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Cumulative storm rainfall distributions: comparison of Huff curves

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Abstract

Watershed models of hydrology and water quality often require a long record of precipitation measured in short time increments. Huff curves, a probabilistic representation of storm intensities using isopleths of probability, can provide this information stochastically or as design storms. However, many factors affect the development of Huff curves, and an objective method for comparing them is needed. This study investigated two methods of comparing Huff curves, a "curve approach" in which the isopleths of probability of Huff curves were compared using a measure of disparity between isopleths, and a statistical test using the Kolmogorov-Smirnov test ("K-S approach"). Fifteen years of precipitation data for May and June from Invercargill, New Zealand were used. The minimum number of storms needed to develop Huff curves that were stable (i.e. a set of curves that does not change with added storms) was used to illustrate the two methods. Both methods used three regions within Huff curves to detect the location of differences. The curve approach showed that a sample size of 110–140 storms was sufficient to achieve stability. The K-S approach showed that a sample size of about 145 storms was adequate, but lowering the significance probability to 5% from 10% revealed that a sample size of 110 to 120 storms was sufficient. The data suggest that a minimum storm sample size of 120 storms was sufficient to develop a stable set of Huff curves based on the results of both methods. The results suggest that either method can be used to identify the minimum number of storms. The two methods can be used to evaluate other factors important in Huff-curve construction for quality control.

Keywords

precipitation, rainfall intensity, storms, time between storms, stochastic simulation

Introduction

Models of hydrology and water quality often require long records of short-time increment precipitation data. These data are generally not available except at research experimental watershed facilities. One method for determining the time-varying rainfall input in the absence of data is through Huff curves (Huff, 1967). These are probabilistic summaries of dimensionless within-storm rainfall intensities (Fig. 1); their development is explained in the Procedure section. They are useful as design-storm patterns and for stochastically simulating storm intensities. They provide "storm" intensities of varying "storm" durations, as opposed to fixed 24-hour durations often used because of a lack of available data.

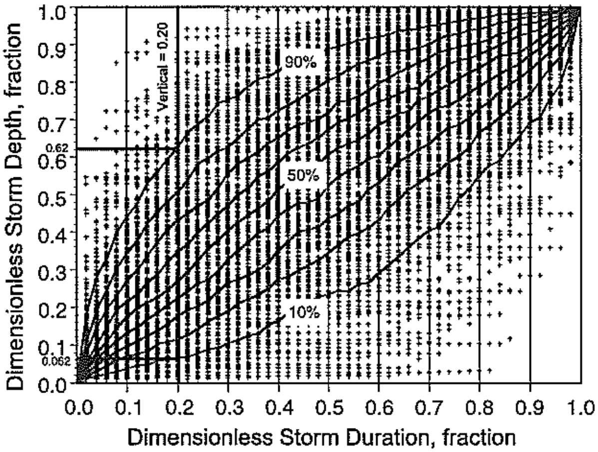

Figure 1 – Dimensionless storm mass-curve inter-sections with isopleths connecting equal probabilities of dimensionless storm depths (i.e., Huff curves) for a sample size of 322 May and June storms at Invercargill (base/smooth curves).

A problem with developing Huff curves is that they can be affected by factors such as season of year, climatic region, sampling-time interval of raw data, storm size, number of storms, etc. (Bonta and Rao, 1987; Bonta, 1997). To determine the effects of these factors on the stability of Huff curves, an objective method for comparing them is needed. A "stable" set of curves is one in which the curves do not change significantly if more storm data are added to the data set that is used to develop them.

Objectively determining stable Huff curves is important not only for quality control of curves, but also to identify regions, in a large geographic area, with similar curves. For example, Bonta and Rao (1989) showed that there was not much difference between published curves for the large central area of the USA from Texas to Illinois to Ohio, in spite of differing methods used to construct the curves. Bonta (1997) showed that the disparity in Huff curves that were developed from two rain gauges 1.6 km apart in Ohio was of the same order as two rain gauges about 640 km apart in Illinois and Ohio. A method of comparing Huff curves is important for identifying where regional boundaries in Huff curves occur over large areas.

In this study we investigate two methods of comparing Huff curves to determine factors affecting them. The focus is on determining the minimum number of storms needed to develop a stable set of Huff curves. The primary purpose for investigating Huff curves in this study is to develop an aid to parameterizing a stochastic storm-generation model called StormGen (Bonta, 1997, 2001). However, the methods will be useful for developing quality-control guidelines for construction of stable Huff curves in general, and for determining boundaries of regional extent of curves.

