

Comparison of a deterministic and a statistical model for predicting streamflow recession curves

George A. Griffiths and Alistair I. McKerchar

NIWA, P.O. Box 8602, Christchurch, New Zealand.

Corresponding author: g.griffiths@niwa.co.nz

Abstract

Prediction of the master baseflow recession curve at an ungauged site is an important and difficult problem in water resource management. Two approaches, one deterministic and one statistical, are developed and their relative performance as predictors is compared to find which is better to apply at sites where streamflow is supplied from bed and bank storage. Both models are used to calibrate a given inverse square recession formula. The basis of the calibration is estimation of the time for the median outflow rate to halve.

The deterministic model expresses this drainage time as net storm rainfall inflow, less quick flow, divided by the average exfiltration rate from channel bed and bank storage.

The statistical model uses six basin characteristics to calculate the relative degree of similarity between an ungauged basin and each one of a set of reference basins. Similarity is expressed as a weight and these are combined with measured drainage times to form a pooled estimate for the ungauged basin.

Data from ten gauged basins in the greywacke geologic terrain of Canterbury, New Zealand are used to calibrate and test the models. These basins have no significant lakes or springs and master curves were constructed using recessions occurring in the January to March period.

Reliable predictions for practical purposes can be made in the specified terrain using the statistical model in its present form. In comparison, improved definitions and measurement of some key variables, and definition of an exfiltration function, are necessary before the deterministic model may be usefully employed.

While further development and testing of the models is needed, particularly in other geologic terrains, results indicate that both types have the potential to supply reasonably accurate predictions of master recession curves in basins where little is known about water storage behaviour.

Keywords

streamflow recession, recession curve, bank storage, deterministic model, statistical model, low flow, baseflow

Introduction

Prediction of master or average baseflow recession curves is an important and difficult problem in hydrology (Brutsaert, 2005; Botter *et al.*, 2009). The equation of the curve and the values of constants depend mainly on basin geology, geomorphology and the density of the drainage network (Smakhtin, 2001).

Definition of baseflow recession relations is often essential in many aspects of water resources management including, with

particular reference to New Zealand, instream flow requirements for aquatic ecosystems, irrigation, water allocation, wastewater disposal and dilution of contaminant discharges (Pearson and Henderson, 2004). Recession curves may also be fundamental components of hydrological models concerned with, for example, reservoir storage, frequency of low flows and rainfall-runoff forecasting (Stedinger *et al.*, 1993).

In earlier work Griffiths and Clausen (1997) used a linear hydrologic model to rout inflows from independent storages of different kinds through a stream channel storage. In the two parameter case an inverse square recession formula explained most variance in recession data from seven basins in Canterbury, New Zealand. Griffiths and McKerchar (2010) obtained the same recession formula from nonlinear hydrological routing of inflows from stream channel bed and bank storage through a channel storage. This equation provided a good fit to each of 10 master baseflow recession curves from Canterbury basins.

Herein we adopt this formula and calibrate it to apply at an ungauged site using two approaches – one deterministic, the other statistical. The basis of the calibration is determination of the elapsed time for stream outflow to drain from median flow to half median flow. The deterministic model is an extension of previous work noted above with explicit definition of inflow to storage and the exfiltration process. The statistical model is a region of influence type (Burn, 1990) previously used in flood hydrology but apparently not in low flow studies. These two different models were selected based on success with flood peak estimation employing both deterministic (rainfall-runoff models) and statistical (flood frequency analysis) approaches.

The purpose of this study is to employ an idealised, basin-scale deterministic model and, separately, a statistical model to

provide recession curve calibration at a given ungauged site. The deterministic analysis uses a simple, nonlinear model derived from the conservation of mass equation; the statistical analysis uses a region of influence approach involving pooling of information from basins with similar hydrologic behaviour to predict the required master baseflow recession curve. The motivation for the study is to gain further understanding of a complex problem in which highly variable field conditions are constrained as much as practicable; to explore some of the limitations of the two approaches; to examine their relative performance and to determine which one is better suited for use at present.

