the difference between the simple model predictions of water level and the measured data from the complex natural system. It is usually serially correlated, and can be described in terms of another transfer function:

$$v_t = \frac{D(z^{-1})}{C(z^{-1})} e_t \tag{13}$$

which operates on a series of uncorrelated, normally distributed, random variables e_t with zero mean and variance σ_e^2. The complete transfer function model relating measured input and output data is obtained by combining equations (10), (12) and (13) to give:

$$H_t = \frac{B(z^{-1})}{A(z^{-1})} R_t + \frac{D(z^{-1})}{C(z^{-1})} e_t \tag{14}$$

MODEL IDENTIFICATION AND PARAMETER ESTIMATION

Identification is the process of determining the number of terms in each of the polynomials A, B, C, D, i.e., whether the transfer function (10) is a first, second, or higher order system, and the nature of the correlation in v_t. Estimation of the parameters of the identified model structure then completes the process.

Simple linear difference equations such as (8) superficially look amenable to multiple least-squares regression, but the effects of the lagged variables and the noise term v_t usually lead to poor parameter estimation. A description of the problems can be found in texts such as Wonnacott and Wonnacott (1979).

The algorithms used to identify and estimate equation (14) were the recursive estimation techniques incorporated in MICROCAPTAIN software (Young and Benner, 1991). The recursive nature of the algorithms enables parameter estimates to be updated as new data are processed. This can be a valuable aid to developing the model structure, as described by Young (1984). The three algorithms available in MICROCAPTAIN are Least Squares (LS), Basic Instrumental Variables (BIV), and Simple Refined Instrumental Variables (SRIV).

The shortcomings of least squares has been mentioned, but use of the instrumental variable techniques overcomes most difficulties (Young, 1984).

Jakeman *et al* (1990) compare properties of LS, BIV, and SRIV, noting that SRIV is particularly effective at identification of slowly decaying components in a second-order hydrological system model. The three algorithms (LS, BIV, SRIV) were compared in the initial stage of the present study, before selecting SRIV as the best.

The recursive instrumental variable techniques are reported (Young, 1985) to be robust and consistent parameter estimators where the system noise is not well structured in statistical terms. This can be important for environmental systems where the noise term may include the effects of other deterministic, but unrecognised, causative inputs.

Identification of the complete transfer function model (14) is a two stage process within the MICROCAPTAIN algorithms. The deterministic component (10) is first identified and the noise series v_t is then identified if required. There are two quantitative criteria for selection of the best model, incorporated in MICROCAPTAIN (Young and Benner, 1991):

(1) The coefficient of determination R^2.
(2) A structure identification criterion (referred to as YIC) which takes into account the variance of the parameter estimates.

The best model usually has the lowest YIC value.

A useful qualitative assessment of a model is the variation of parameter values with the amount of data added to the recursive estimation. A well structured model has parameters that converge to stable values (consistency of estimation). Another important quality is the physical reality of the model. This can be assessed by examining the impulse response (equivalent to the hydrologist's unit hydrograph) of the system model, an option available in MICROCAPTAIN. For example, oscillating impulse responses are not usually characteristic of hydrological systems.

The model structure notation (n, m^1, b) output from MICROCAPTAIN differs slightly from the definition in (11a, 11b) in that n, m^1 are the number of a_i and b_i parameters to be estimated. The inclusion of b_o makes $m^1 = m + 1$. The (n, m^1, b) notation is used in this paper.

SIMULATION AND FORECASTING

Simulation of the aquifer system involves making the best estimate of bore water levels in response to determined values of rainfall recharge. The estimated series L_t is obtained by applying the transfer function in equation (10) to the input series R_t.

For forecasting future values of bore water level, given the known present water level and assumed future values of R_t, a forecast equation can be developed (Box and Jenkins, 1970) from the full system description of equation (14). The e_t series is the one-step-ahead forecast error which becomes known when a new value of H_t is measured, and then is part of the input data to the forecast equation:

$$A(z^{-1})\, C(z^{-1})\, H_t = B(z^{-1})\, C\, b(z^{-1})\, R_t + D(z^{-1})\, A(z^{-1})\, e_t \qquad (15)$$

obtained by cross-multiplying equation (14).