Procedure

Approach

The approach used in this study is statistical and empirical. Two methods were investigated: one used a measure of the sum of deviations between two sets of Huff curves ("curve approach"), and the other used the Kolmogorov-Smirnov method to test for statistically significant differences between Huff curves (K-S approach). The number of storms needed to develop a stable set of Huff curves was used to investigate the performance of the two methods. The smaller the number of storms, the more visually rough or jagged Huff curves are, compared to the smooth curves that are developed from over 300 storms (Fig. 1). To account for variation in Huff-curve shapes along the dimensionless-duration axis in Figure 1, three regions along this axis in the dimensionless depth-duration grid of Huff curves were compared. For the curve approach, differences between corresponding isopleths for each region were compared, whereas only representative "verticals" (vertical lines at specific dimensionless durations) were used for the K-S approach (Fig. 2).

Data

A 15-year record of precipitation data for Invercargill, New Zealand was used; it was obtained from the National Institute of Water and Atmospheric Research. Precipitation data for the months of May and June in the 15-year record were combined into a single data set. The long-term monthly means for precipitation for Invercargill for May and June are about 100 mm and 113 mm, respectively.

Table 1 - Characteristics of May and June *storms*[1] for Invercargill, New Zealand.

	Sample size	**Min**	**Max**	**Mean**	**Median**	**Std Dev**
May						
Time between storms, minutes	223	507	13514	2129	1277	2126
Storm duration, minutes	161	60	7271	1264	762	1366
Storm depth, mm	161	0.15	114.0	11.3	3.9	17.3
June						
Time between storms, minutes	200	565	13045	2374	1413	2370
Storm duration, minutes	161	64	8715	1088	743	1159
Storm depth, mm	161	0.10	70.1	8.0	4.2	10.9

[1] *Storms* defined by minimum time between storms (critical duration, CD) using exponential method. CD = 507 min for May; CD = 564 min for June.

Development of Huff curves

Bonta (1997) explained the construction of Huff curves in detail. Briefly, Huff curves are developed by first separating bursts of rainfall in a measured precipitation record into independent storms. The method used for this study is one suggested by Restrepo and Eagleson (1982). In their method, a minimum dry-period duration is found such that dry periods greater than this minimum dry period statistically separate storms from one another, under the assumption that dry-period durations larger than the minimum are exponentially distributed. This minimum duration is found by first computing the mean and standard deviation of dry time between continuous rainfall bursts for an entire set of data. If the coefficient of variation (CV=standard deviation / mean) is greater than unity, then the CV of dry-period durations greater than those data remaining after omitting the smallest dry-period duration is computed. This iterative procedure continues until the smallest dry-period duration is interpolated at CV=1. The equivalence of the mean and standard deviation (CV=1) is a property of the exponential distribution, and the exponential distribution separates statistically independent events in a Poisson process (modeling the occurrence of storm events).

Storms were identified and extracted using the minimum time between storms for May and June, and individual mass curves of *storm* precipitation were computed using the Invercargill data. Storms with four or more points were used to minimize the possibility of selecting many mass curves having a linear trend. Linear mass curves skew the shape of Huff curves. Storms were nondimensionalized by dividing all storm depths in an individual mass curve by the total storm depth, and storm durations were nondimensionalized similarly using total storm duration. All nondimensional mass curves were graphically superimposed and dimensionless mass-curve intersections along verticals at 0.02 increments along the dimensionless-duration axis were computed (Fig. 1). The empirical probability values of mass-curve intersections from 10% to 90% along each vertical in increments of 10% were interpolated. For example, along the 0.20 vertical (the first 20% of storm durations), 10% of the dimensionless mass-curve intersections occur at a dimensionless depth of 0.062, and 90% of intersections occur at a dimensionless depth of 0.62 for Invercargill (Fig. 1). Connecting equal probabilities with lines yields the nine isopleths of probability that constitute "Huff curves" (Fig. 1). The storm-duration quartile of the maximum storm intensity was determined for each storm and summarized.