Theory

To enable calibration of a recession curve for an ungauged site we first adopt the theoretical formula of Griffiths and McKerchar (2010) given by:

$$Q = Q_m / (1 + bt)^2, t \geq 0 \quad (1)$$

in which Q is stream outflow, Q_m is median flow, b is a constant, t is time and $Q = Q_m$ at $t = 0$. With Q_m assumed known at the ungauged site and with $Q = 0.5 Q_m$ at $t = t_r$, then from Equation 1:

$$b = 0.414 / t_r \quad (2)$$

where t_r is the time for flow to drain from Q_m to $0.5Q_m$ (Martin, 1973).

It follows that if t_r can be estimated at the ungauged site then b can be calculated and the recession curve sought is then given by Equation 1.

In the following sections two approaches are employed to predict t_r : one deterministic, the other statistical.

Deterministic model

A conceptual, lumped, time invariant model is derived based on the conservation of mass equation. After the occurrence of a typical storm over a catchment (taken here as that

which generates a mean annual flood) when outflow is less than or equal to median flow, we presume that flow is chiefly supplied from channel bed and bank storage because quick flow, or rapid runoff, has ceased. Then t_r is given by net rainfall minus quick flow divided by the average exfiltration rate, Q_s , that is:

$$t_r = (IA - L_c - 1.5 Q_p t_c) / Q_s \quad (3)$$

where I is total storm rainfall depth (of 2-year return period and duration equal to the time of concentration), A is catchment area, L_c is loss (dominantly from interception and evapotranspiration), Q_p is the mean annual flood peak and t_c is time of concentration – taken to be equal to time of rise of the quick flow hydrograph where the time to fall is $2t_c$.

Assuming that Q_s is composed of both rapid and slow subsurface runoff where rapid exfiltration can be characterised by a Stephenson type formula (Stephenson, 1979; Sedghi-Asl *et al.*, 2014) and slow exfiltration by the Darcy formula (Maidment, 1993, p. 5.17) then we may write:

$$Q_s = f [mL_s y (gd_{50}S)^{0.5} + kL_s y S] \quad (4)$$

in which f is some specified function, m is porosity of subsurface storage, $L_s y$ is the area of stream bank and bed where exfiltration is occurring and where L_s is stream length and y is the depth, g is the acceleration of gravity, d_{50} is a surrogate for particle size in subsurface flow channels and is the median size of surface streambed material measured at the halfway point on the main channel above a site, S is the slope of the main channel, and k is hydraulic conductivity. Substitution of Q_s from Equation 4 into Equation 3 yields the deterministic model:

$$t_r = (IA - L_c - 1.5 Q_p t_c) / \{f [mL_s y (gd_{50}S)^{0.5} + kL_s y S]\} \quad (5)$$

Statistical model

A region of influence type model (Burn, 1990; Kjeldsen *et al.*, 2014) is derived to estimate t_r using pooled information from

reference basins in the area with similar hydrological behaviour owing to their having similar meteorological, geological and geomorphic characteristics. The degree of similarity or affinity between a reference site and a nominated ungauged site is measured by Euclidean distance, d_p , defined by:

$$d_i = \left\{ \sum_{i=1}^n w_c \left[\frac{(\ln c_i - \ln c_u)}{\sigma(\ln c_a)} \right]^2 \right\}^{0.5} \quad (6)$$

in which c_i is the value of a reference basin characteristic, c_u is the value of the same characteristic at the ungauged site, $\sigma(\ln c_a)$ is the standard deviation of the logs of the same characteristic for the reference basins and w_c is a characteristic weight factor. Finally, the $n = 6$ characteristics selected here are I , m , d_{50} , S , D_d and Q_p/A , where Q_p/A is mean annual flood runoff and D_d is drainage density, equal to L_s/A .

To allow for the different record lengths used to compute drainage time in the reference basins, t_{rb} , a power law is adopted to specify a record weighting factor, w_r , given by:

$$w_r = 0.410 T^{0.262} \quad 10 \leq T \leq 30 \quad (7)$$

$$w_r = 1 \quad T > 30 \quad (8)$$

in which T is record length in years so that when $T = 10$ then $w_r = 0.75$, and for $T = 30$ then $w_r = 1$. We assume here that a record of 10 or more years is required in a reference basin to calculate an acceptable estimate of t_r .

