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COMMENT ON "A LINEAR MODEL OF STORM RUNOFF FROM SOME URBAN CATCHMENTS IN NEW ZEALAND" BY PAUL BROOME AND ROBERT H. SPIGEL

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The approach proposed by Broome and Spigel (1982) of characterizing the storm runoff from urban catchments is a valuable one. Some additional details about the derivation of unit hydrographs by harmonic analysis may be useful. When the effective rainfall $i(\tau)$ and direct runoff $Q(t)$ are expressed as continuous time functions, they may be related by the convolution integral

$$Q(t) = \int_0^t h(t-\tau)\, i(\tau) d\tau \qquad (1)$$

where h(t) is the instantaneous unit hydrograph. As discrete-time functions the equivalent summation is

$$Q_n = \sum_{m=1}^{n} h_p I_m ;\ p = n - m + 1 \qquad (2)$$

where Q_n, $n = 1, 2, \ldots, N$; and I_m, $m = 1, 2, \ldots, M$ are the direct runoff rate (m^3/s or cm/hr) and effective rainfall depth (cm), respectively, in successive time intervals of duration $\Delta\tau$ and h_p, $p = 1, 2, \ldots, P$ is the unit hydrograph of duration $\Delta\tau$. Harmonic analysis uses a Fourier Series representation of Q_n and I_m to create a Fourier Series representation of h_p.

The key point is that harmonic analysis, when applied to discrete-time data, produces the unit hydrograph h_p, of duration equal to the sampling interval, not the instantaneous unit hydrograph h(t). Thus Hall (1977) refers to the result of harmonic analysis as the TUH, or unit hydrograph of duration T time units.

When the sampling interval is small, e.g. 0.1 hr or 0.2 hr, the difference between the IUH and UH's of these time intervals is not great, but when the sampling interval increases, the unit hydrograph peak begins to fall and to be delayed in time significantly. An example is shown in Fig. 1 which compares the unit hydrographs for $\Delta\tau = 0.5$ hr and 1.0 hr developed by the S-hydrograph method from a given unit hydrograph having $\Delta\tau = 0.05$ hr, with the results of harmonic analysis for the same time intervals. For input data to the harmonic analysis in this sample, the effective rainfall was assumed uniformly distributed over the duration $\Delta\tau$ and the direct runoff hydrographs for $\Delta\tau = 0.5$ hr and 1.0 hr were obtained by sampling the direct runoff hydrograph produced using Eq. (2) and the given 0.05 hr unit hydrograph. Figure 1 shows that the unit hydrograph produced by harmonic analysis is coincident with the given values for $\Delta\tau = 0.05$ hr and closely approx-

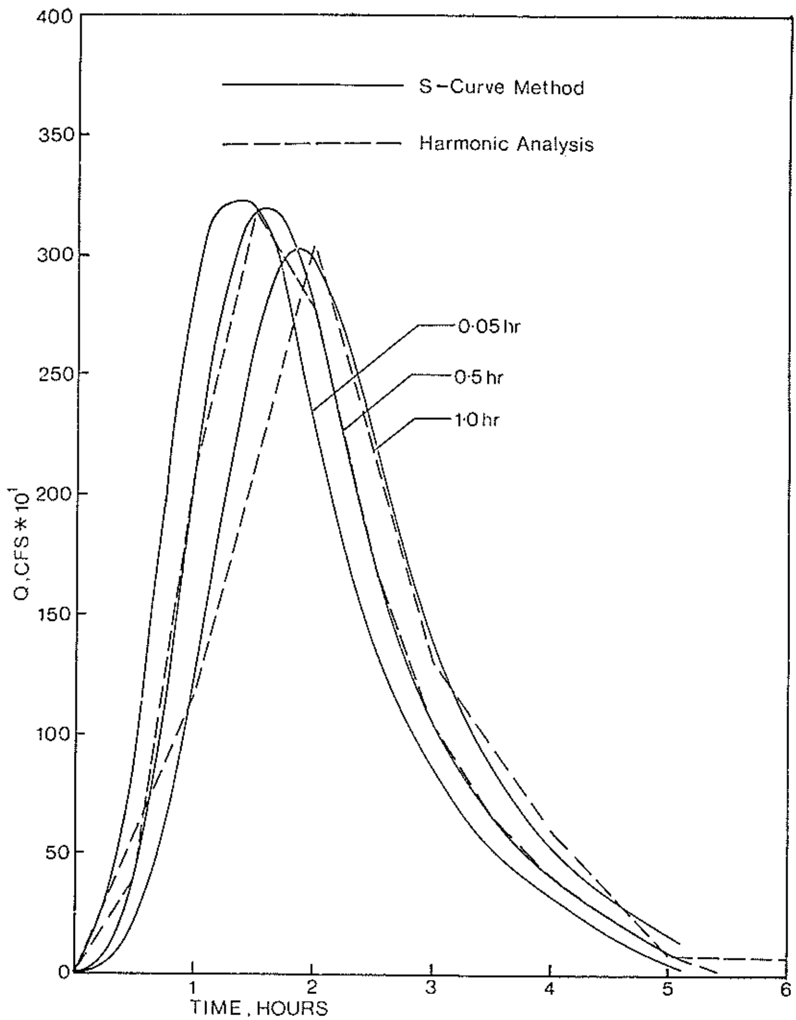

FIG. 1—Hydrograph Comparisons.

imates the unit hydrographs developed by the S-hydrograph method for $\Delta\tau = 0.5$ hr and 1.0 hr.

