AN EMPIRICAL RELATIONSHIP BETWEEN RAINFALL AND RUNOFF

R. J. Pittams*

ABSTRACT

A relationship has been derived for the prediction of runoff from rainfall. The basic hypothesis is that a rainfall increment, when suitably modified by factors representing catchment response and antecedent rainfall effect, can be used as an index for discharge so that the sum of all the index increments gives the total storm runoff.

The various parameters are explained and examples given to demonstrate the feasibility of the technique and the flexibility of its application. The ultimate success of the relationship depends on the ability to identify accurately the parameters for individual catchments from historical runoff and rainfall records. Subsequent calculation by a large computer can provide the generality and complexity often lacking in "lumped" catchment models.

INTRODUCTION

Individual pairs of catchments usually do not correlate sensibly with each other either for peak discharges or for storm runoff. Differences in storm rainfall pattern, and time distribution differences due to dissimilar catchments, are the prime causes. Before records can be extended, peak discharges predicted, or representative basin data applied to other catchments within a hydrological region, the accurate relationship of rainfall to discharges and runoff is required for each catchment.

Four years of accurate data are available from the catchments of Otutira Experimental Basin situated at the north-west corner of Lake Taupo in the central volcanic region of the North Island of New Zealand. In addition, rainfall and runoff have been measured for one year in the Purukohukohu Experimental Basin established within the Mangakara Representative Basin at Reporoa between Taupo and Rotorua, also in the central volcanic region of the North Island but in a different geological phase. These catchments are part of the experimental basin programme of the New Zealand Ministry of Works.

* Water and Soil Division, Ministry of Works, Hamilton, New Zealand.

At Ototira, standard techniques for data analysis give disappointing results because of the minute runoff volumes even from catchments of up to 300 hectares with perennial flow. The Ototira soils are sandy loams derived from a very porous pumice breccia, and the vegetation, on all except one small (grass) catchment, is a dense native scrub. However, certain trends can be obtained by plotting combinations of certain rainfall parameters against peak discharges and total runoff.

The maximum discharges recorded from the ephemeral 4.5-hectare grass catchment, Otutaru, are 13–14% of the maximum rainfall intensities, and although total storm runoff is only about 4% (one storm — on 7 December 1968 — did record 30% runoff), almost every fluctuation in the intensity pattern from the single automatic rain gauge is reflected in the flow hydrograph, albeit in a very complex way.

Comparable data for the marginally perennial scrub catchments Otukanuka (83 ha) and Otumoko (37 ha) are peak discharges of 0.2% and total runoff of 2%, while for the perennial grass catchment Puruki (34.4 ha) and its marginally perennial grass neighbour Purutaka (22.5 ha) data are peak discharges of 9–10% of maximum rainfall intensity and total runoff of 40%. All of these last four catchments show very sensitive response to rainfall but the hydrographs, especially from the scrub catchments, are heavily 'damped'. Such response 'damping', as expected, increases with increase in catchment size.

METHOD

The following observations can be used as a guide in formulating a relationship:

1. Rainfall parameters of intensity and duration can be eliminated by taking a constant time interval. In this way storm rainfall is described as a series of increments.

2. The time unit used for a particular catchment can be selected by analysis of a number of storms, especially those with several peaks. A good estimate is the time lag between the start of significant rainfall and the start of runoff (referred to in this paper as "storm lag"). Sharply defined rainfall is more satisfactory than a storm beginning with a prolonged period of very light rainfall. The time the hydrograph appears to remain at its peak is also useful, being the shortest discernible time interval that can be used. This value is used as the maximum time base. Owing to the fact that for large rivers the rate of change of the peak is less rapid than

for small catchments, the apparent time over which peak flow occurred increases from about 3 minutes for a small catchment (say 5 ha) to perhaps 2 hours for a very large basin (say 1500 km²).

3. A constant time base results in a spread of increments from 0.1 mm to — for central North Island conditions — 10 mm or more (depending on the time base), whereas discharge increments are spread from 0.000001 mm to 1.5 mm or more (again depending on the time base and also on the catchment). Only the largest rainfall increments — from about 2.5 mm upwards for a 3-minute time base — result in very large peak discharges (0.2 mm or more). The simplest relationship linking a rainfall increment (p) with its runoff index (Q) is the exponential equation

$$Q \propto p^x \qquad (1.1)$$

where x is some parameter dependent on p.