To convert the weighted distance values, $d_i w_r$, to basin weights $0 < w_{fi} < 1$ the relation:

$$w_{fi} = d_i w_r / \sum_{i=1}^s d_i w_r \quad (9)$$

is employed where s is the number of reference basins. Then, lastly, the predicted value of t_r at the ungauged site is estimated as the geometric mean of basin values:

$$t_r = \exp \left\{ \sum_{i=1}^s w_{fi} (\ln t_{rb})_i \right\} \quad (10)$$

Note that other statistics such as the 7-day mean annual low flow (7dMALF) may also

be estimated at an ungauged site using this statistical model.

Application

To reduce complexity this study is restricted to natural basins within a specific geological terrain in terms of lithology, structure and recent geomorphic history, namely, rivers and streams located in the steep foothill and mountainous area of the eastern Southern Alps of Canterbury. All have coarse boulder or gravel beds composed almost exclusively of greywacke and are incised in glacial outwash gravels or mass movement deposits. Water supply to the river and streams for low flows less than, say, the median is dominantly by exfiltration from channel bed and bank storages supplied by rapid subsurface flow (Hayward, 1976; Pearce and McKerchar, 1979); there are no significant lakes or springs within the chosen basins. To avoid seasonality effects and any influence of snow and ice storage, only recessions occurring during the period January to March were selected to construct master recession curves.

Data selection

Ten basins of differing size and hydrologic and physiographic properties were selected from the foothills and mountains of Canterbury (see Griffiths and McKerchar, 2010). Details of hydrological recording stations or sites, record length and hydrologic and catchment characteristics are given in Table 1. Basin area was obtained from Walter (2000). Stream length above each site was estimated from computerised 1:50,000 scale topographic maps (NZMS 260 Series) on which the stream network is defined by blue lines. Main channel slope was calculated by the equal-area method (NWASCO, 1975) using the above topographic maps.

Porosity of storage was estimated using information in Chow (1964, p. 13-14) supported by field inspection.

Median size of the surface streambed material was measured using the Wolman method (Wolman, 1954). Rainfall intensity of 2-year return period and duration equal to the time of concentration was estimated at the centroid of a basin by employing HIRDS (V3) (available on the Internet at <http://hirds.niwa.co.nz>) (Thompson, 2002). Median flow and mean annual flood were calculated from site flow records. Time of concentration for a basin was estimated using a formula given in Griffiths and McKerchar (2012, Eq. 5). Time to drain from median to half median flow was measured from the master baseflow recession curve, obtained as described below. Average exfiltration rate during the time to drain was also obtained from the relevant master baseflow recession curve.

Master baseflow recessions

At least eight flow recessions were selected from the January to March periods of flow record, and master baseflow recession curves were constructed using the tabular method of Toebes and Strang (1964). In applying this method mean daily discharges were employed and a check was made to ensure that there was no significant rainfall in a basin during a selected recession. Indeed, the occurrence of rainfall events in the basins severely restricted the number of recessions available for analysis. The master baseflow recessions used in this study are the same as those of Griffiths and McKerchar (2010).

Calibration of deterministic model

Before the deterministic model (Eq. 5) can be employed it is necessary to specify f . Moreover, it was not clear how γ could be measured so its value was taken as unity. Also, the actual relative proportion of rapid to slow subsurface runoff and how this might vary with time is unknown, as is the value of k . In fact, if it is assumed that only slow subsurface runoff occurs then back calculation using

Table 1 – Hydrologic and physiographic characteristics of selected Canterbury basins and recession flow data.