Simulated water level data from the monitoring bore used in system identification may be transferred to other bores in the aquifer using the separation of variables assumption. Since the water level at any location is assumed to be L = L(x) L(t), the whole aquifer has a common time record L(t) which is simply scaled by the factor L(x) for the bore at x. Simple regression of concurrent water levels (at zero time lag) between a bore N and the monitoring bore M accounts for the ratio of scale factors, as well as differences in measurement datum (usually local ground level), so that:

$$L_N = b_N L_M + k_N \tag{16}$$

Only a relatively short record of data at bore N is needed to estimate equation (16).

The difference equations derived for simulation and forecasting were implemented using standard spreadsheet software (GRAPHIC OUTLOOK).

DATA

Monthly rainfall recharge was estimated using a daily water balance model operating on daily rainfall and estimated daily evapotranspiration. The water balance model is a component of a crop irrigation model developed by Heiler (1982) and has the form:

$$\text{Recharge} = \text{Rainfall} - \text{Evapotranspiration} + \text{Soil moisture}$$
$$- \text{Available water capacity}$$

for positive values of the expression, and zero otherwise. Surface runoff is not a significant component of the regional water balance of the Central Canterbury Plains and has not been included in the model.

Available water capacity was set at 80 mm on the basis of knowledge about soils in the area, and was not a variable parameter.

$$\text{Evapotranspiration} = \text{Potential evapotranspiration}$$
$$\times \text{ Soil moisture factor} \times \text{Crop factor}$$

Potential evapotranspiration was estimated from the Penman method using daily data from the Lincoln meteorological station (Fig. 1). The particular Penman formulation used is adapted from French and Legg (1979):

$$E = \frac{1000 \times \Delta \times NR/\lambda + \gamma \times A}{\Delta + \gamma}$$

E = potential evaporation mm/day

Δ = slope of saturation water vapour pressure curve against air temperature (kPa/K) at air temperature

γ = psychrometric constant (kPa/K)

λ = latent heat of vaporization of water (J/kg) at air temperature

A = 2.6 x V x (1 + 0.0062 x U)

V = vapour pressure deficit (kPa) at the mean daily temperature

U = wind run (km/day) corrected to 2 m height

The net radiation NR is estimated from measured solar radiation SR by means of a regression equation (Jamieson, 1982):

$$NR = -0.25 + 0.59\ SR\ (J/m^2/day)$$

The soil moisture factor is given by the expression (Minhas *et al* 1974):

$$\frac{1 - \exp(-ap)}{1 - 2\exp(-a) + \exp(-ap)}$$

where a is a parameter to be set and p is fraction of total soil moisture capacity. For this study a = 30, and the expression is near to unity except for $p < 0.1$.

The crop factor has the following values for each month (Heiler, 1982) reflecting the dominant agricultural land use of the Canterbury Plains:

J	F	M	A	M	J	J	A	S	O	N	D
0.9	0.8	0.7	0.7	0.6	0.6	0.6	0.7	0.7	0.7	0.8	0.9

The water balance procedure was operated with daily rainfall data from Darfield (Fig. 1).

Monthly totals of recharge obtained by the above procedure were treated as model input data and no fitting or optimisation of parameters was done as part of the subsequent time-series analysis.

Monthly instantaneous water level data (for the start of each month) for bore L36/92 (92 in Figure 1), were obtained from manual readings (up to May 1975) and subsequently from an automatic water-level recorder.

The database used for the model analysis were 285 monthly values of rainfall recharge as input and monthly bore level as output for the period April 1967 to December 1990. These data, for January 1968–December 1990, are shown in Figure 4(a).

Monthly water level data from bores M36/45, L36/58, L36/63, L36/66, L36/92 and L36/93 (45, 58, 63, 66, 92, 93 in Figure 1) were used for estimation of equation (16) for spatial extension of water level records. There were many missing values in the 13–15 years of record for some sites, so sections of concurrent records were selected for the regression equation (16).

