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# EXPERIMENTAL EVALUATION OF INFILTRATION MODELS

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## 1. INTRODUCTION

Infiltration of water into the soil is a major component in a hydrological model for runoff from a watershed. During a storm, rainfall intensities in excess of infiltration rate produce surface runoff. Since the percentage of rainfall which infiltrates into the soil is usually large, the accuracy with which surface runoff can be predicted is dependent on an accurate evaluation of infiltration. There are two basic deterministic approaches to infiltration, the soil physics, and the hydrological approach. In soil physics modeling, too much data and extensive computations are required; the additional accuracy that can be obtained doesn't justify the time, labour, and expense associated with using such a model. Several hydrological models have been reported in the literature. Hydrologists generally estimate infiltration from an analysis of hydrographs, from the results of infiltrometers, field tests, and from the derivation of basin indices for a complete watershed. In addition to these, empirical formulae for infiltration have been developed by Green and Ampt (1911), Kostiakov (1932), Horton (1940), Philip (1957), Holtan (1961), and others. However, a lack of knowledge of the equation parameters for different soils and locations makes the use of these formulae difficult. The purpose of this study is to obtain numerical values for these parameters from field tests with infiltrometers, and to compare the different infiltration models. The study is restricted to infiltration under ponded surface conditions, and does not cover the boundary condition of constant surface flux.

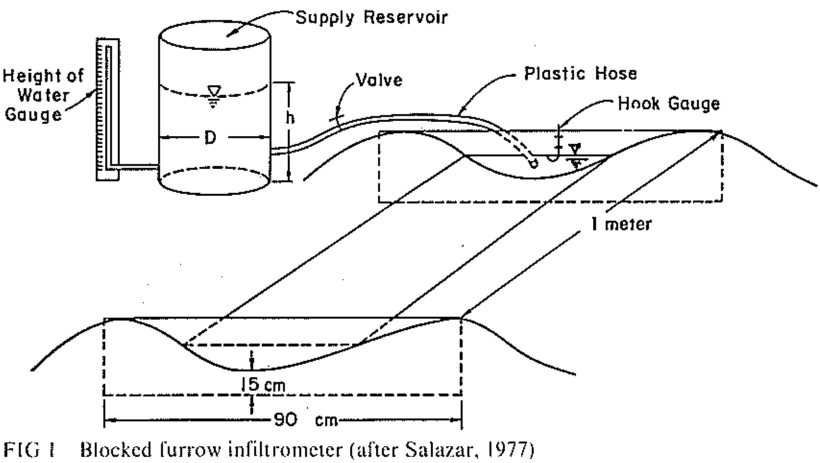


FIG 1 Blocked furrow infiltrometer (after Salazar, 1977)

## 2. EXPERIMENTAL PROCEDURE

Infiltration data from an experimental plot, at Northern Colorado Research Demonstration Centre, Greely, Colorado USA, were collected during the summer of 1983. Nine "Blocked Furrow Infiltrometers" were installed in three rows. The infiltrometer (Fig. 1) consists of a supply reservoir with a water gauge for reading the water levels. Water is supplied to a furrow from the reservoir through a plastic hose fitted with a control valve. The water in the furrow infiltrates into the soil and its level is maintained constant by a means of a hook gauge. A detailed description of the infiltrometer is given by Salazar (1977).

The test was carried out on a relatively homogenous, coarse-textured loamy sand soil. The initial moisture condition can be considered uniform for the top 20 cm of the soil. Before each infiltrometer run, soil samples were taken at 7 cm depth intervals. From these samples, average values of the initial moisture content, moisture content at field capacity, bulk density, porosity, and bulk specific gravity for the soil were determined (Table 1).