4. A factor dependent on antecedent rainfall is required.

5. Antecedent rainfall effect (to be quantitatively defined as a) increases with continued rain, but when rain stops the effect follows a decay series such that after a period of, say, 10,000 time intervals (20 days if the time interval is 3 minutes) without rain the parameter a becomes negligible.

6. To obtain the peak discharges and to synthesize hydrographs, it is necessary to convolute each increment index of runoff over a number of successive time units. Allowance must be made for the fact that antecedent rainfall will influence the shape of the hydrograph. The summation of proportions for a specific time increment will give the average discharge for that increment.

7. The sum of the increment indices of runoff will be equivalent to the total storm runoff.

Two equations are proposed, one to calculate the increment indices of runoff and one to convolute them for hydrograph purposes. The second of these two equations is not essential to establish the method, and has been ignored for this paper — it is very complex, and access is required to a large computer for application to storms involving more than, say, 10 increments. However, the basic form of the equation can be approximated by a family of straight lines plotted on log-probability graph paper as in Figure 1.

Each line represents a hydrograph where the peak coincides with the mean of the distribution at a given number of time increments after the rainfall increment that caused it. Selection of a line for a particular runoff-increment index is dependent on the antecedent rainfall value a used in the calculation of that index — where

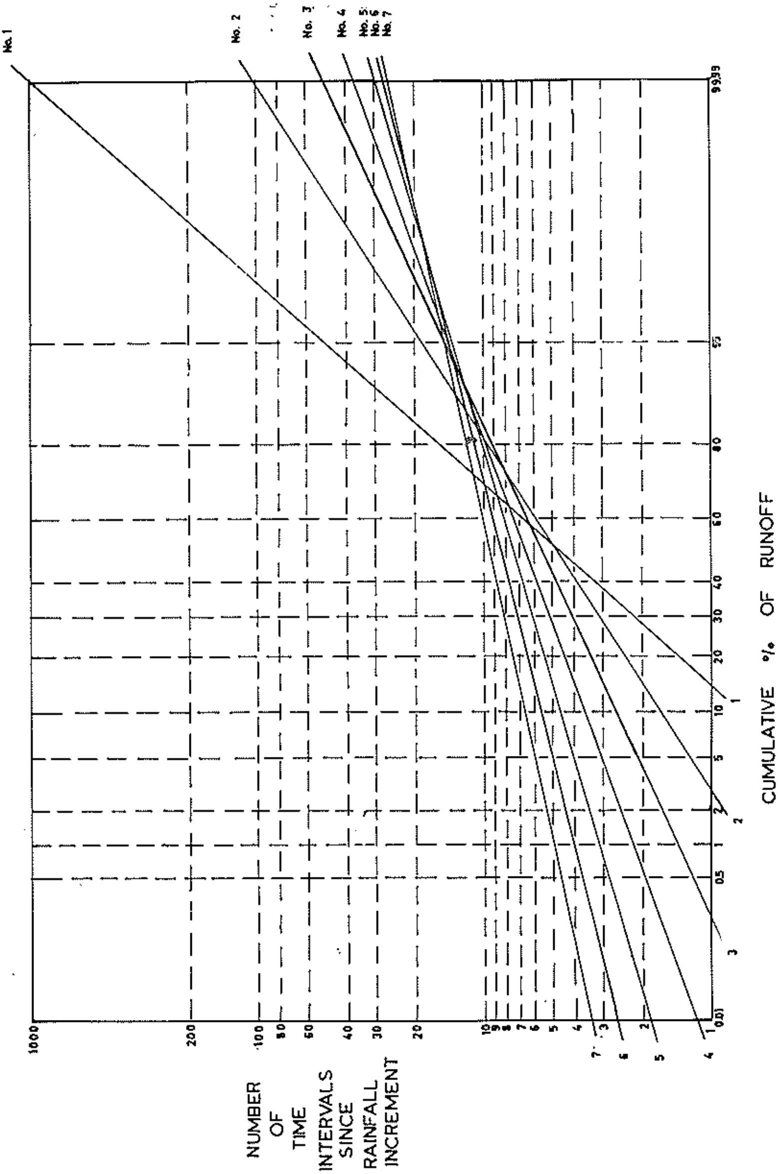

FIG. 1 — Approximations of increment-index convolution equations.

the relationship can most simply be described by plotting the logarithm of *a* against the line numbers, with the smallest line numbers associated with the largest *a* values. The few total storm hydrographs that have been synthesized (of which two small storms are given as examples in Table 1) are the summations of the component convolutions obtained in this way.