RESULTS

Comparative Algorithm Test

The three algorithms, Least Squares (LS), Basic Instrumental Variables (BIV), and Simple Refined Instrumental Variables (SRIV) were used in model structure identification runs on the full data set of 285 input-output pairs. The data were expressed as departures from their mean values of recharge (18.24 mm) and water level (–42169 mm).

The range of models considered were all combinations of first, second and third order with time delays of up to three months, i.e.

$$\text{all } ((n, m^1, b)\ n, m^1, b = 1, 3)$$

All three algorithms selected the eight combinations of first- and second-order models with time delay of one or two months as the best eight models, but not ranked in the same order according to the YIC criterion. The best four models selected by each algorithm are shown in Table 1.

The SRIV algorithm performs best on the basis of the lowest YIC values, and the highest R^2 values, the physical consistency of one first-order model ranking first and second according to time delay, and one second-order model ranking third and fourth according to time delay. For these reasons the SRIV algorithm was selected for the remainder of the analyses.

TABLE 1—Algorithm Comparison by Model Identification

Algorithm	Model (n, m^1, b)	YIC	R^2
Least Squares (LS)	2, 2, 1	–2.833	0.852
	1, 1, 2	–2.577	0.597
	1, 2, 1	–2.323	0.733
	1, 2, 2	–2.054	0.732
Basic Instrumental Variables (BIV)	2, 1, 1	–2.859	0.839
	2, 2, 1	–2.817	0.870
	1, 1, 2	–2.633	0.607
	1, 2, 1	–2.389	0.741
Simple Refined Instrumental Variables (SRIV)	1, 1, 2	–10.111	0.884
	1, 1, 1	–9.893	0.864
	2, 1, 1	–7.997	0.901
	2, 1, 2	–7.386	0.900

Datum Level Sensitivity Test

The use of mean values as the bases from which data departures are measured, depends on good estimates of the means being obtained from the available record. For a slowly decaying system such as the aquifer being studied, the estimate of mean water level becomes less precise when the record sample is split for calibration/validation tests. The sensitivity of the model identification criteria to changes in datum level was assessed for the full 285–pair dataset by setting the input datum to the mean value 18.24 mm and varying the output datum in 500 mm steps from –39000 mm to –45000 mm.

The variation of YIC and R^2 for the best first-order and second-order models (using SRIV) are shown in Figure 3.

For the full 285-pair dataset the optimal YIC and R^2 do occur at datum values near the output mean. However, this is not the case when the first 143

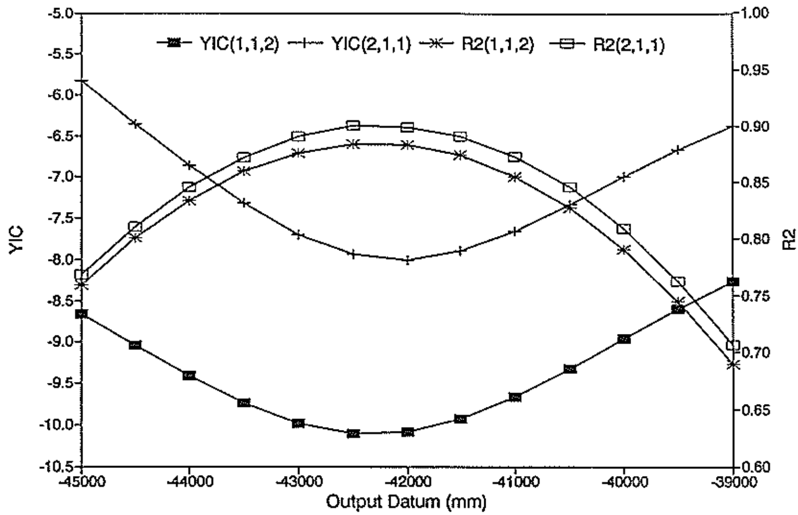

FIG. 3—Sensitivity of YIC and R^2 to changes in output datum for an input datum set to the mean value for the full data set.

data pairs are analysed for model calibration. The estimated input mean (19.91 mm) is higher and the output mean (-43475 mm) is lower, an obvious lack of equilibrium datum match. The effect is demonstrated in Table 2, which shows the best four models for the datum pairs: input and output means of data 1–143; input and output means of data 1–285; input mean of 1–143 and optimised output datum.