Infiltration into the soil was measured at various time intervals. In all, 12 observations were taken for each infiltrometer, making a total of 108 observations for the experimental plot. The average values of the infiltration data from the nine infiltrometers are given in Table 2. A plot of infiltration against elapsed time (Fig. 2) shows that the constant infiltration rate after prolonged wetting is $f_c$ = 1.1 cm/hr, after an initial infiltration rate $f_o$ = 18.75 cm/hr.

## 3. DATA ANALYSIS

Since infiltration data are available from several infiltrometers, the data may be pooled before analysis to fit each infiltration equation (Elliot and

Table 1    Physical Properties of the Soil

| Property | | Value | Remark |
|---|---|---|---|
| soil type, | | | loamy sand |
| bulk specific gravity, | $\gamma_b$ | 1.550 | |
| bulk density, | $\rho_b$ | 1.550 | |
| moisture content: | | | |
| initial, | $\theta_i$ | 0.125 | |
| at field capacity, | FC | 0.261 | $\theta = \gamma_b . W$ |
| porosity, | $\phi$ | 0.420 | |

TABLE 2 — Measured Infiltration Rates

| elapsed time (min) | 0 | 5 | 10 | 15 | 20 | 30 | 45 | 60 | 75 | 90 | 120 | 140 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| cumulative infiltration, F (cm) | — | 1.50 | 2.75 | 3.88 | 4.73 | 6.15 | 8.10 | 9.90 | 11.15 | 11.95 | 12.74 | 13.11 |
| infiltration rate, f (cm/hr) | — | 18.0 | 15.0 | 13.6 | 10.2 | 8.50 | 7.80 | 7.20 | 5.00 | 3.20 | 1.58 | 1.10 |

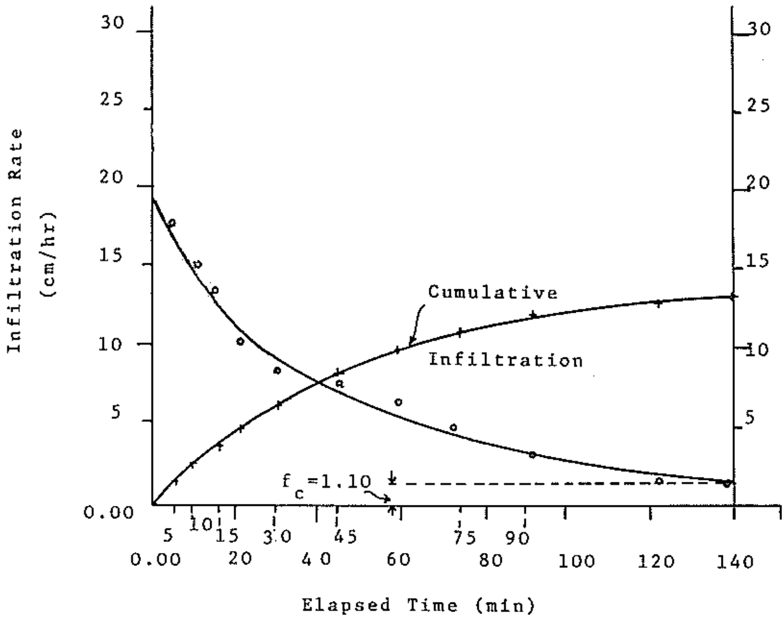


FIG 2—Infiltration characteristic curves

Walker 1980). Average infiltration rates from the nine infiltrometers are presented in Table 2.

Six infiltration models were examined to evaluate their paramters: Kostiakov, Modified Kostiakov, Philip, Green and Ampt, Horton, and Holtan-Overton.

### *3.1 Kostiakov Model*

One of the simplest infiltration models was proposed by Kostiakov (1932):

$$F = K_1 t^{\alpha_1} \quad (1a)$$

$$f = dF/dt = \alpha_1 K_1 t^{\alpha_1 - 1} = \eta_1 t^{\gamma_1} \quad (1b)$$

where F is the total depth of water infiltrated into the soil (cm), t is the elapsed time in minutes, $K_1$ is cumulative infiltration at unit time, $\alpha_1$ is an empirically-determined coefficient which is positive and always less than unity, f is the rate of infiltration up to the time when the infiltration rate would be equal to the saturated hydraulic conductivity (cm/hr), $\eta_1 = \alpha_1 . K_1$, and $\gamma_1 = \alpha_1 - 1$ which is always negative. Equation (1) is used for the initial high rate of infiltration.