TABLE 1 — Otutaru rainfall analyses (all in mm/3 min).

	29 March 1968					*17 May 1968*			
t	*p*	*i*	*fi*	*q×10⁶*	*t*	*p*	*i*	*fi*	*q×10⁶*
1345	2.4	1989	–	–	1403	0.1	1.00	–	–
1348	–	–	–	–	1406	0.8	52.00	0.003	tr
1351	–	–	–	–	1409	0.2	2.92	0.0103	tr
1354	–	–	0.199	–	1412	0.6	44.26	0.255	tr
1357	–	–	1.989	–	1415	0.6	48.67	2.218	1
1400	–	–	17.901	tr	1418	1.3	861.60	9.065	72
1403	–	–	79.560	27	1421	1.3	1500.00	86.877	195
1406	–	–	179.010	–	1424	0.1	1.00	273.785	271
1409	–	–	278.46	195	1427	–	–	350.575	271
1412	–	–	298.35	271	1430	–	–	332.589	–
1415	–	–	298.35	271	1433	–	–	286.758	–
1418	–	–	258.57	–	1436	–	–	234.841	72
1421	–	–	198.90	–	1439	–	–	177.244	72
1424	–	–	139.23	129	1442	–	–	142.260	72
1427	–	–	89.505	–	1445	–	–	127.902	–
1430	–	–	59.670	72	1448	–	–	86.678	–
1433	–	–	33.813	–	1451	–	–	73.545	–
1436	–	–	23.868	27	1454	–	–	56.872	27
1439	–	–	13.923	–	1457	–	–	43.231	–
1442	–	–	7.956	–	1500	–	–	35.697	–
1445	–	–	3.978	–	1503	–	–	29.768	–
1448	–	–	2.983	–	1506	–	–	24.010	–
1451	–	–	1.989	8	1509	–	–	20.662	–
			etc.					etc.	
Total:	2.4	1989	1990	2600		5.0	2511	2510	3100

t=time (hours); *i*=rainfall-increment index

The relationship considered in this paper enables a runoff-increment index Q to be calculated from an equivalent rainfall increment p. This relationship began with the simplest possible equation and became progressively more complex as new parameters were required to explain errors in prediction. Early attempts to represent mathematically catchment response and antecedent rainfall effect concentrated on exponential or hyperbolic functions.

An exponential equation of the form

$$Q=(Fp)^{xy} \tag{1.2}$$

where x is a catchment response function,
y is an antecedent rainfall effect function, and
F is a proportionality constant,

satisfactorily accounts for observed runoff from low-intensity rainfall but tends to infinity very quickly as rainfall intensity increases, so that such an equation cannot be used for high-intensity storms.

Similarly, a hyperbolic equation of the form

$$Q=p-(1+xy)^{-1} \qquad (1.3)$$

satisfactorily accounts for runoff from high-intensity rainfall but is much too sensitive for low-intensity increments.

A combination of both, where the x and y functions have a hyperbolic form but are applied as exponents on the function of rainfall intensity, is equally unsatisfactory.

The final equation chosen combines both catchment response and antecedent rainfall effect in a single function which contains the inverse tangent relationship so that the function is asymptotic to both upper and lower limits while maintaining maximum flexibility in between. This function is applied to a dimensionless function of rainfall, introduced to enable calculations in any system of units. Unfortunately, the dimensionless function requires estimation of the maximum expected rainfall in an increment — an arbitrary choice influenced by length of record and the presence of high-intensity storms. Although the ratio must not exceed unity, a rainfall event of single increment approaching the chosen maximum value will produce a flow volume approaching total runoff. Such an event will probably occur in a short-duration high-intensity storm. Under such conditions the peak runoff increment in a hydrograph will be the same proportion of total runoff as it is of the relevant rainfall increment.