TABLE 2—Comparison of Input-Output Datum for Sample 1–143

Input datum Output datum	Model (n, m^1, b)	YIC	R^2
19.91 mm -43475 mm	2, 2, 2 2, 1, 1 2, 1, 2 1, 1, 2	-8.776 -6.157 -5.603 -4.215	0.933 0.673 0.670 0.333
18.24 mm -42169 mm	1, 1, 2 1, 1, 1 2, 2, 2 2, 1, 1	-10.603 -10.394 -9.651 -8.373	0.934 0.926 0.972 0.940
19.91 mm -39550 mm	1, 1, 2 1, 1, 1 2, 2, 2 2, 1, 2	-10.535 -10.368 -10.179 -8.158	0.922 0.914 0.922 0.927

Mismatched datum levels caused poorer model performance in terms of YIC and R^2, especially for the otherwise best ranked (1, 1, 2) model. The optimal output datum (–39550 mm) was selected at the highest YIC value for the (1, 1, 2) model, which had remained first ranked over a range of datum values. The (2, 2, 2) model maintained high YIC values and was first (–10.617) at a datum of –39400. However, the recursive parameter estimates showed poorer convergence than for the (1, 1, 2) model, and the impulse response did not show any advantage in terms of smoother initial response.

Selection of a Simulation Model

The SRIV results (Table 1) show that the (1, 1, 2) model is best according to the established criteria. However, since the original physical model is a second-order system, it is worth considering the best second-order model (2, 1, 1) as a possible simulation model in case any physical reality remains untested. Both models were calibrated over the full 285 data pairs: data 1–143 and data 143–285. The means of the full 285 data series were used as datum levels throughout. The estimated parameter values for each model and dataset are compared in Table 3. The confidence limits on the parameter values in the table are only an approximate measure of the standard deviation of the parameter estimates, but will be referred to as "standard deviation". The approximation arises from the structure of the noise series.

From Table 3, the (1, 1, 2) model parameters for data 143–285 are within one standard deviation of the estimated values for data 1–285, whereas the parameters for data 1–143 differ by several standard deviations. The (2, 1, 1) model shows similar behaviour for the reverse order of datasets. This behaviour suggests a change in aquifer system dynamics, perhaps related to water levels, and may also demonstrate the ability of the SRIV algorithm to detect the "correct" parameter values for a specified model. The (2, 1, 1) model parameters for data 143–285 are not physically realistic because $a_1^2 - 4a_2 < 0$, which implies an oscillatory impulse response. A detailed discussion of these parameter properties, in a hydrological context, is provided by Spolia and Chander (1974).

The (2, 1, 1) model based on the full data 1–285 is:

$$L_t = \frac{9.79\ R_{t-1}}{(1 - 1.705z^{-1} + 0.714z^{-2})}$$

which, by factorising the denominator, can be written:

$$L_t = \frac{9.79\ R_{t-1}}{(1 - 0.965z^{-1})\ (1 - 0.740z^{-1})}$$

This is the transfer function of two first order systems in series: a relatively slow response storage (a_1 = 0.965); and a faster response storage (a_1 = 0.740). The slower response system has a coefficient (0.965), close to the (1, 1, 2) model coefficient for any part of the data set. The second, faster response storage apparently was active only during the earlier portion of the record when water levels were lower, and could not be adequately identified during the later portion. For these reasons, the first order model is the more reliable single model predictor. The explained variance (Table 1) of R^2 = 0.88 is almost as good as the second-order model (R^2 = 0.90), as demonstrated by the two simulations in Figure 4(b), (c).