A better representation of the depth infiltrated over a long period of time is given by the modified Kostiakov equation as

$$F = K_2 t^{\alpha_2} + Ct \quad (2a)$$

$$f = \eta_2 t^{\gamma_2} + C \quad (2b)$$

where C is a constant equal to the basic infiltration rate in cm/hr.

*3.2 Philip Model*

Philip (1957) proposed an equation for the depth of penetration, x, of a given moisture content, $\theta$, represented by the series:

$$x = a_1(\theta).t^{1/2} + a_2(\theta).t + a_3(\theta).t^{3/2} + \ldots + a_m(\theta).t^{m/2} + \ldots \quad (3)$$

The above series converges so rapidly that only a few terms are required for a satisfactorily accurate solution. The volume of infiltration can be obtained by integrating the depth of penetration over the range of change in moisture content. Therefore, the total amount of infiltration up to a given time t is given as

$$F = \int_{\theta_i}^{\theta_s} x d\theta + K_o t \quad (4)$$

where $K_o$ is the unsaturated hydraulic conductivity corresponding to $\theta_i$. Philip suggested that, for most practical purposes, only the first two terms of Eq. (3) are required, so that Eq. (4) can be written as:

$$F = S.t^{1/2} + A.t \quad (5)$$

where S is called the sorptivity and is given by

$$S = \int_{\theta_i}^{\theta_s} a_1(\theta)\, d\theta \quad (6)$$

and the second parameter, A, is given by

$$A = K_o + \int_{\theta_i}^{\theta_s} a_2(\theta)\, d\theta \quad (7)$$

Thus, the infiltration rate can be obtained by differentiating Eq. (5) with respect to t, giving:

$$f = \frac{S}{2} t^{-1/2} + A \quad (8)$$

The parameters S and A have the dimensions $L/T^{1/2}$ and $L/T$ respectively. Parameter A is considered roughly identical to steady infiltration. Parameter S is sensitive to antecedent soil moisture condition, and relatively less sensitive to the parameter A.

*3.3 Green and Ampt Model*

Green and Ampt (1911) proposed a formula for infiltration into the soil based on a model of uniform parallel capillary tubes. The model assumed a piston-type movement of soil water downward into the soil. Their approximate treatment assumed that the advancing moisture profile consists of two parts: an upper zone of higher moisture content separated from the original dry soil by a sharp discontinuity (Childs, 1967). Direct application of Darcy's law yields the following form of the Green and Ampt equation:

$$f = K_s \left[\frac{H_o + H_f + x}{x}\right] \quad (9)$$

where $K_s$ is saturated hydraulic conductivity of the transmission zone, $H_o$ is pressure head at the ground surface, $H_f$ is the effective suction head at the wetting front, and x is the depth of penetration of the higher moisture content. The rate of infiltration is

$$f = dF/dt = \frac{d(\Delta\theta.x)}{dt} \tag{10}$$

where: $\Delta\theta = \theta_s - \theta_i$.
Combining Eq. (9) and (10) obtains

$$\Delta\theta.\frac{dx}{dt} = K_s.\left[\frac{H_o + H_f + x}{x}\right] \tag{11}$$

which can be integrated to yield a relation between wetting-front position and time in the form

$$\frac{K_s.t}{\Delta\theta} = x - (H_o + H_f)\,ln\left[1 + \frac{x}{H_o + H_f}\right] \tag{12}$$

