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A NOTE ON THE USE OF INFILTRATION EQUATIONS IN INFILTRATION ANALYSIS

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ABSTRACT

Some modern thoughts on the processes of infiltration are given and a brief mention made of the available infiltration equations. Rainfall and runoff data on some small experimental catchments in New Zealand have been analysed and infiltration curves derived. The various equations are tested against the curves obtained by analysis to investigate if the curves can be expressed by any of the existing equations. It is concluded that for the curves tested the Horton equation gives the best fit.

INTRODUCTION

Infiltration is the entry of water into the soil. The rate of entry may be expressed by a curve, showing a decreasing rate with time and such a curve is called an infiltration curve. Infiltration is of great interest to a variety of sciences and the possible wide use of infiltration curves demands the characterization of these by a small number of parameters. A simple algebraic equation may express the curves and two empirical equations and two theoretically derived ones are available. Watson (1959) tested some of these against infiltration data obtained by an infiltrometer. In this paper the suitability of three equations for infiltration curves obtained by analysis of rainfall and runoff records from a natural catchment is considered.

INFILTRATION PROCESS

Infiltration into the soil is an important operator in the hydrological cycle. A detail of this cycle is shown in Fig.1 illustrating the movement of water after infiltration has taken place

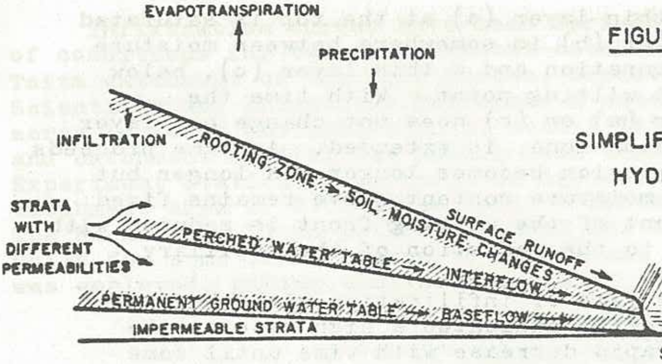

FIGURE 1.

SIMPLIFIED DETAIL OF THE HYDROLOGICAL CYCLE

In unsaturated soils infiltration will change the moisture profile, the development of which, in turn, will determine the rate of entry of water. Initially a thin layer at the surface will be saturated and the water will advance into a dryer layer underneath along a so called "wetting front". Behind the front the soil is nearly saturated - beyond the front it remains unchanged. There is a sharp line of demarcation between wet and dry parts. Bodman and Colman (1943) studied the downward movement of water in Yolo sandy loam by applying water to the surface of a soil column so there was always a 5 mm layer of water standing on the surface. Their results are shown in Fig. 2 which illustrates the movement of the wetting front.

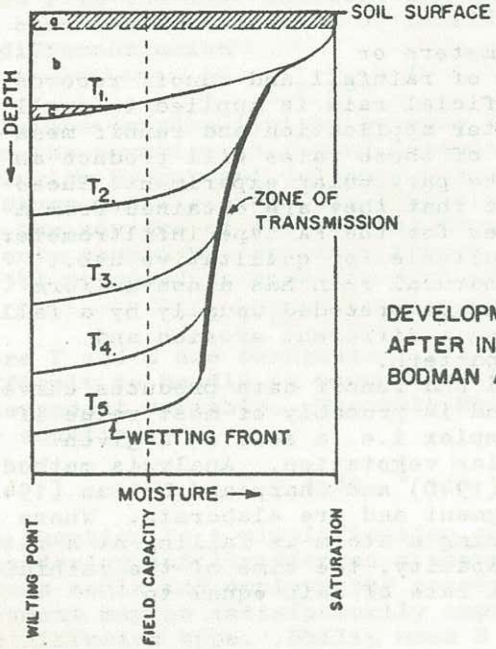

FIGURE 2.