TABLE 3—First- and Second-Order Model Parameters

Model Parameter	Data Set		
	1–143	143–285	1–285
a_1	–0.980 +/– .001	–0.968 +/– .003	–0.970 +/– .001
b_0	30.33 +/– .83	32.38 +/– 1.64	32.46 +/– .86
a_1	–1.700 +/– .03	–1.843 +/– .01	–1.705 +/– .03
a_2	0.706 +/– .03	0.853 +/– .01	0.714 +/– .03
b_0	9.57 +/– .93	10.28 +/– .41	9.79 +/– .90

The preferred choice of simulation model, calibrated on the full dataset is:

$$L_t = \frac{32.46}{1 - 0.970\, z^{-1}} R_{t-2}$$

$$\text{i.e. } L_t = 0.970\, L_{t-1} + 32.46\, R_{t-2}.$$

If L^*_t and R^*_t are raw data values:

$$L_t = L^*_t + 42169\ ,\ R_t = R^*_t - 18.24$$

and

$$L^*_t = 0.970\, L^*_{t-1} + 32.46\, R^*_{t-2} - 1857 \qquad (17)$$

is the simulation equation for use with raw data.

Selection of a Forecast Model

The performance criterion for a forecast model is minimization of the one-step-ahead error variance σ_e^2. This is the variance of the series e_t which is the uncorrelated input to the noise component of the transfer function (14):

$$H_t = \frac{B(z^{-1})}{A(z^{-1})} R_t + \frac{D(z^{-1})}{C(z^{-1})} e_t$$

The relationship of e_t to forecast error is explained by Box and Jenkins (1970). For each model (1, 1, 2 and 2, 1, 1) the noise component v_t was modelled by a simple first-order equation:

$$v_t = c_1\, v_{t-1} + e_t$$

or

$$v_t = \frac{1}{(1 - c_1 z^{-1})} e_t$$

$$\text{i.e. } D(z^{-1}) = 1 \text{ and } C(z^{-1}) = (1 - c_1 z^{-1})$$

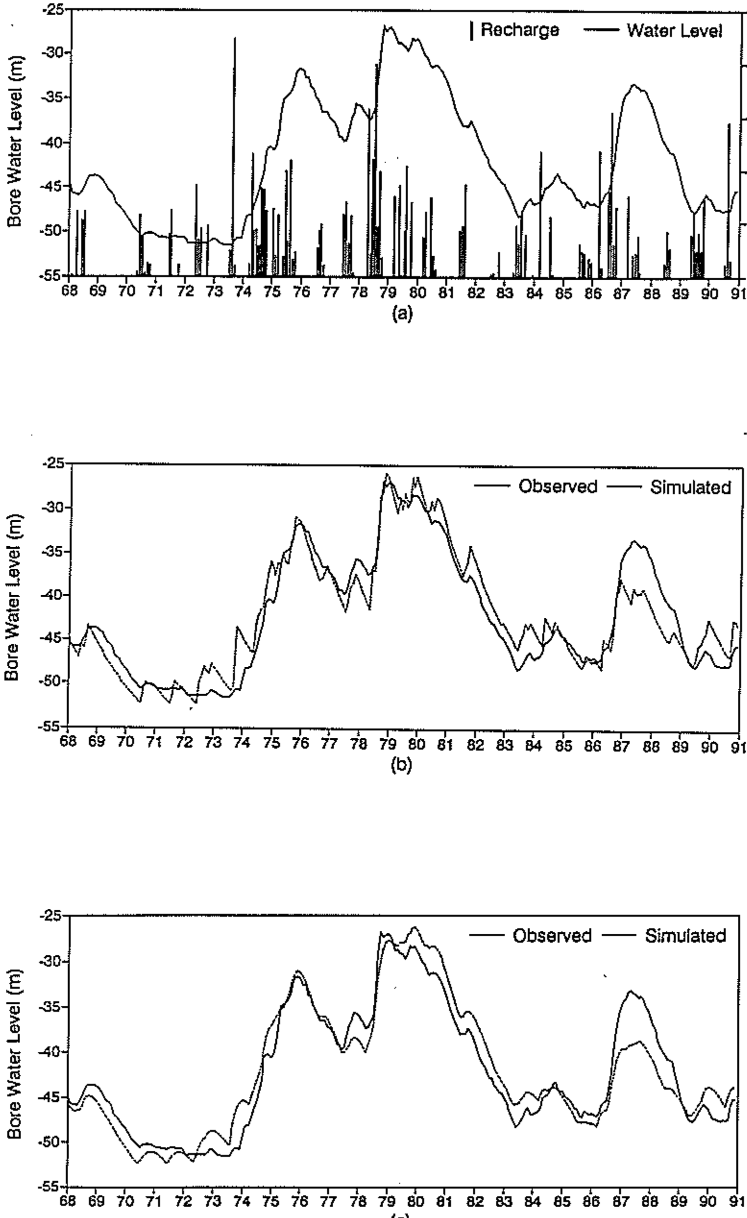

FIG. 4—The (a) water level and recharge series, (b) (1, 1, 2) simulation, and (c) (2, 1, 1) simulation, for monthly data January 1968–December 1990.

This simple model generated residuals e_t which were not significantly correlated for the purpose of model comparison. The values of the c_1 parameter for the models and data sets of Table 3 are shown in Table 4.

TABLE 4—First-Order Noise Model Parameter c_1

Model	Data Set		
	1–143	143–285	1–285
(1, 1, 2)	0.863 +/– .04	0.963 +/– .02	0.925 +/– .02
(2, 1, 1)	0.958 +/– .02	0.991 +/– .01	0.978 +/– .01

A calibration/validation exercise of the first-order and second-order forecast equations produced the results in Table 5. The performance measure is the residual variance σ_e^2.

TABLE 5—Model Calibration and Validation — σ_e^2

Model	Calibration		Validation	
	1–143	143–285	1–143	143–285
(1, 1, 2)	836494	573916	999774	708035
(2, 1, 1)	240050	166456	446783	171749

The consistent superiority (lower σ_e^2) of the second-order model is obvious from the results in Table 5. This probably reflects the more natural and smoother impulse response of the second-order system. The parameter instability with respect to data set is overcome to a large extent by the "self correcting" effect of dependence in the noise component.

When the second-order (2, 1, 1) model was fitted to the full data set, the resulting noise series was described by a second-order autoregressive process:

$$v_t = 1.241\, v_{t-1} - 0.268\, v_{t-2} + e_t \tag{18}$$

The residuals e_t were tested for serial correlation by means of the "portmanteau" test (Box and Jenkins, 1970, 8.2.2). The resulting chi-square value was acceptable only at the 2.5% significance level. Improvement to this rather marginal acceptance could be obtained by including terms at time lags of 12 and 15 months, but the small improvement to the forecast was not worth the increased complexity of a practical forecast equation. The distribution of the residuals from equation (18) was accepted as normal under a Kolmogorov-Smirnov test. The noise and residual series for equation (18) are shown in Figure 5.

The complete forecast model is:

$$H_t = \frac{9.79\, R_{t-1}}{1-1.705z^{-1}+0.714z^{-2}} + \frac{e_t}{1-1.241z^{-1}+0.268z^{-2}} \tag{19}$$

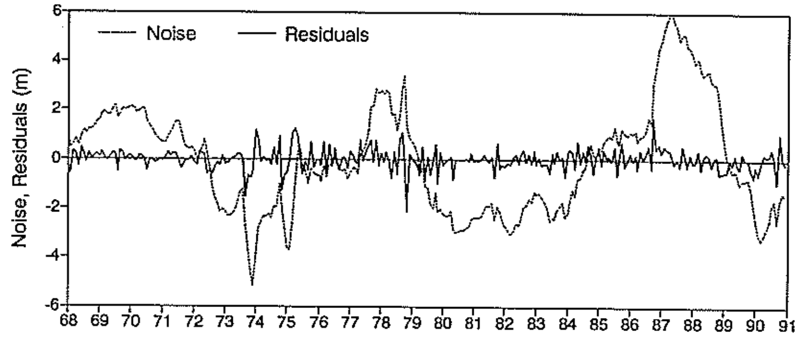

FIG. 5—The noise (v_t) and residuals (e_t) of the (2, 1, 1) model.