Figure 2 of Broome and Spigel's paper shows the same phenomenon. The unit hydrographs for $\Delta\tau = 0.1$ hr and 0.2 hr are very similar while the 1-hr unit hydrograph shown has a lower peak than the other two.

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REPLY

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Maidment and Lee (1983) have focussed on an important consideration in hydrograph analysis, that of sampling resolution. However, they are not strictly correct in stating that the method used by Broome and Spigel (1982) produces the TUH rather than the IUH. In fact, the method used by Broome and Spigel is recommended by O'Donnell (1966) for deriving an approximation to the IUH; it does not yield the TUH. The method used by Broome and Spigel is not the harmonic analysis method used by Maidment and Lee, and referenced by Hall (1977). The confusion arises because of the use of the term "harmonic analysis" to describe two closely related but different methods for approximating the excess rainfall and storm runoff using Fourier series. For the purposes of this reply we will use the term "harmonic series" to apply to the case in which one fits N data points (N odd) with exactly N sine and cosine terms plus a constant A_0, as:

$$f(j\Delta t) = A_0 + \sum_{r=1}^{p} \left[A_r \cos \frac{2\pi rj}{N} + B_r \sin \frac{2\pi rj}{N} \right] \qquad (1)$$

where $p = (N-1)/2$, Δt is the sampling interval, and f is the function being sampled. Such an N-term series can be made to pass through the N sampled points by properly choosing the coefficients A_0, A_r, B_r; $r = 1, \ldots, p$. We will use the term "Fourier series" to apply to the second case, in which one approximates the function f at all points t within the time span T by an infinite-term Fourier series, as:

$$f(t) = A_0 + \sum_{r=1}^{\infty} \left[A_r \cos \frac{2\pi rt}{T} + B_r \sin \frac{2\pi rt}{T} \right] \qquad (2)$$

In practice, of course, one uses only a finite number of terms, say R, for the Fourier series, and one knows the function f at only a finite number of points, say N. As R approaches infinity, the series in (2) converges in the mean to f(t) over the interval T (Hilderbrand, 1965, p87), whereas the harmonic series in (1) only passes through the N sampled points. The difficulty in practice is that one does not usually know the form of f(t) between the N sampled points, so that it is impossible to carry out the necessary integrations for evaluation of the coefficients A_0, A_r and B_r in (2). As will be explained below, the way in which one approximates these coefficients determines whether one derives the TUH or an approximation of the IUH.

Maidment and Lee reference Hall (1977) to support their criticism. However Hall references O'Donnell (1966) as the basis for his work with TUHs. O'Donnell (1966) does indeed show that if one uses harmonic series to fit both the rainfall and runoff, one derives the TUH rather than the IUH. However, a much larger part of the O'Donnell (1966) paper (the theory used by Broome and Spigel) is devoted to the problem of using Fourier series to derive the IUH from the convolution integral. If one uses Fourier series for both rainfall and runoff, then O'Donnell's method leads to an exact solution of the convolution integral for the Fourier series coefficients of the IUH. This is stated plainly by O'Donnell (1966) and is readily verified by direct substitution in the convolution integral. Because of the difficulty, discussed earlier, of deriving the Fourier series coefficients for the rainfall and runoff when these are sampled at only a finite number of points, O'Donnell (1966) suggests the following method for deriving an approximation to the IUH:

(1) Assume that the rainfall histogram represents the actual rainfall pattern at any point in time, and fit a Fourier series to the histogram; truncate the series after N terms, where N is the number of sampled points in the runoff hydrograph. The coefficients so derived will be those of a Fourier series, and will not be the same as those of the harmonic series representation of the rainfall histogram.

(2) Use the harmonic series representation of the runoff hydrograph as an approximation to the Fourier series. O'Donnell points out that this method will lead only to an approximation of the IUH, whereas use of harmonic series for both rainfall and runoff will lead to the TUH. A better approximation would possibly result if, instead of following step (2) above, one assumed that the runoff hydrograph was approximated by a series of linear segments joining the N sampled points. A Fourier series could then be fitted to this function, in the same way that a Fourier series is fitted to the rainfall histogram. Not only would this provide a better approximation to the Fourier series for the runoff hydrograph, but it would also be more consistent with step 1 for approximating the rainfall Fourier series. Sample calculations for implementation of such a modification are now under way.

We thank Maidment and Lee for their interest in our paper and for providing the incentive to improve the method of O'Donnell (1966) for estimating the IUH.

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