DEVELOPMENT OF MOISTURE PROFILE AFTER INFILTRATION ACCORDING TO BODMAN AND COLMAN (1943)

At time 1 a thin layer (a) at the top is saturated while a second layer (b) is somewhere between moisture equivalent and saturation and a thin layer (c), below that, is nearly at wilting point. With time the thickness of layer (a) or (c) does not change but layer (b), the transmission zone, is extended. As time proceeds the zone of transmission becomes longer and longer but the wetting front moisture content curve remains fixed. The rate of movement of the wetting front is reduced with time probably due to the reduction of the capillary potential gradient.

Comparing the rate of infiltration with the change of the moisture profile indicates a high rate at the beginning with a rapid decrease with time until some point where the rate of infiltration becomes appreciably constant. This constant rate is thought to be due, in some degree, to the effect of the large gravity channels present in most natural soils, providing a relatively constant rate of percolation downwards (Toebes, 1962).

Infiltration rates are normally expressed in curves as shown in Fig. 3. There is a definite limit to the rate of infiltration at any one instant and limiting curves are termed infiltration capacity curves. The initial rate of infiltration is termed fo and the ultimate rate fc. The (fo-fc) values are a function of antecedent moisture conditions as shown by Toebes (1962).

INFILTRATION ANALYSIS

Infiltration curves have been obtained by laboratory experiments by Youngs (1957) and others, but are of limited value because of the disturbed soil profile. Field methods for the derivation of curves for actual conditions may be done by

(i) infiltrometers or
(ii) analysis of rainfall and runoff records.

With infiltrometers artificial rain is applied to small plots and the rate of water application and runoff measured. A simple analysis of these rates will produce an infiltration curve for the particular experiment. These data suffer from the fact that they are obtained from a small area (12 x 30 inches for the FA type infiltrometer) and, as such, are only suitable for qualitative use. Unlike artificial rain, natural rain has a non-uniform intensity and dropsize and is preceded usually by a fall of low intensity producing a different erosion and subsequent infiltration pattern.

Analysis of rainfall and runoff data produces curves for an integrated area and is probably of most value if obtained for a single complex i.e. a soil in a given condition with a particular vegetation. Analysis methods are by Horner and Lloyd (1940) and Sharp and Holtan (1942). Both methods require judgment and are elaborate. Where rainfall, at any time during a storm is falling at a rate less than infiltration capacity, the time of the rainfall is condensed to produce a rate of rain equal to infiltration capacity.

Infiltration curves have been derived for a variety of conditions for two experimental catchments, viz. the Taita catchment of the Soil Bureau, Department of Scientific and Industrial Research, a catchment of 37.8 acres in scrub and native bush, soil - Taita silt loam; and catchment No.1 of the Makara Soil Conservation Experiment Station, Department of Agriculture, with a catchment area of 2.02 acres, soil - Korokoro silt loam. The Horner and Lloyd method was used for both catchments while for some Taita storms the Sharp and Holtan method was employed, giving similar results.

INFILTRATION EQUATIONS

Gardner and Widtsoe (1921) suggested an inverse exponential equation to fit derived infiltration curves. Horton (1940) modified the formula and it subsequently became known as the Horton equation:

$$f = fc + (fo - fc)\, e^{-t/c}$$

where f is the rate of infiltration at time t, e the base of the natural logarithm and c a constant. The equation has been widely used because it is simple to apply and has the advantage that, in the limit as t approaches infinity, the infiltration rate does not become zero. It gives a bad fit when infiltration rates decrease rapidly and three parameters are needed to express any particular curve.