which is cross-multiplied to the form of equation (15), and terms collected to yield the difference equation for use with raw data H_t^*, R_t^*:

$$H_t^* = 2.946\, H_{t-1}^* - 3.098\, H_{t-2}^* + 1.343\, H_{t-3}^* - 0.191\, H_{t-4}^*$$
$$+ 9.79\, R_{t-1}^* - 12.15\, R_{t-2}^* + 2.62\, R_{t-3}^*$$
$$+ e_t - 1.705\, e_{t-1} + 0.714\, e_{t-2} \qquad (20)$$

The residual variance σ_e^2 = 189362 (0.0041 of total variance), which means that the one-step-ahead forecast standard deviation is 435 mm.

If equation (20) is used at time t-1 to forecast H_t^*, the only unknown on the right hand side is the value e_t, the one-step forecast error. For forecasts, made at time t, of H^*_{t+n}, which the n-step-ahead forecast, equation (20) is used successfully to forecast H^*_{t+1}, H^*_{t+2} . . . etc. using as data:

(1) known values of H_t^* (otherwise use the most recent forecasts),
(2) known R_t (otherwise assumed future values), and
(3) known e_t (otherwise assume $e_t = 0$.)

Box and Jenkins (1970) describe the mathematical background of forecast equations, updating forecasts, and the variance of forecasts. The variance of the n-step forecast obtained from equation (20), given assumed future recharge data (such as $R_t = 0$, during summer), is obtained from the noise term of equation (19). The standard deviations of forecasts up to six months ahead, calculated by the method in Box and Jenkins (1970, 5.2.4), are shown in Table 6.

TABLE 6—Standard Deviation of Bore Water Level Forecast

Lead time (months)	1	2	3	4	5	6
Standard Deviation (mm)	435	693	812	897	966	1025

Data Transfer Between Bores

The regression equation (16) relating water level L^*_N at bore N to water level L^*_{92} at bore 92, for raw data, is:

$$L^*_N = b_N L^*_{92} + k_N$$

Values of the coefficients b_N, k_N and the associated R^2 for each bore are shown in Table 7.

TABLE 7—Regression Coefficients for Levels at Bore N from Levels at Bore 92

Bore N	b_N	k_N	R^2
45	0.467 +/- .015	6336 +/- 611	0.876
58	0.865 +/- .018	-18773 +/- 704	0.947
63	1.141 +/- .018	14208 +/- 749	0.962
66	0.917 +/- .040	12908 +/- 1677	0.851
93	0.917 +/- .024	22 +/- 986	0.935

DISCUSSION

The high degree of explained variance (R^2 = 0.90) obtained in this study demonstrates that the Greendale aquifer is recharged predominantly from rainfall on the Plains. The response of aquifer water levels at monitoring bores can be quantified by simple equations based on linear system theory. The direct relationship between the system theory and physical concepts provides managers and users of the aquifer with simple, quantifiable aids to understanding the resource.

For simulations the water level at bore 92 is described by the transfer function:

$$L_t = \frac{32.46R_{t-2}}{(1 - 0.970z^{-1})} \qquad (21)$$

The term $(1-0.970z^{-1})$ describes the time response of the whole aquifer. The coefficient 0.970 is the recession coefficient for water level records at any location in the aquifer, under the separation of variables assumption. The Greendale aquifer is a very slow responding system. On the basis of standard calculations from the first order model (21), the response time is about six years to reach 90% of a new steady state after a hypothetical step change in steady recharge input.