Equation (12) can be written in terms of the cumulative infiltration up to time t, since $F = \Delta\theta.x$,

$$K_s.t = F - \Delta\theta(H_o + H_f)\,.\,ln\left[1 + \frac{F}{\Delta\theta(H_o + H_f)}\right] \tag{13}$$

Expressing the depth of penetration as $x = F/\Delta\theta$, and assuming the depth of ponding at the surface is shallow, so that $H_o \approx 0$, Eq. (9) and Eq. (13) may be written respectively as

$$f = K_s.\left[1 + \frac{S_f}{F}\right] \tag{14a}$$

$$K_s.t = F - S_f.ln\left[1 + \frac{F}{S_f}\right] \tag{14b}$$

in which $S_f$ is the storage-suction factor defined as

$$S_f = H_f.\Delta\theta \tag{15}$$

*3.4 Horton Model*

Horton (1940) proposed an infiltration equation derived from work and energy principles:

$$f = f_c + (f_o - f_c)\,e^{-\beta t} \tag{16a}$$

Integrating Eq. (16a) yields

$$F = f_c.t + \frac{f_o - f_c}{\beta}(1 - e^{-\beta t}) \qquad 16(b)$$

where $f_t$ is the infiltration rate at time t (cm/hr), $f_o$ and $f_c$ are initial and final infiltration rates (cm/hr) respectively, and $\beta$ is an infiltration decay parameter. It is assumed in this equation that rainfall is always greater than infiltration capacity rates, and hence some ponding will always result. This is a major disadvantage in the use of Horton's equation. The steepness of the initial portion of the infiltration curve is reflected in the magnitude of the $\beta$ value. Field tests showed that $\beta$ is, in general, larger for very wet conditions and for crusted soil conditions, than for cultivated and sod plots. For an initial soil moisture content, $\theta_i$, equal to moisture content at natural saturation, $\theta_s$, one expects $\beta$ to become infinity, since immediately $f_c$ should take the value of saturated hydraulic conductivity, $K_s$.

### *3.5 Holtan Model*

A conceptual model based on a storage concept was proposed by Holtan (1961). The final rate of infiltration $f_c$ was associated with the gravity force at field capacity. The model was then formulated to relate capacity infiltration rate to the volume remaining as

$$f = a(S_P - F)^n + f_c \qquad (17a)$$

or

$$(f - f_c) = a(S_P - F)^n \qquad (17b)$$

where $S_P$ = storage potential of the soil above the control depth (cm). The constants a and n depend on the soil type and surface and cropping conditions. They should be determined experimentally from infiltrometer plot data. The constant dictates the steepness of the infiltration at the beginning of the infiltration process. Holtan and Creitz (1967) suggested that n may be taken as constant (n = 1.4) for all soils. The major difficulties in applying equation (17) are in defining the control depth, and in using an exponent of 1.4 which cannot be integrated in order to obtain a time distribution of capacity rates. Therefore, Overton's simplifications are used in order to fit the Holtan model. Overton (1964) showed that for a value of n = 2 in Eq. (17), the cumulative infiltration and the rate of infiltration could be expressed as a function of time in the following form

$$F = S_P - [f_c/a]^{1/2} . \tan [a.f_c(t_c - t)]^{1/2} \qquad (18a)$$

and

$$f = f_c . \sec^2 [(a.f_c)^{1/2}(t_c - t)] \qquad (18b)$$

where $t_c$ is the time to constant rate of infiltration expressed as

$$t_c = [\frac{1}{a.f_c}]^{1/2} . \tan^{-1} [S_P .(a/f_c)^{1/2}] \qquad (19)$$

and the parameter a is given as

$$a = \frac{f - f_c}{[S_P - F]^2} \qquad (20)$$

The storage potential is defined as the depth of pore space available for water to saturation in a depth Y of soil and is given by:

$$S_P = (\phi - \theta_i).Y \qquad (21)$$

where Y is the control depth, and $\phi$ is the porosity.