River & site name	Site number (Walter, 2000)	Record length yrs	Area km ²	Stream length km	Main channel slope	Estimated porosity of storage %	Median of surface bed material mm	Drainage density 1/km	Rainfall intensity, (2 yr, tc) mm	Median flow m ³ /s	Mean annual flood m ³ /s	Time of conc. hr	Time to drain hr	Average exfiltration rate m ³ /s
			A	L_s	S	m	d_{50}	D_d	I	Q_m	Q_p	t_c	t_r	Q_s
Acheron at Clarence	62103	51	973	1400	0.0067	25	40	1.44	40	15.9	331	11	7	11
Ashley at Lees Valley	66210	22	121	170	0.024	15	50	1.40	20	2.69	72.7	4.5	6.4	2
Waimakariri at Old Highway Br.	66401	43	3210	4630	0.0035	25	25	1.44	65	94.0	1480	21	12	71
Camp Stm at Craightburn	66405	42	0.9	2	0.22	15	100	2.22	24	0.025	0.577	2	20	0.019
Selwyn at Whitecliffs	68001	46	164	250	0.019	15	50	1.52	35	2.02	76.9	8.5	12	1.52
Dry Acheron at Water Race North	68529	12	6.19	8.7	0.133	30	80	1.41	30	0.163	2.76	2.5	18	0.122
Ashburton at Old Weir	68810	27	276	370	0.021	30	50	1.34	55	6.72	148	9	25	5.04
Orari at Gorge	69505	28	522	760	0.016	20	40	1.46	33	6.20	195	7	20	5.27
Rocky Gully at Rockburn	69621	45	23	21	0.05	30	60	0.913	20	0.173	14.6	4	13	0.13
Forks at Balmoral	71129	47	98	145	0.014	40	35	1.48	60	2.50	22.6	6	45	1.88

data from Table 1 yields a range in k of three orders of magnitude with no pattern evident. Accordingly, as k is essentially unpredictable all exfiltration was assumed to occur as rapid flow. Finally, L_c is small and is ignored given the great difficulty in trying to estimate its value. Consequently, given the above, and for practical purposes, Q_s was modelled as a simple power law using the data in Table 1. The result is:

$$Q_s = 1.7 \times 10^{-5} [mL_s(gd_{50}S)^{0.5}]^{1.38} \quad (11)$$

so that t_r becomes (from Equation 5):

$$t_r = (IA - 1.5Q_p t_c) / 1.7 \times 10^{-5} [mL_s(gd_{50}S)^{0.5}]^{1.38} \quad (12)$$

Recession curve prediction

Within the defined geologic terrain a master baseflow recession curve may be predicted deterministically for an ungauged natural basin using Equation 1 where b is determined from Equation 1 and t_r from Equation 12. To undertake this calculation, values of Q_m , I , A , Q_p , t_c , m , L_s , d_{50} and S must be known for the basin. Of these, Q_m can be estimated from information given in, for example, Woods *et al.* (2006) and Q_p from McKerchar and Pearson (1989) or Griffiths *et al.*, (2011). (Note that for the basins in Table 1 the ratio of median to mean flow is approximately 0.65.) Values for the remaining variables may be obtained as described previously.

With the statistical model Equation 1 is used where, again, b is determined from Equation 2 and t_r from Equation 10. To undertake this calculation, values of I , m , d_{50} , S , D_d , Q_p/A , T and t_r must be known for the reference basins and I , m , d_{50} , S , D_d , Q_p/A and Q_m for the ungauged basin all estimated as described above. Of these variables, I , m and d_{50} are assumed to be of more importance in assessing degree of similarity amongst the reference basins. Thus, in Equation 6 we put $w_c = 0.5$ when dealing with these variables and $w_c = 1$ otherwise – as the smaller the value of d_i the greater the degree of similarity.

To further sharpen the value of the pooled estimate we select the five basins with the smallest values of d_p , that is we put $s = 5$ in Equation 10.

Estimation of the accuracy of model predictions is problematic. Formal calculation of the error in t_r in Equations 10 and 12 is possible but the errors in I , m and d_{50} for both models are unknown and would have to be guessed. More importantly, the model errors reflecting the suitability of the models themselves in Equations 1, 10 and 12 are likely to dominate and these are quite uncertain. We believe a measure of the relative accuracy of the two models can perhaps best be determined from the results of many comparisons in gauged basins of predicted versus actual values of t_r in particular. As a start to this approach, we now apply the two models at two sites in Canterbury with flow records.

Example 1: Jollie at Mt Cook Station (Site No. 71135, T=45 yrs)

Values of relevant variables for this site are $Q_m = 6.43 \text{ m}^3/\text{s}$, $A = 139 \text{ km}^2$, $Q_p = 74 \text{ m}^3/\text{s}$, $t_c = 5 \text{ hrs}$, $m = 0.35$, $L_s = 175 \text{ km}$, $d_{50} = 70 \text{ mm}$, $S = 0.029$, $I = 100 \text{ mm}$, $D_d = 1.26$, $t_r = 22 \text{ days}$, $7\text{dMALF} = 4.67 \text{ m}^3/\text{s}$.