The overall expression can be most conveniently considered as five component functions involving 19 quantities:

a_o	antecedent rainfall effect in increment o,
a_n	antecedent rainfall effect in increment n,
b, c, d, e	catchment response constants used in the calculation of —
f, f'	dimensionless functions of rainfall,
h, j, k	catchment constants defining the reduction of antecedent rainfall effect,
m	the number of increments representing the incremental time control on the reduction of antecedent rainfall effect,
n	the number of increments between successive rainfall increments,
p	rainfall in an increment,
q	runoff in an increment,
r	ratio of $p:p_{\max}$ in f and f',
s, s'	lower limits of f and f',

x inverse tangent function of rainfall intensity and antecedent rainfall effect.

Factors n and p are the two basic variables obtained from the rain-gauge record. From these the runoff-increment index (Q) can be obtained from the expression

$$Q = p(f)^{1/x} \quad (1)$$

The component f represents the basic manner in which the catchment responds to rainfall intensity. The form of f selected for Otutaru is:

$$f = 2r^2/(1+r)+s \quad (2.1)$$

(where s can represent "runoff" from initial rainfall) which is sensitive to change in r only at values of r approaching unity.

The equation for x represents the complex way in which catchment response is modified by antecedent rainfall conditions and includes a rainfall intensity factor, as very high intensity rainfall can be shown to dominate completely the runoff response. The chosen form for x is:

$$x = (b/\pi)\ \tan^{-1}[(pa_n)d^{-1} - e] + c \quad (3)$$

where computation is made in radians, and b, c, and e are dimensionless constants controlling the limits, symmetry and slope of the equation, while d is a rainfall constant with the units of p, so that x is a dimensionless function.

The antecedent rainfall effect for the nth increment (a_n) is calculated from the exponential curve:

$$a_n = a_o \exp[-h(m+n)^i] + k \quad (4)$$

where the value m allows the antecedent moisture condition to recover with rainfall. Each increment of rain contributes to the antecedent rainfall condition for the next increment. For Otutaru this contribution was able to be calculated from:

$$f' = 2r/(1+r^2)+s' \quad (2.2)$$

and from equation (4)

$$m' = [h^{-1}\ln a_o(a_n - k + f'p - Q)^{-1}]^{1/i} \quad (5)$$

Adding $f'p$ and subtracting Q moves the antecedent moisture effect $m-m'$ increments back along the recovery curve. The value m' is now used as m in equation (4) for the next interval. Errors in estimating the initial value of m are quickly reduced because of the form of equation (4).

The variable factors are controlled by requirements of maxima and minima. Equations (2.1) and (2.2) must be established from existing runoff records using very short duration storms following several days without rain. All quantities in equations (3) and (4) must be considered in pairs, so that the maximum value for x can be calculated (approximately) from the extreme values of f. The constants b and c control the range and symmetry of runoff response, and d and e regulate the effect of rainfall intensity and a change in antecedent rainfall effect, while a_o and k control the influence of a given rainfall increment on the antecedent rainfall effect, and h and j control the rate of change in antecedent rainfall effect between storms.

RESULTS

The method has been applied to 12 storms from Otutaru, 8 storms from Otukanuka and 21 storms from Puruki. In addition a brief comparison has been made between Otukanuka and Otumoko, and between Puruki, Purutaka and Purukohukohu.

Otutaru is a catchment of extremes. It responds spectacularly to high-intensity rainfall even though of very short duration, yet it is little affected by low-intensity storms. Storm lags, and most other time parameters for large storms, are three to six minutes — indicating three minutes to be the time base for Otutaru. The charts are accurate to about three minutes, and this time interval seems to have additional significance as the approximate duration of any short sharp shower.

Very low intensity rain does not produce runoff, but the highest rainfall intensities recorded (6.4 and 6.7 mm/3 min) resulted in almost total runoff. In addition, runoff volumes from rainfall intensities below 2.5 mm/3 min require f and x to remain almost constant when antecedent rainfall effect is negligible. The value of s in equation (2.1) must be 0.09 and x^{-1} must tend to about 4.0. The value of p_{max} is arbitrarily set at 10 mm/3 min.

On 29 March 1968, 2.4 mm of rain fell in (approximately) one 3-minute increment after 15 days without rain. This isolated shower resulted in 0.0026 mm of runoff being recorded at the flow recorder. The estimated value, from equations (1) and (2.1) when x^{-1} is 4.0, is 0.0020 mm. Using line 7 of Fig. 1, which was drawn using the assumption that the peak discharge is 14% of the storm flow for a given increment, the peak discharge on 29 March is estimated at 0.00030 mm/3 min compared with 0.00027 mm/3 min recorded. This isolated shower was especially valuable for defining the convolution function (see Table 1).