## 4. EVALUATION OF EQUATION PARAMETERS

Least squares regression analysis is chosen to evaluate the six concise infiltration models and to determine the values of the equation parameters (Salazar, 1977).

The Kostiakov model, Eq. (1a), is a power function with $K_1$ and $\propto_1$ constants. Taking the logarithms of both sides of Eq. (1a) results in a straight-line equation. The value of log $K_1$ corresponds to the intercept of the line at log t = 1, and the value of $\lambda_1$ represent the slope of the line. Thus, Kostiakov's model becomes

$$F = 0.625\ t^{0.653} \tag{22a}$$

and

$$f = 0.408\ t^{-0.347} \tag{22b}$$

The coefficient of determination is $r^2 = 0.992$.

To facilitate regression analysis, the Modified Kostiakov model, Eq. (2a) is written in the form

$$\log (F - Ct) = \log K_2 + \propto_2 \log t \tag{23}$$

In this case, log (F-Ct) is plotted against elapsed time, log t. As was stated previously, C is the basic infiltration rate after a long time, which can be determined from the infiltration curve. From this initial estimate the value of C should be adjusted by trial and error until log (F-Ct) vs. log t data plots approach a straight line. By regressing log (F-Ct) vs. log t, the derived slope value will be exponent $\propto_2$. The value of log $K_2$ corresponds to the intercept of the line at the log t = 1 position.

The curve-fitting process can also be achieved analytically. The C-value should be adjusted until a value of the coefficient of determination of close to one is reached. Thus, the modified Kostiakov model becomes

$$F = 0.648\ t^{0.612} + 1.08\ t \tag{24a}$$

and

$$f = 0.400\ t^{-0.388} + 1.08 \tag{24b}$$

The coefficient of determination is $r^2 = 0.986$.

Philip's model, Eq. (8), is a linear function and its parameters are determined either by plotting f vs. $1/t^{1/2}$, or by using linear regression analysis. The results obtained are A = 0.426, S/2 = 5.84, and a coefficient of determination, $r^2 = 0.886$. Hence, Philip's model becomes

$$F = 11.68\ t^{1/2} + 0.426\ t \tag{25a}$$

and

$$f = 5.84\ t^{-1/2} + 0.426 \tag{25b}$$

The Green and Ampt model, Eq. (14), is a simple linear function. By redefining the independent variable as $1/F$, the parameters $K_s$ and $S_f$ are calculated using linear regression analysis. The slope and intercept represent $K_s.S_f$ and $K_s$ respectively. The results obtained are: $K_s$ = 1.17 cm/hr, $K_s.S_f$ = 37.74, $S_f$ = 32.26, and a coefficient of determination, $r^2$ = 0.964. Hence, the Green and Ampt model is

$$t = \frac{1}{1.17}\left[F - 32.26\, ln\left(1 + \frac{F}{32.26}\right)\right] \tag{26a}$$

and

$$f = 1.17\left[1 + \frac{32.26}{F}\right] \tag{26b}$$

Horton's model, Eq. (16a), represents a nonlinear function, and the data points were fitted to an exponential curve, applying the least-squares method to the transformed equation. To facilitate the analysis, Eq. (16a) is written in the form:

$$f - f_c = (f_o - f_c)\, e^{-\beta t} \tag{27}$$

The slope and intercept represent $\beta$ and $f_o$-$f_c$ respectively. The results obtained are $\beta$ = 1.4, $f_o$-$f_c$ = 16.88, and a coefficient of determination, $r^2$ = 0.91. This procedure tends to minimize the square of deviations of the dependent variable, f, and consequently the equation closely fits the data during the entire run (Horton, 1940). Thus, Horton's model becomes

$$F = 1.87t + 12.06\,(1 - e^{-1.4t}) \tag{28a}$$

and

$$f = 1.87 + 16.88\, e^{-1.4t} \tag{28b}$$

Holtan's model, Eq. (17) is a nonlinear function with parameters: a and n. To determine these parameters, a control depth on which to base $S_P$ was assigned. This depth was computed from the initial soil water content, the soil porosity, and the volume of water that had infiltrated at the time the infiltration rate reached a constant value. The model is very sensitive to the value of the control depth. The initial moisture condition for each test plot can be considered uniform for the top 20 cm of the soil depth, so it was decided to base soil characteristics and equation parameters on the top 20 cm of soil. This control depth was less than the thickness of the A-horizon which extends up to 25 cm.