Curve approach

After Huff curves were developed for the Invercargill precipitation data, three dimensionless-duration regions within the Huff curves were defined (Fig. 1). These regions were designated as the "first" region (dimensionless storm durations from 0-0.33), the "middle" region (dimensionless durations between 0.33 and 0.66), and the "third" region (dimensionless duration between 0.66 and 1.0). The nine Huff-curve isopleths (10% to 90% in increments of 10%) that were developed from using *all* storm data extracted, formed the base set of curves with which all other curves were compared. Other sets of curves were developed using sample sizes from 5 to 300, in increments of 5, 10, 20, or 25 storm samples. For each storm sample size, Huff curves were developed, and relative deviations were computed between the base curves and the curves developed from the different storm sample sizes (Fig. 2). Preliminary calculations showed that relative deviation, computed as

(current isopleth - base isopleth) /
(maximum of current isopleth and
base isopleth) (1)

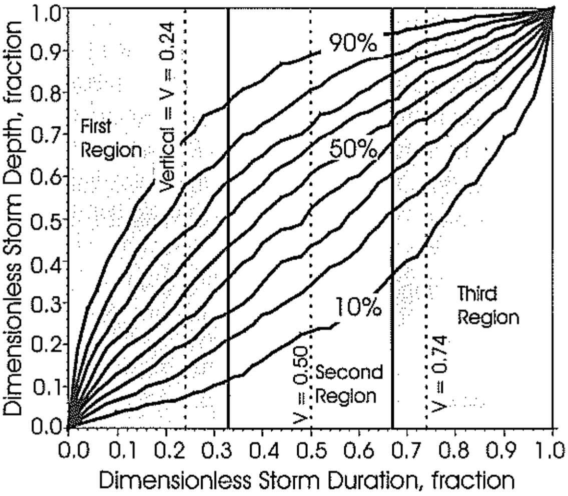

Figure 2 – Three Huff-curve regions in the dimensionless depth-duration grid for the curve approach, and three verticals for the K-S approach for Invercargill.

was a good indicator of disparity between Huff curves. Relative deviations were computed between dimensionless mass-curve intersections at verticals in increments of 0.02 for the nine isopleths. Relative deviations were computed in each of the three Huff-curve regions. Mean, standard deviation, and coefficient of variation of deviations were plotted against sample size. Trends in the graphs were inspected to determine the sample size above which larger storm sample sizes did not notably reduce relative error.

K-S approach

Three verticals at 0.24, 0.50, and 0.74 were chosen to represent the variation of differences in Huff curves along the dimensionless-duration axis (Fig. 2). These verticals were chosen at points near the nominal dimensionless storm duration verticals of 0.25, 0.50, and 0.75 (they were the closest verticals to the 0.02 increment of dimensionless duration used in Figure 1). Sample sizes ranging from 5 to 300 storms were selected randomly from the entire set of May and June storms used in the curve approach. Forty samples of each sample size were tested. The frequency distribution of mass-curve intersections along each of the three representative verticals were compared with the corresponding distribution from the base data set of storms. The Kolmogorov-Smirnov (K-S) test for differences between frequency distributions was used for each comparison. Graphs of K-S significance probability versus sample size for each of the 40 comparisons were inspected for each representative vertical to determine the minimum sample size needed for stable Huff curves at the 5% and 10% significance probabilities.

Results and discussion

Characteristics of storms

The minimum dry-period durations for May and June data for Invercargill that yielded an exponential distribution of times between storms were 507 min and 564 min, respectively. A total of 322 storms were identified using these minimum times between storms. Huff curves developed from the 322 storms formed the base set of curves with which all Huff curves developed from varying storm sample sizes were compared. Of these, 131 storms (40.7%) had the largest quartile intensity during the first quartile, 60 storms (18.6%) had their maximum intensity during the second quartile, 51 storms (15.8%) had their maximum intensity during the third quartile, 59 storms (18.3%) had their maximum intensity during the fourth quartile, and 21 storms (6.5%) had maximum quartile intensities occurring equally in two quartiles.

Times between storms calculated using the exponential method ranged from 507 min to 13514 min, with a mean of 2129 min and a median of 1277 min for May. For June, time between storms ranged from 565 to 13065 min, with a mean of 2374 min and a median of 1413 min. Standard deviations for both months were nearly identical with means, a property of the exponential distribution and exponential method used to identify storms.

Storm durations for May ranged from 60 to 7271 min with a mean of 1264 min and a median of 762 min. For June, durations ranged from 64 to 8715 minutes, with a mean of 1088 min and a median of 743. Storm durations tended to be slightly shorter for June than for May storms.

Storm depths for May ranged from 0.15 mm to 114 mm, with a mean of 11.3 mm and a median of 3.9 mm. For June, storm depths ranged from 0.10 mm to 70.1 mm, with a mean of 8.0 mm and a median of 4.2 mm. Mean storm depths were larger for May, but median storm depths were larger for June.