Kostiakov (1932) proposed the empirical equation:

$$F = ct^{a}$$

where F is the mass infiltration and c and a are constants. The equation for the rate of infiltration is obtained by differentiation

$$f = act^{a-1}$$

Its scope is also limited because the value of a obtained when the equation is fitted depends on the range of t. Following Bodman and Colman's classification of the moisture profile, van Duin (1955) derived an equation for the advance of the wetting front assuming that flow is only caused by capillary forces and gravity. Philip (1954) proposed an equation similar to van Duin

$$t = Y\,(F - Z \log\,(1 + F/Z))$$

where Y and Z are constants. The equation is a little difficult to handle on account of the use of t as the independent variable. Recently Philip (1957a) derived the equation:

$$f = \tfrac{1}{2}\, St^{-\frac{1}{2}} + A$$

The equation is a simplification of one derived by considering infiltration as a phenomenon of flow in porous media and employs the concept that soil water movement may be satisfactorily expressed by equations of the diffusion type. Philip uses S to denote sorptivity, which is a measure of the absorption and desorption of water (capillary uptake or removal of water). A is an approximate correction to the exponent - $\frac{1}{2}$. The equation, although advantageous in that it has only two parameters, must fail when t becomes very large because it is unable to express a constant rate of infiltration.

TESTING INFILTRATION EQUATIONS

The equations have been tested by Philip (1957b) using laboratory procedures. Excellent agreement was found for the Philip/van Duin and the Philip equations; the Kostiakov equation fitted moderately well while the Horton equation failed badly. Recently Watson (1959) tested the Philip and the Horton equations against curves derived by a sprinkling infiltrometer since he considered that in laboratory experiments air is free to escape from the soil pores during the downward movement of the wetting front and there is no counterpart of this in infiltration into soils in situ. He found that the Philip equation fitted the field curve particularly well up to the time when f became fc and for greater times f should be taken as fc. The Horton equation fitted less well. However, air escape in sprinkled plot experiments is also non-natural.

The escape of air from a soil during infiltration takes place chiefly through the large pores, especially from the tops of soil surface irregularities where the surface detention is slight or non-existent. In sprinkled plot experiments the surface detention is uniform and air escape is sideways. It is considered, therefore, that testing of curves obtained by analysis of rainfall and runoff data should be done to test the field use.

Some seven curves derived from the Taita catchment and five curves from the Makara catchment No.1, both sets of curves extending over a full range of antecedent conditions, were tested. The parameters of the Kostiakov and Philip equations were computed by the method of least squares using the TRAP programme on the IBM 650 computer. Because of the difficulty in assigning an exact value to the initial infiltration capacity fo in many analyses, the curves were tested from time f1 hr for Taita and F$\frac{1}{4}$hr for Makara to time fc. The parameters of the Horton equation were derived graphically.

RESULTS

Fig. 3 shows a typical infiltration curve for the Makara catchment No.1 with the best fit curves obtained by the Horton, Kostiakov and Philip equations. The Philip and Kostiakov equations fit better in the upper range while the Horton one gives a better agreement in the lower range.