Parameter a was found utilizing Eq. (20) and Eq. (21) at t = 0,

F = 0 , $f_o$ = 18.75 cm/hr, and $S_P$ = 5.9 cm, hence,
$a = (18.75 - 1.1)/(5.9)^2 = 0.51\ (cm.hr)^{-1}$. From Eq. (19),
$t_c$ = 1.78 hrs and from Eq. (18b), $f_o$ = 19.86 cm/hr. Thus, the Holtan-Overton model becomes:

$$f = 1.1\, sec^2\, [(0.75)(1.78 - t)] \tag{29}$$

## 5. DISCUSSION AND CONCLUSION

Of the six infiltration models evaluated, the Horton model gives the most satisfactory results in terms of the sum of cumulative infiltration, mean error, and percentage of error (Table 3). Very good representations of the infiltration rate-time relationships can be obtained from the Kostiakov, Modified Kostiakov, and Green and Ampt models. The infiltration rate in the Kostiakov model decays to zero, rather than to a basic infiltration rate. Thus, Eq. (1) may be descriptive only of earlier stages of the rainfall or irrigation. The Modified Kostiakov model represents the soil's basic infiltration rate. Although the addition of constant C increases the number of parameters in the model, the flexibility of Eq. (2) allows the infiltration to be applied to a wider variety of soils. In this analysis, the Modified Kostiakov model fits best the shape of the curve of cumulative infiltration vs. time. For the parameter values tested, Philip's model is sensitive to soil characteristics, and to boundary conditions $\theta_i$ and $\theta_s$. The resulting value for A in Philip's model is approximately one third of the saturated hydraulic conductivity of the soil. This value also agrees with the recommendations made by Youngs (1968). Hence, Philip's equation will not be physically consistent for lengthy periods if it is used to define C.

TABLE 3 — Statistics of the Infiltration Models

| | Kostiakov | Modified Kostiakov | Philip | Green Ampt | Horton | Holtan-Overton |
|---|---|---|---|---|---|---|
| Sum of cumulative infiltration, cm | 14.24 | 14.30 | 17.37 | 13.90 | 13.53 | 10.43 |
| mean of error,* cm | 1.13 | 1.19 | 4.26 | 0.79 | 0.42 | −2.68 |
| % of error** in sum | 8.62 | 9.08 | 32.50 | 6.03 | 3.20 | −20.44 |
| coefficient of determination, $r^2$ | 0.992 | 0.986 | 0.886 | 0.962 | 0.91 | 0.864 |

* error = calculated — measured,
** % error = 100 (calculated — measured)/measured

The two parameter Green and Ampt model performed nearly as well as the three parameter Modified Kostiakov and Horton models. The Green and Ampt model was the easiest to use, and gave predictions sufficiently accurate for most field problems. The flexibility of this model for describing infiltration under varied initial boundary and soil profiles conditions makes it an attractive method for field applications. An advantage of the model is the equation parameters have physical significance.

For Holtan-Overton model a reliable estimate of the control depth on which $S_P$ is based must be made. The infiltration process was inadequately described by the Holtan-Overton model, the process is controlled by soil moisture gradients and hydraulic conductivity rather than storage capacity or soil porosity (Smith, 1976).

Table 4 summarizes the computed values of the parameters fitted in the different models, and the main concept involved in each. Symbols in Table 4 are defined in Appendix 1.