Curve approach

Figure 3 shows that, above a sample size between 110 and 140 storms, there does not appear to be rapid improvement in the trend of the mean-deviation lines. It is expected that as sample size becomes larger, the lines would converge to the zero-deviation line. This converging trend is apparent after a sample size of about 175 storms. The mean relative deviation varies greatly for the three regions, particularly for the less-than-0.33 dimensionless-duration region for sample sizes less than 110. In this region, about 41% of the storms had their maximum storm intensity during the first quartile of storm duration. The third region (dimensionless duration ≥ 0.66) showed the least variability

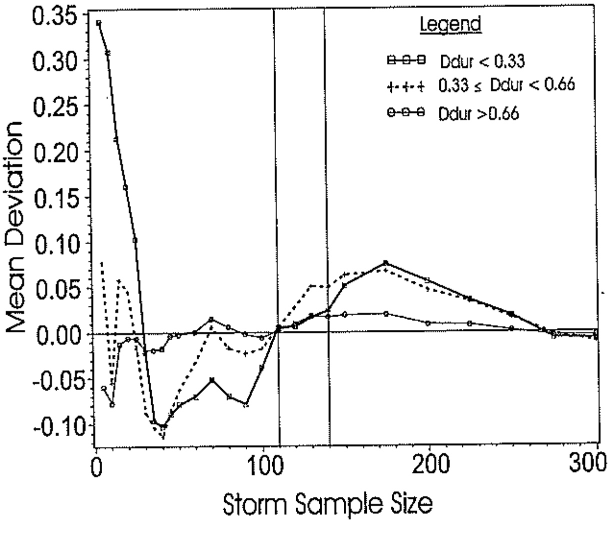

Figure 3 – Mean of deviations versus storm samples size for the curve approach for each dimensionless-duration (Ddur) region.

in mean deviation for nearly all sample sizes. The middle region (0.33 ≤ dimensionless duration < 0.66) shows moderate variability. The primary reason that variability decreases from the first through the third regions is that few storms remain in the 322-storm-sample data set in which the maximum quartile intensity has not occurred. For example, by the end of the second quartile (50% of storm duration), 59% of the storms had experienced the quartile of maximum intensity. By the end of the third quartile (75% of storm duration), 75% of the storms had experienced the maximum quartile intensity. Consequently, deviations between the base set of curves (322 storms) and remaining sample sizes would tend to be small in the last quartile. This is reflected in the mean deviation in the third region. Figure 3 shows the importance of separating the data into dimensionless-duration regions. If deviation statistics were computed from a single data set by collapsing the three regions, then the additional information on the regions of maximum deviations in the first and second regions would not be available.

Figure 4 shows that standard deviation of deviations appears to become stable at a sample size of about 110 to 140. The standard deviation is region-dependent, as was the mean in Figure 3, highlighting the desirability of separating Huff curves into regions for comparison using the "curve" approach. As with the mean, the first region (<0.33) shows the most variability, while the third region (> 0.66) shows the least variability.

The coefficient of variation (CV, Fig. 5) shows that a sample size of about 130 storms is the smallest sample size that should be used to construct Huff curves using CV as the criterion. Sample sizes between 40 and 70 storms are associated with large negative and positive CV because the mean (Figure 3) was nearly zero for the third (>0.66) region and the standard deviations were moderately large for these sample sizes in this Huff-curve region (Fig. 4). The sensitivity of CV to small means suggests that CV is not a good measure with which to judge Huff-curve stability.

K-S approach

Figure 6 shows that significance probability increases with increasing sample size. Nearly all probabilities of the 40-storm samples for a given sample size increase beyond the 10% level after a sample size of about 145 storms in all three Huff-curve regions. Only three samples lie in the first region (Fig. 6, top) between probabilities of 0.0 and 0.1, for storm

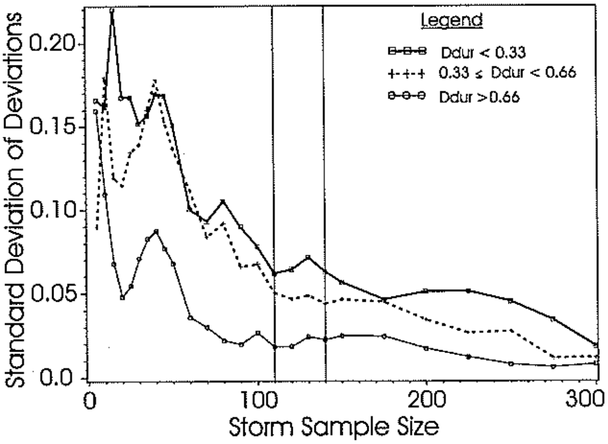

Figure 4 – Standard deviation of deviations versus storm samples size for the curve approach for each dimensionless-duration (Ddur) region.