Another small storm on 17 May 1968, with twice the rainfall volume but much lower intensities spread over eight increments, also resulted in a peak discharge of 0.00027 mm/3 min with a total storm runoff of 0.0031 mm. This storm was useful for estimating the constants *b*, *c*, *d* and *e* in equation (3), although subsequent adjustment was necessary when larger storms were calculated. The final values (see Table 7) allow moderate influence from antecedent rainfall effect at low rainfall intensities, so that for the storm of 17 May total storm runoff is estimated at 0.0025 mm. The influence of antecedent rainfall while being controlled by *d* and *e* is also affected by *k* in equations (4) and (5). The value of *k* determines the rate of change in antecedent rainfall effect during a storm. When *k* is large (1 or more) the antecedent rainfall effect will change very slowly. When *k* is small, even a small rainfall will cause a big change. Values for *h* and *j* are not critical, though certain combinations are more probable than others. The ones chosen for Otutaru (0.05 and 0.95 respectively) give a fast decline to a relatively stable value determined by *k*.

Estimates and recorded values of total runoff and peak discharges for the 12 storms analysed are tabulated in Table 2. These include the five largest storms recorded and five small storms, and the predicted values compare well with the respective recorded values.

TABLE 2 — Otutaru rainfall analyses (all in mm and mm/3 min).

Date	*P*	*Q(est)*	*Q(rec)*	*p*	*q(est)*	*q(rec)*
2/ 2/67	167.4	1.7243	6.0955	1.65	0.0568	0.0298
1/ 3/67	6.3	0.0004	0.0001	0.5	0.0001	tr
23/ 4/67	4.3	0.0003	0.0001	0.5	0.0001	tr
18/ 1/68	42.4	3.5632	4.3936	3.9	0.2055	0.5673
29/ 1/68	12.6	1.4586	0.5113	3.8	0.2016	0.0688
1/ 2/68	2.2	0.0001	–	0.3	0.0001	tr
3/ 2/68	7.2	0.0004	0.0001	0.4	0.0001	tr
29/ 3/68	2.4	0.0020	0.0026	2.4	0.0003	0.0003
1/ 5/68	3.0	0.0002	–	0.3	0.0001	tr
17/ 5/68	5.0	0.0025	0.0031	1.3	0.0003	0.0003
7/12/68	61.3	12.8247	18.8913	6.1	1.3521	0.8018
24/ 1/70	23.7	4.3312	3.9935	6.4	0.5661	1.1454

Accuracy of prediction for extreme peak discharges appears to be limited at both ends by chart accuracy. Rainfall intensities above 4 or 5 mm/3 minutes cannot be read off the charts with any certainty, and runoff volumes below 0.0001 or 0.0002 mm are really only estimates indicating that water did flow.

Otukanuka is almost the opposite of Otutaru in response pattern. Each change in rainfall intensity is reproduced as a change in the hydrograph, but the peak discharges, never very large, are caused almost entirely by rainfall volume. After consideration of the various parameters, a time base of 20 minutes was selected, the larger time unit greatly simplifying analysis.

A constant value is required for f, most simply achieved by letting p_{max} tend to infinity, so that f tends to s (again 0.09). A small variation in x is required to account for differences due to changes in antecedent rainfall effect. For this purpose b (0.1), and c (0.42), are kept very small while d (5.0) and e (3.5) are comparatively large. In equations (4) and (5), h and j have been changed to 0.3 and 0.5 respectively to give a much slower decay in antecedent rainfall effect.

A period of 17 days in late August and early September 1968 and the high-intensity storms of 7 December 1968 and 24 January 1970 have been analysed with very good results. Estimates and recorded values of total runoff and peak discharges are tabulated in Table 3.

TABLE 3 — Otukanuka rainfall analyses (all in mm and mm/20 min).