## SUMMARY

This investigation was conducted to evaluate experimentally the infiltration models proposed by Kostiakov, Modified Kostiakov, Philip, Green-Ampt, Horton, and Holtan-Overton. Infiltration data were obtained from nine infiltrometers installed in an experimental plot. The parameters in the six equations were obtained by regression analysis of the infiltration data.

TABLE 4 Comparative Study of the Models

| Investigator | Model | Value of Parameters | Remarks |
| --- | --- | --- | --- |
| Kostiakov | Eq. (1): $F = K_1 t^{\alpha_1}$ $f = \eta_1 t^{\gamma_1}$ | Eq. (22): $K_1 = 0.625$ $\alpha_1 = 0.653$ | simplest model, basic infiltration rate of soil not accounted. |
| Modified Kostiakov | Eq. (2): $F = K_2 t^{\alpha_2} + Ct$ $f = \eta_2 t^{\gamma_2} + C$ | Eq. (24): $K_2 = 0.648$, $C = 1.08$ $\alpha_2 = 0.612$, | flexible model, basic infiltration rate of soil is considered. |
| Philip | Eq. (5) & Eq. (8): $F = St^{1/2} + At$ $f = S/2.t^{-1/2} + A$ | Eq. (25): $S = 11.68$ $A = 0.426$ | moisture content represented by an infinite series. |
| Green and Ampt | Eq. (14): $F = K_s t - S_f.ln[1 + \frac{F}{S_f}]$ $f = K_s[1+S_f/F]$ | Eq. (26): $K_s = 1.17$ $S_f = 32.26$ | parallel capillary tubes, easiest to use, good prediction. |
| Horton | Eq. (16): $F = f_c t + \frac{f_o - f_c}{\beta}[1 - e^{-\beta.t}]$ $f = f_c + [f_o - f_c]e^{-\beta.t}$ | Eq. (28): $\beta = 1.4$ $f_o - f_c = 16.88$ | based on work-energy principle, ponding is assumed. |
| Holtan and Overton | Eq. (18): $F = S_p - [\frac{f_c}{a}]^{1/2}.\tan[a.f_c (t_c - t)]^{1/2}$ $f = f_c.\sec^2[a.f_c]^{1/2}.(t_c - t)]$ | Eq. (29): $S_p = 5.9$ $a = 0.51$ | based on storage concept, soil porosity taken into account. |

The three-parameter Horton model appears to predict infiltration rates that agree closely with experimental values. However, the two-parameter Green-Ampt model performed nearly as well as the three parameter Modified Kostiakov and Horton models (in terms of cumulative infiltration, mean error, percentage error, and coefficient of determination.)

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## APPENDIX I

List of Symbols:

C : basic infiltration rate.
F : cumulative infiltration.
f : infiltration rate at time t.
$f_o$ : initial infiltration rate.
$f_c$ : final infiltration rate.
FC : field capacity of the soil.
$H_o$ : pressure head at the ground surface.
$H_f$ : effective suction head at the wetting front.
K : hydraulic conductivity of the soil.
$K_o$ : unsaturated hydraulic conductivity of the soil.
$K_s$ : saturated hydraulic conductivity of the soil.
S : sorptivity.

$S_f$ : storage suction factor.
$S_p$ : storage potential of the soil above the control depth.
t : elapsed time.
$t_c$ : time to constant rate of infiltration.
W : water content of the soil on a dry weight basis.
x : depth of moisture penetration.
Y : control depth.
$\beta$ : infiltration decay parameter.
$\gamma_b$ : bulk specific gravity.
$\theta$ : moisture content of the soil.
$\theta_i$ : initial moisture content.
$\theta_s$ : moisture content at natural saturation.
$\rho_b$ : bulk density.
$\phi$ : porosity.
$\propto$ : model parameters.
$\gamma$
$\eta$