The coefficient 32.46 in (21) is specific to bore 92. The corresponding coefficients for other bores can be obtained by scaling with the regression coefficients b_N (Table 7). These bore coefficients are directly proportional to the range of water levels at the bore. Under the assumptions of the aquifer model, the water level range increases with distance from Lake Ellesmere which controls the assumed downstream boundary condition. This relationship is obvious for bore 45 (b_N = 0.467) which is about 10 km downstream of bore 92, but somewhat contradictory for bore 58 (b_N = 0.865). The explanation requires further investigation.

The 23 years of water level data include only three or four events of a significant range in response to the relatively high frequency recharge data. This high degree of system inertia makes perception of cause and effect difficult for the aquifer user, and also provides the systems analyst with some problems.

The estimation algorithm comparison in Table 1 shows that a good fit to a simple robust model was only achieved by the SRIV method. This is due to the low level of "excitement" in this slow responding system and the nature of the error component. Robust algorithms for time-series analysis of noisy systems are not commonly available in standard multipurpose statistical software packages. The recent availability of commercial software to address difficulties in analysing noisy environmental systems is a significant technological step.

The most significant property of these estimation techniques is their ability to obtain the "true" values of model parameters in the presence of a poorly determined noise structure. The noise series then can be examined with some confidence to determine other possible inputs to the system. For example, the significant departure of simulated from measured water levels in 1986–88 (Figure 4) provoked an investigation into the flow of the Selwyn River as a possible explanation. The highest mean monthly flow since records began in 1964 occurred in August 1986, and the possibility of additional recharge from alpine rainfall should be considered.

The slowly varying water-level data required an additional analytical step in establishing the datum for further data processing, especially when the sample was split for calibration/validation trials. This is not usually required with other higher frequency hydrological data.

Although the second-order model (2, 1, 1) showed the highest R^2 (0.90) for the 23 years data, the first-order model (1, 1, 2) (R^2 = 0.88) was better overall in terms of the special criterion (YIC) included in the algorithm. Its superiority was also demonstrated in the split sample calibration/validation trials, when a significant system parameter change seemed to occur with both models (Table 3). The second-order model could be adequately identified only during the first half of the data record, and when decomposed into its first-order constituents showed that a faster-decaying storage modulates the slow-decaying first-order model. This behaviour seems to occur at lower water levels. The physical explanation requires further investigation and, at this stage, cannot be conclusively linked with the storativity components of the simple physical model described in this paper.

The ability of time-series analysis to quantify the structure of the noise component of the models allows the upgrading of a simulation model to a forecast model, providing optimal estimates of future events based on present information, and places confidence limits on the forecasts. The second-order model was preferred for forecasting because of a more realistic impulse response, and high dependence in the noise component, resulting in smaller forecast error.

The simplicity of the simulation and forecast models means that their implementation is ideally suited to standard spreadsheet software. The combination of commercial time-series analysis and spreadsheet software enables many environmental systems to be analysed, and resource management tools to be implemented, without the need for specialist software development.

The models are limited to defining the natural water level variation at a bore due to rainfall recharge. This knowledge forms the baseline against which departures due to abstraction or supplementary recharge from other sources may be assessed. Water levels at several bores can be related by simple linear equations and these relationships can be used to assess the cost effectiveness of the bore monitoring programme. The good fit of the models suggests that the estimates of recharge from the soil-water balance model are suitable for resource assessment, although the overall fit has not been tested by varying the water-balance parameters.

The ability to predict likely natural aquifer levels under various scenarios of rainfall recharge allows development of baseline recommendations for well depths and drawdown interference. During periods of low water level the forecast models could provide farmers with early warning, so that management decisions can be implemented in time to avoid catastrophic loss. The noise component includes the effect of other recharge sources and variations in abstraction, whether seasonal or long term. A detailed time-series analysis of the noise has not been included in the present study, but would be an obvious source of additional information for management of the resource.

CONCLUSIONS

Water levels in a semi-confined aquifer in Central Canterbury Plains can be related to rainfall recharge by simple linear system models. These models provide resource managers with the ability to estimate aquifer water levels under specified recharge scenarios, and forecast water levels several months ahead using available commercial software.

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