Date	*P*	*Q(est)*	*Q(rec)*	*p*	*q(est)*	*q(rec)*
22/ 8/68 (a)	20.4	0.0639	0.0446	4.1	0.0054	0.0050
(b)	6.4	0.0206	0.0130	2.6	0.0020	0.0019
23/ 8/68	16.4	0.0463	0.0560	2.7	0.0038	0.0039
27/ 8/68	6.6	0.0116	0.0106	0.9	0.0004	0.0005
30/ 8/68	53.6	0.1837	0.3611	4.2	0.0062	0.0065
6–7/12/68 (a)	8.2	0.0144	0.0141	1.0	0.0009	0.0008
(b)	18.2	0.0394	0.0409	3.0	0.0039	0.0042
(c)	46.8	0.2496	0.3286	20.5	0.0170	0.0132
24/ 1/70	25.3	0.0772	0.0463	12.4	0.0075	0.0071

It is necessary to subtract a constant base flow from the recorded hydrograph for each storm — this being flow contributed from previous storms. The January storm was the ephemeral type in a period of exceptionally dry weather, so that no deduction is required. Of particular interest is the way the predicted input must be distributed over the days after rain falls.

The time base of 20 minutes may be a little large for Otukanuka. The total runoff estimated will not be significantly affected, but peak discharges will be underestimated if the time base is too long — because a 20-minute unit will predict a peak discharge lasting 20 minutes (unusual in such a small catchment), unless special provision is made in the convolution function. To demonstrate the effect of changing the time base two of the storms have been recalculated for Otukanuka on a 6-minute time base, and the results are tabulated in Table 4. Results from calculating the same two storms on a 6-minute time base for the very similar Otumoko catchment are included in Table 4 for comparison.

TABLE 4 — Otukanuka and Otumoko analyses (all in mm and mm/6 min)

Date	P	$Q(est)$	$Q(rec)$	p	$q(est)$	$q(rec)$
Otukanuka						
22/ 8/68	20.4	–	0.0446	2.0	–	0.0015
24/ 1/70	25.3	0.0416	0.0463	5.6	0.0026	0.0018
Otumoko						
22/ 8/68	20.4	0.0173	0.0449	2.0	0.0012	0.0020
24/ 1/70	25.3	0.0271	0.0243	5.6	0.0021	0.0014

The observed discrepancy is due to inadequate allowance for antecedent rainfall effect, and is present (to a much lesser degree) also in the calculations for Ototaru — for example the storms of 2 February 1967 and 7 December 1968. Unlike Ototaru, however, the response to rainfall in both Otukanuka and Otumoko is very nearly a simple proportion of that rainfall, with the proportion dependent on antecedent conditions. The storm of 24 January 1970 following a prolonged dry period can be closely approximated by taking 0.183% of each rainfall increment for Otukanuka and 0.096% of each rainfall increment for Otumoko, but for the storm of 22 August 1968 which followed a particularly wet period, 0.220% of each increment must be used for both catchments. Equation (1) is still required to explain other basic hydrograph differences, especially in Otumoko, but in future calculations of Ototira data, equation (3) must be modified for increased sensitivity to antecedent rainfall effect. Equations (4) and (5) are not affected.

The Purukohukohu catchments are likely to be much more useful than the Ototira trio for prediction of runoff on other catchments. Results from Puruki and Purutaka may be very similar to those from any small steep grass catchment with perennial flow.

Puruki and Purutaka, like Ototaru, show an extreme response to rainfall intensity, so that many of the equation (3) and (4) parameters derived for Ototaru can be applied. A time base of 12 minutes has been selected for Puruki and used in calculations for the other two to simplify comparison. The constants in equations (2.1) and (2.2) cannot be used for catchment response. Two new equations of the same form:

$$f = 1.91r^2/(1+r) + 0.045 \quad (2.3)$$

and

$$f' = r/(1+r^2) + 0.5 \quad (2.4)$$

have been adopted for the Purukohukohu group and a value of 30 mm has been chosen for p_{max}. Reference to Otutaru suggests 40 mm for p_{max} but the longer time base requires an adjustment for the lower frequency of rainfall increments approaching the maximum. Purukohukohu (186 ha) contains dense indigenous forest in some 25% of its area so that with the combination of larger catchment and reduced response a time base of 30 minutes and a much higher p_{max} should give more satisfactory agreement between predicted and recorded storm data.

Table 5 comprises 21 storms analysed for Puruki with good results. Data from two storms at the end of 1968 have also been analysed for Purutaka and Purukohukohu and summarized in Table 6. Storm runoff for both is less than for Puruki so that a reduction in c is required for both. Purutaka requires increased sensitivity to high-intensity rainfall (decrease in d and e) and Purukohukohu requires decreased sensitivity.