Substitution of pertinent values in Equation 12, the deterministic model, yields $t_r = 30 \text{ days}$ which, not surprisingly in view of all the approximations, is rather different from the value determined from the measured master base flow recession curve for the site of $t_r = 22 \text{ days}$. From Equation 2, $b = 0.014$ and the predicted recession curve (Eq. 1) is shown in Figure 1. The predicted time of occurrence for 7dMALF is 12.7 days; measured value is 8.9 days.

Calculations for application of the statistical model (Eq. 12) are displayed in Table 2. The predicted value of drainage time, t_r , determined from the $s = 5$ reference basins is 18 days, which is close to the measured value of 22 days. From Equation 2, $b = 0.023$

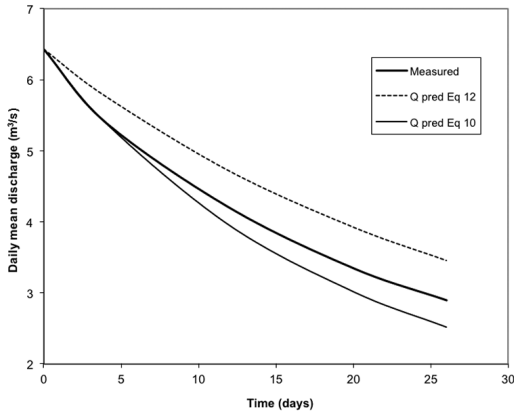


Figure 1 – Predicted and measured baseflow recession curves for Jollie at Mt Cook Station (Site No 71135). Eq. 10 is the statistical model; Eq. 12 is the deterministic model.

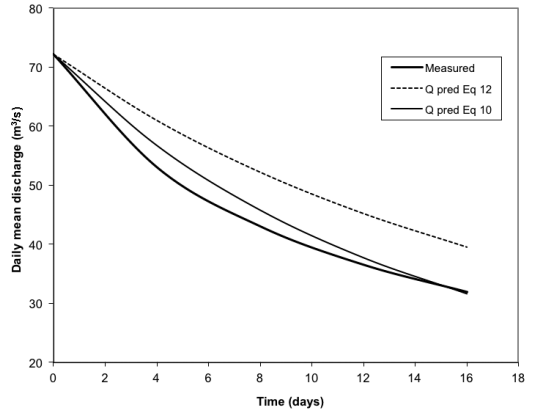


Figure 2 – Predicted and measured baseflow recession curves for Waiiau River at Marble Point (Site No 64602). Eq. 10 is the statistical model; Eq. 12 is the deterministic model.

and the predicted recession curve is illustrated in Figure 1. The predicted time of occurrence for 7dMALF is 7.7 days; measured value is 8.9 days.

Example 2: Waiiau at Marble Point (Site No. 64602, T = 43 yrs)

Values of relevant variables for this site are $Q_m = 72.1 \text{ m}^3/\text{s}$, $A = 1980 \text{ km}^2$, $Q_p = 1000 \text{ m}^3/\text{s}$, $t_c = 9 \text{ hrs}$, $m = 0.25$, $L_s = 2990 \text{ km}$, $d_{50} = 50 \text{ mm}$, $S = 0.0067$, $I = 60 \text{ mm}$, $D_d = 1.47$, $t_r = 13 \text{ days}$, $7\text{dMALF} = 34.4 \text{ m}^3/\text{s}$.

Substitution of pertinent values in Equation 12, the deterministic model, yields $t_r = 19 \text{ days}$ which as with Jollie at Mt Cook is rather different from the measured value of $t_r = 13 \text{ days}$. From Equation 2, $b = 0.022$ and the predicted recession curve (Eq. 1) is shown in Figure 2. The predicted time of occurrence for 7dMALF is 21 days; measured value is 14.5 days.

Calculations for the application of the statistical model (Eq. 12) are displayed in Table 2. The predicted value of t_r determined from $s = 5$ reference basins is 13 days, the same as the measured value. From Equation 2, $b = 0.032$ and the predicted recession curve is illustrated in Figure 2. The predicted

time of occurrence for 7dMALF is 14.5 days; measured value is 14.5 days.