sample size greater than 110 storms. Five samples lie in the middle region (Fig. 6, middle), and one sample lies in the third region (Fig. 6, bottom). Figure 6 shows that the minimum number of storms required for stable Huff curves is not as dependent on the Huff-curve-region as the number of storms found for the curve approach, and that any vertical could be used alone.

Probabilities of the 40 samples increase beyond the 5% significance level after about 110-120 storm samples. This suggests that the curve and K-S approaches give similar results, and that the minimum sample size of storms required to construct stable May-June Huff curves for Invercargill is about 120 storms.

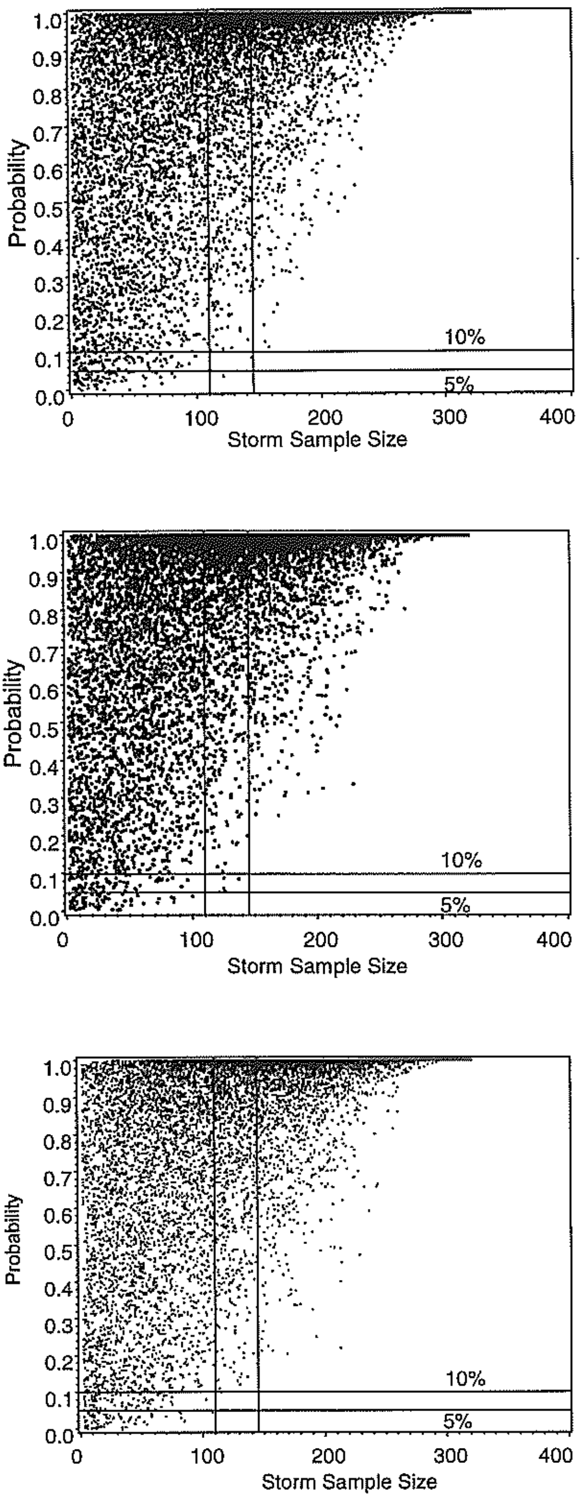

Figure 6 –

(top) Kolmogorov-Smirnov significance probabilities for different sample sizes of 40 storms for the dimensionless-storm-duration vertical of 0.24.

(middle) Kolmogorov-Smirnov significance probabilities for different sample sizes of 40 storms for the dimensionless-storm-duration vertical of 0.50.

(bottom) Kolmogorov-Smirnov significance probabilities for different sample sizes of 40 storms for the dimensionless-storm-duration vertical of 0.74.