TABLE 5 — Puruki rainfall analyses (all in mm and mm/12 min).

Date	*P*	*Q(est)*	*Q(rec)*	*p*	*q(est)*	*q(rec)*
23/12/68	2.5	0.0594	0.0275	0.8	0.0028	0.0028
24/12/68	13.4	2.0118	1.3729	3.3	0.0743	0.0953
25/12/68	2.9	0.1191	1.0622	1.2	0.0097	0.0509
26/12/68	4.9	0.1682	0.3666	1.0	0.0066	0.0059
27/12/68	5.2	0.2663	0.6146	1.3	0.0281	0.0378
28/12/68	1.5	0.0511	0.2026	1.1	0.0056	0.0011
29/12/68	13.1	2.8459	0.7572	6.6	0.3421	0.0409
30/12/68	2.6	0.3386	0.2678	2.4	0.0467	0.0013
31/12/68	0.2	0.0045	0.0227	0.1	0.0003	0.0003
5/ 1/69	3.9	0.0944	0.2066	0.8	0.0026	0.0015
6/ 1/69	34.0	3.8487	2.3850	1.5	0.0397	0.0127
7/ 1/69	22.3	4.9517	3.7624	2.2	0.0914	0.1263
8/ 1/69	6.0	0.2471	1.5175	1.0	0.0098	0.0062
21/ 3/69	3.8	0.0957	0.3155	0.9	0.0035	0.0041
31/ 3/69	1.6	0.0352	0.0862	0.5	0.0016	0.0007
3/ 4/69	28.3	7.0892	7.9738	7.4	0.4917	0.5588
24/ 6/69	43.4	8.1649	16.1310	3.5	0.3453	0.5587
13/ 9/69	56.5	18.2842	20.2200	10.6	0.8850	0.7374
25/ 9/69	86.3	21.9883	26.4880	6.4	0.6316	0.6815
26/ 9/69	27.3	5.6224	20.8490	3.5	0.3591	0.3390
12/12/69	34.2	19.4706	19.4970	16.1	2.6505	2.1200
2/ 1/70	29.2	15.0642	11.1790	13.9	2.0329	2.0330

TABLE 6 — Purukohukohu and Purutaka rainfall analyses (all in mm and mm/12 min).

Date	*P*	*Q(est)*	*Q(rec)*	*p*	*q(est)*	*q(rec)*
Purukohukohu						
27/12/68	5.2	0.1496	0.2316	1.3	0.0146	0.0148
28/12/68	1.5	0.0421	0.0181	1.1	0.0044	0.0016
29/12/68	13.1	0.8146	0.2943	6.6	0.0879	0.0140
30/12/68	2.6	0.0953	0.0454	2.4	0.0126	0.0087
Purutaka						
27/12/68	5.2	–	0.5025	1.3	–	0.0627
28/12/68	1.5	–	0.0292	1.1	–	?
29/12/68	13.1	–	0.2423	6.6	–	0.0466
30/12/68	2.6	–	0.0390	2.4	–	0.0004

The relationship between peak discharge and peak rainfall intensity may not be able to be held constant at 14–15% for Purutaka and perhaps for Puruki, as it was for the Otutira catchments, so the recorded peak is not as useful in checking a prediction. The lines in Fig. 1 will only apply to low-intensity storms. As rainfall intensity increases, so the proportion of runoff contributed to the peak discharge by each increment also increases. In the storm of 12 December 1969 some 20–25% of the runoff produced by the peak rainfall increment appears to have been contributed to the peak at Puruki, and an even higher proportion may have been measured at Purutaka, where the junction of the two component subcatchments is immediately up stream of the flume.

TABLE 7 — Equation components.