It is evident from the above examples that while the deterministic model (Eq. 5) has a firm theoretical basis its performance in practice, even with calibration (Eq. 12) and perhaps some degree of fortuity, is not good enough for useful prediction. In particular much better definition of the exfiltration function, f , is needed and a more precise alternative to the surrogate variable, d_{50} , is required. Moreover the model is sensitive to the value of I which is difficult to estimate especially without information on areal reduction factors in larger basins. Indeed, model performance should markedly improve by using a distributive approach in these basins, that is, by integrating outputs from sub-basins.

The performance of the statistical model (Eq. 10) shows promise although the basin characteristics that best define degree of similarity are problematic and a means of determining the optimum number of reference basins to employ in generating the pooled estimate for an ungauged basin needs to be found.

Table 2 – Application of statistical model

River and site name	Record (yrs)	Record weight factor, w_r (Eqs 7 & 8)	Euclidean distance, d_i (Eq. 6)	Basin weight (w_B) (Eq. 9)	Weighted drainage time $w_B \ln(t_{rt})_i$
Jollie at Mt Cook					
Acheron at Clarence	51	1	3.096		
Ashley at Lees Valley	22	09.22	3.102		
Waimakariri at Old Highway Br.	43	1	2.856	0.168	0.417
Camp Stm at Craigieburn	42	1	4.069		
Selwyn at Whitecliffs	46	1	2.673	0.180	0.447
Dry Acheron at Water Race	12	0.786	2.142	0.177	0.512
North Ashburton at Old Weir	27	0.972	1.455	0.321	1.033
Orari at Gorge	28	0.982	3.075	0.154	0.461
Rocky Gully at Rockburn	45	1	3.689		
Forks at Balmoral	47	1	3.845		
				1.000	2.87
					t = 18 days
Waiau at Marble Pt					
Acheron at Clarence	51	1	0.934	0.258	0.502
Ashley at Lees Valley	22	09.22	1.996		
Waimakariri at Old Highway Br.	43	1	1.152	0.209	0.519
Camp Stm at Craigieburn	42	1	4.122		
Selwyn at Whitecliffs	46	1	1.425	0.169	0.506
Dry Acheron at Water Race	12	0.786	2.542		
North Ashburton at Old Weir	27	0.972	1.302	0.180	0.579
Orari at Gorge	28	0.982	1.286	0.184	0.457
Rocky Gully at Rockburn	45	1	3.598		
Forks at Balmoral	47	1	1.801		
				1.000	2.563
					t = 13 days

Future work

Apart from the further development of model structure, and better parameter definition and measurement, refinement of the models is desirable in at least three areas. First, further testing is needed with data from other basins within the same geologic terrains to explore the effect of employing different basins for calibration and in determining pooled estimates. Second, it would be useful

to apply the models to other geologic terrains to assess the influence of this major factor. Third, much field work is needed to better understand the structure and properties of channel bed and bank storages.

Conclusions

Calibration of a master baseflow recession curve in an ungauged basin can be achieved

in principle using either a deterministic or a statistical model.

At this stage both the derived models are limited in their application to natural basins in Canterbury in greywacke geologic terrain having no significant lakes or springs; and to that section of a master recession curve between the median and lesser flows, for the period January to March.

Much improved definitions and measurement of key variables, such as rainfall intensity, particle size and flow rates in subsurface flow channels, as well as definition of the exfiltration function is needed before the deterministic model will be useful in prediction.

The statistical model could be employed for prediction in the specified terrain and with some refinement, mainly in basin characteristic parameters that best measure degree of similarity between catchments, even better results may be obtained.

Further testing of the two predictive models when further developed is required especially in different geologic terrains, and more knowledge of the operation of channel bed and bank storage is needed if the models are to be substantially refined.