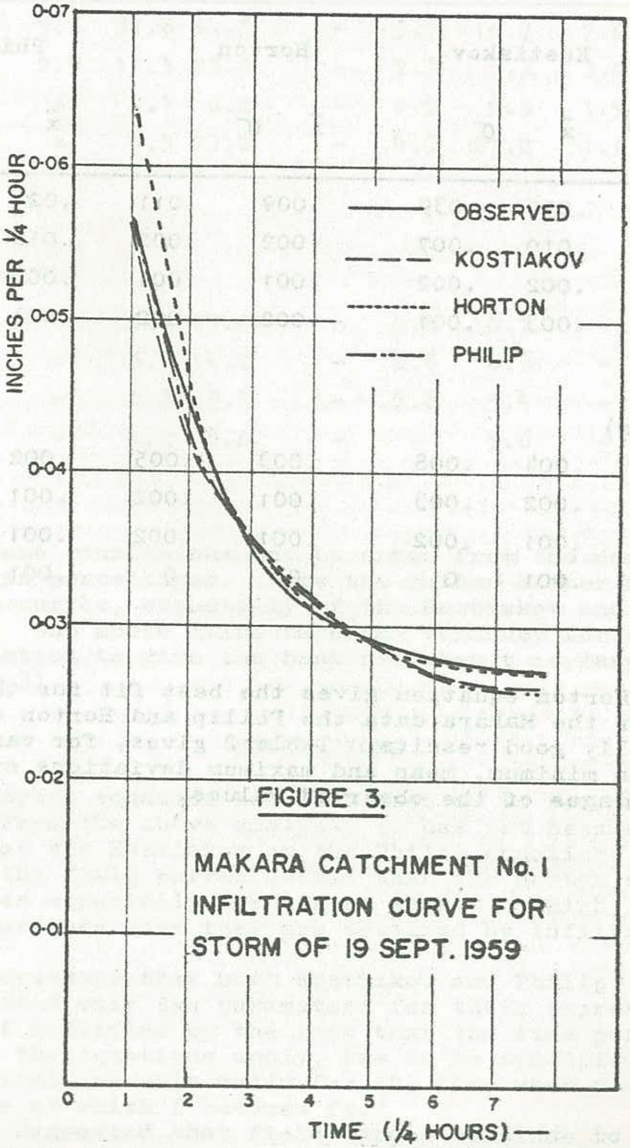

FIGURE 3.

MAKARA CATCHMENT No. I INFILTRATION CURVE FOR STORM OF 19 SEPT. 1959

Table 1 shows, for various times, the mean differences ($\bar{x}$) and the standard deviations (σ) of the equations tested against the observed curves.

TABLE 1:

TAITA -

Time (hrs)	Kostiakov $\bar{x}$	Kostiakov σ	Horton $\bar{x}$	Horton σ	Philip $\bar{x}$	Philip σ
1	.050	.039	.009	.011	.021	.010
5	.010	.007	.002	.002	.010	.004
10	.002	.002	.001	.001	.003	.001
15	.003	.001	.002	.002	.004	.002

MAKARA -

Time ($\frac{1}{4}$hrs)	Kostiakov $\bar{x}$	Kostiakov σ	Horton $\bar{x}$	Horton σ	Philip $\bar{x}$	Philip σ
1	.004	.008	.003	.005	.002	.002
3	.002	.003	.001	.002	.001	.002
5	.001	.002	.001	.002	.001	0
7	.001	0	0	0	.001	.001

The Horton equation gives the best fit for the Taita data. For the Makara data the Philip and Horton equations give equally good results. Table 2 gives, for various times, the minimum, mean and maximum deviations expressed in percentages of the observed values.

TABLE 2:

TAITA - Percentage Deviations

Time (hrs)	Kostiakov min.	Kostiakov mean	Kostiakov max.	Horton min.	Horton mean	Horton max.	Philip min.	Philip mean	Philip max.
1	5.6	31.8	64.5	-	5.7	16.7	7.0	13.4	17.6
5	2.9	11.5	25.0	-	2.3	6.3	3.0	11.5	16.9
10	-	3.7	6.2	-	1.9	9.5	1.9	5.6	13.6
15	-	6.5	23.8	-	4.3	27.2	1.6	8.7	38.0
MAKARA -									
Time ($\frac{1}{4}$hrs)									
1	-	6.7	17.0	-	5.0	10.7	-	3.3	5.3
3	-	5.6	14.2	-	2.8	6.3	-	2.8	9.5
5	-	2.3	5.4	-	3.8	5.4	-	2.3	5.0
7	-	4.2	8.0	-	-	5.6	-	4.2	12.0

The same conclusions may be drawn from the deviations expressed in percentages. Note the rather better fit for the Makara curves, especially of the Kostiakov and Philip equations. The above illustrate the tendency for the Horton equation to give the best fit when t is large (refer Fig.3).

CONCLUSIONS

The Horton equation has been used up to now in New Zealand. From the above analyses it has not become certain that the Kostiakov or the Philip equations will represent the field curves better than the Horton one. This applies especially for curves when t is high, a common occurrence when they are obtained by infiltration analyses.

The advantage that both Kostiakov and Philip equations need only two parameters for their expression, is somewhat nullified by the fact that the time period, over which the equations apply, has to be specified; both equations are only valid for the time when $f = f_1$ to the time at which f becomes fc.

It is suggested that field curves continue to be expressed by the Horton equation and that work be done to obtain an infiltration equation which allows for varying time periods of the infiltration curve.

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