Huff curves developed from different sample sizes of storms

Figure 7 illustrates the visual improvement in stability of Huff curves as sample size increases (sample sizes of 5, 120, and 322 storms). The curves for a sample of five storms show more jaggedness and greater disparity from the smoother base set developed from 322 storms. Those Huff curves for the suggested minimum sample size of 120 storms lay close to the smooth base set of curves. The purpose of the present study was to investigate methods to compare Huff curves using a small subset of precipitation data for May and June. It has been shown that Huff curves vary seasonally (Bonta, 1997; Bonta and Rao, 1987). Consequently, the storm sample size may also vary with season of year and also across large geographic areas.

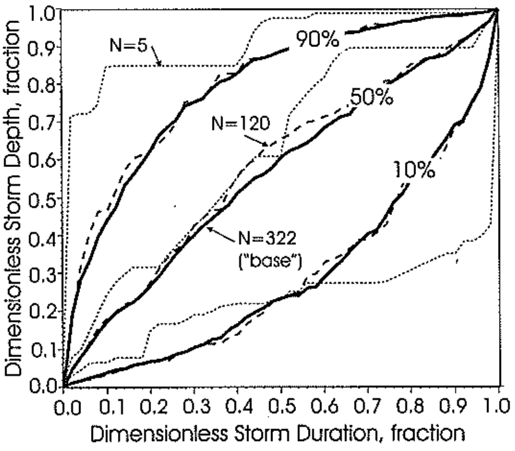

Figure 7 – Example Huff-curve isopleths resulting from three different sample sizes for Invercargill for the 10%, 50%, and 90% isopleths.

Conclusions

Huff curves are probabilistic representations of dimensionless within-storm intensities, and are useful as design storms and for stochastic simulation of storms. Storms were identified by computing a minimum dry time that separates storms. Two methods of comparing Huff curves were investigated to determine the minimum number of storms needed to develop a stable set of Huff curves (a set of curves developed from a smaller sampling of storms that are not different from a set of curves developed by using all data for a given rain gauge). The curve approach yields the mean and standard deviation of a measure of the deviations between Huff curves using all data (base data set) and Huff curves developed with differing sample sizes of storms. Three regions of Huff curves were evaluated (dimensionless storm duration < 0.33, $0.33 \leq$ dimensionless duration <0.66, and dimensionless duration ≥ 0.66). The K-S approach uses the Kolmogorov-Smirnov statistical test to determine statistically significant differences between the base set of curves and those of sets of 40 Huff curves developed using differing sample sizes. Three representative verticals were investigated for the K-S method. The following conclusions are made regarding the two methods, from using 15 years of storm-precipitation data for May and June, for the rain gauge at Invercargill, New Zealand:

1. The minimum dry-period durations for May and June data for Invercargill that yielded an exponential distribution of times between storms were 507 min and 564 min, respectively.
2. Mean storm time between storms was 2129 min for May and 2374 min for June.
3. Mean storm duration was 1264 min for May and 1088 min for June.
4. Mean storm depth was 11.3 mm for May and 8.0 mm for June.

5. A total of 322 storms were identified. Of these, 131 storms (40.7%) had the largest quartile intensity during the first quartile, 60 storms (18.6%) had their maximum intensity in the second quartile, 51 storms (15.8%) had their maximum intensity in the third quartile, and 59 storms (18.3%) had their maximum intensity in the fourth quartile.
6. The first two regions are needed to determine the region in which Huff curves may differ for the curve approach, but any of the representative verticals can be used with the K-S approach.
7. The region with dimensionless duration <0.33 shows the most variability in differences between Huff curves using the curve approach. This region is associated with rapidly varying Huff curves.
8. The curve approach results in a minimum sample size of about 110 to 140 storm samples for Invercargill.
9. The K-S approach results in sample sizes of about 145 storm samples for the 10% significance probability for Invercargill.
10. A smaller minimum sample size of 110 to 120 storms is acceptable using the K-S approach at the 5% significance level. There are few comparisons (one to five samples out of 40 storms) that lie between the 5% and 10% significance levels for storm sample sizes exceeding 110.
11. Based on the two methods, storm sample sizes greater than about 120 storms samples do not contribute significantly to stable Huff curves, and 120 storms is the minimum number of storms suggested for the data investigated.

The two methods investigated can be applied to develop quality-control procedures for Huff-curve development for a variety of factors such as season of year and storm size, and for identifying regions of similar Huff curves. Such quality control is important for parameterizing storm generator models such as StormGen (Bonta, 2001). Both methods appear to give similar results.

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