Component	*Otutaru*	*Otukanuka*	*Otumoko*	*Puruki*	*Purutaka*	*Purukohukohu*
a_o	1000	1000	1000	1000	1000	1000
b	3.4	0.1	0.1	3.4	3.4	1.0
c	1.303	0.42	0.33	1.303	1.00	0.05
d	2.82	5.00	5.00	1.00	0.10	9.00
e	1.478	3.500	1.478	0.500	0.500	0.500
h	0.05	0.30	0.99	0.30	0.30	0.30
j	0.95	0.50	0.23	0.50	0.50	0.50
k	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
p_{max}	10 mm	infinite	infinite	30 mm	30 mm	30 mm
s	0.09	0.09	0.09	0.045	0.045	0.045
s'	0.09	0.30	1.00	0.50	0.50	0.50
time base	3 min	20 min or 6 min	6 min	12 min	12 min	12 min

In each example used for this study, a compromise has been required between the need to match the total measured runoff, and the need to estimate the peak discharge. Towards the end of the study the second requirement was largely eliminated by calculating a large number of storms covering a very wide range of intensity and duration conditions. The study must now become computerized to be able to handle the amount of data required while the equation parameters and the calculations made with them must be regarded as "fascinating approximations". The catchment components used in the equations are summarized in Table 7.

CONCLUSIONS

The model proposed gives an adequate means of predicting discharges and runoff for a catchment with historical records including at least one hydrograph from high-intensity rainfall. Although only a limited number of storms and catchments have been investigated it is apparent that less difficulty is experienced with large perennial basins than with small ephemeral catchments where record accuracy is critical.

On the other hand, large catchments are usually too insensitive for the accurate formulation of a general model. The small size and uniformity of rainfall, soil, and vegetation on Otutaru simplify the total relationship, and the extreme range in values ensures that the equations used are valid. The Otutira catchments all behave independently and all have certain seemingly inexplicable peculiarities, so that a relationship that provides a method of comparative analysis there is likely to be valuable anywhere.

The time base is closely related to catchment area and possibly shape, but it can be estimated fairly accurately from a study of the basin lag, the discharge duration, and the number of storms recorded from a large storm with several changes in rainfall intensity. Changing the time base affects the rainfall increments because 1.0 mm of rain on a 3-minute time base should be equivalent to 10 mm of rain on a 30-minute time base, but the former will occur more frequently.

The function f is a basic catchment response indicator, varying approximately between 0.01 and 1.0. It is possibly a function of soil and geomorphology factors, so that a major change in soil would need a major change in f. The antecedent rainfall effect should be related to soil moisture so that a correlation of the two may produce a means of estimating one from the other. The present interrelationship of antecedent rainfall effect and rainfall intensity is not completely satisfactory. The exponent x is a device for modifying the basic catchment response under variable rainfall and soil moisture conditions. The two major parameters b and c are almost certainly vegetation factors, so that a major change in vegetation would need a major change in b and c. The two minor parameters d and e control the rate of change in x with a change in rainfall and antecedent conditions.

The present set of time distribution functions are perhaps a little oversimplified, but as they are selected by reference to antecedent moisture conditions they do take into account all the known causes of hydrograph variation, including the seasonal effect on recession slopes.

The results are interesting in another way — they highlight possible sources of error in orthodox analyses. Time identification is exceedingly critical. On small catchments (less than 5 ha), three minutes is essential but very close to the limit of definition, especially of the flow recorder. The times at which runoff starts and finishes are also important, especially on an ephemeral-flow catchment. For a small storm on Otutaru, standard data reduction may result in an error of up to 20% of the total runoff, much of the error being caused by the recession "tail" on a weir box. In perennial catchments the subdivision of storms into component parts is a problem, partially solved in this paper by assuming a constant base flow from antecedent rain. Over a period of two or three days this is reasonable but gives problems at the onset of winter — with steadily increasing base flow — and where two or three hydrograph peaks are to be separated from a single storm — with steep hydrograph recessions. In water balance terms these errors are insignificant, but for hydrograph analyses they become important. The adequacy of a single automatic rain gauge, or its siting, is not important as long as a consistent relationship between rainfall and flow can be established. The rainfall errors can be appraised very quickly before any data are calculated. Some flexibility is possible.

This method is a relatively simple method of analyzing complex changes within the catchment because it provides a set of "bench mark" conditions. If the relationship between rainfall and runoff changes in any way, it will be apparent from the progressive inability of the existing parameters to predict discharges and total runoff. However, the calculation of large storms is very tedious and must be computerized if the method is to be widely applicable.

Possible uses are: extension of records, and application of representative and experimental basin data to other regional catchments. The method also provides a starting point for taking a catchment apart to investigate soil, vegetation and geomorphological relationships. The equation components themselves provide a very useful way to compare catchments without having to take rainfall and catchment differences into account.

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