References

- Botter, G.; Porporato, A.; Rodriguez-Iturbe, I.; Rinaldo, A. 2009: Nonlinear storage-discharge relations and catchment streamflow regimes. *Water Resources Research* 45, W10427, doi:10.1029/2008WROO7658: 16 p.
- Brutsaert, W. 2005: *Hydrology: An Introduction*. Cambridge University Press, New York.
- Burn, D.H. 1990: Evaluation of regional flood frequency analysis with a region of influence approach. *Water Resources Research* 26: 2257-2665.
- Chow, V.T. 1964: *Handbook of Applied Hydrology*. McGraw-Hill Book Co. Inc., New York.
- Griffiths, G.A; Clausen, B. 1997: Streamflow recession in basins with multiple water storages. *Journal of Hydrology* 190: 60-74.
- Griffiths, G.A.; McKerchar, A.I. 2010: Recession of streamflow supplied from channel bed and bank storage. *Journal of Hydrology (NZ)* 49(2): 99-109.
- Griffiths, G.A.; McKerchar, A.I. 2012: Estimation of mean annual flood in New Zealand. *Journal of Hydrology (NZ)* 51(2): 111-120.
- Griffiths, G.A.; McKerchar, A.I.; Pearson, C.P. 2011: Review of flood frequency in the Canterbury Region. NIWA Client Report No. CHC2011-045, National Institute of Water and Atmospheric Research Ltd, Christchurch, New Zealand, 22 p.
- Hayward, J.A. 1976: The hydrology of a mountain catchment and its implications for management. Proceedings of the Soil and Plant Water Symposium, DSIR Information Series 126: 126-136.
- Kjeldsen, T.R.; Jones, D.A.; Morris, D.G. 2014: Using multiple donor sites for enhanced flood estimation in ungauged catchments. *Water Resources Research* 59(8): 6646-6657.
- Maidment, D.R. 1993: *Handbook of Hydrology*. McGraw-Hill, New York.
- Martin, G.N. 1973: Characteristics of simple exponential baseflow recessions. *Journal of Hydrology (NZ)* 12(1): 57-62.
- McKerchar, A.I.; Pearson, C.P. 1989: *Flood Frequency in New Zealand*. Publication 20, Hydrology Centre, Division of Water Sciences, Department of Scientific and Industrial Research, Christchurch, New Zealand, 87 p.
- National Water and Soil Conservation Organisation 1975: *Metric Version of Technical Memorandum No. 61*. Water and Soil Division, Ministry of Works and Development, Wellington, New Zealand, 19 p.
- Pearce, A.J.; McKerchar, A.I. 1979: Upstream generation of storm runoff. In Murray, D., and Ackoyd, P. (eds), *Physical Hydrology: New Zealand Experience*. New Zealand Hydrological Society, Wellington, pp. 165-192.
- Pearson, C.P.; Henderson, R.D. 2004: Floods and Low Flows. In: Harding, J.; Mosley, P.; Pearson, C.; Sorrell, B. (eds.). *Freshwaters of New Zealand*, New Zealand Hydrological Society and New Zealand Limnological Society Wellington, pp. 165-192.

- Sedghi-Asl, M.; Rahimi, H.; Farhoudi, J.; Hoorfar, A.; Hartmann, S. 2014: One-dimensional fully developed turbulent flow through coarse porous medium. *Journal of Hydrologic Engineering (Technical Note)* 19(7): 1491-1496.
- Smakhtin, V.U. 2001: Low flow hydrology: a review. *Journal of Hydrology* 240: 147-186.
- Stedinger, J.F.; Vogel, R.M.; Foufoula-Georgiou, E. 1993: Frequency analysis of extreme events. In, Maidment, D.R. (ed), *Handbook of Hydrology*, McGraw-Hill, New York, p. 18.1-18.66.
- Stephenson, D. 1979: *Rockfill in Hydraulic Engineering*. Elsevier, Amsterdam, Netherlands.
- Thompson, C.S. 2002: The high intensity rainfall design system: HIRDS. Proceedings of the International Conference on Flood Estimation, Berne, Switzerland. International Commission for the Hydrology of the Rhine Basin, CHRII-17: 273-282.
- Toebes, C.; Strang, D.D. 1964: On recession curves 1: recession equations. *Journal of Hydrology (NZ)* 3(2): 2-15.
- Walter, K.M. 2000: Index to Hydrological Recording Sites in New Zealand. Technical Report 73, National Institute of Water and Atmospheric Research Ltd, Wellington, New Zealand, 216 p.
- Wolman, M.G. 1954: A method for sampling coarse river bed material. *Transactions, American Geophysical Union* 35(6): 951-956.
- Woods, R.; Hendriks, J.; Henderson, R.; Tait, A. 2006: Estimating mean flow in New Zealand rivers. *Journal of Hydrology (NZ)* 45(2): 95-110