DRAWING STAGE DISCHARGE CURVES THROUGH POINTS SHOWING SCATTER

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Plotted points, providing the relation between stage and discharge and having more or less scatter depending on the quality of the site and the measuring methods and conditions show a curvilinear trend. Normally a free hand mean curve is drawn, a somewhat hazardous procedure when the scatter is significant.

Recent, more skilful, data collection gives an indication of the cause of scatter allowing a family of curves to be drawn through the array of points and/or the individual points may be weighted according to their accuracy.

However, in many cases, no information is available on causes of scatter and the relative accuracy of gaugings is no more than a surmise. In such cases, especially when the data are numerous, the drawing of a mean curve is difficult.

The fitting of a mean curve by mathematical means is incorrect because it presumes a known relation between the stage and the discharge and such a relation is normally not known. It may be argued that the natural cross section may be approximated to a geometrically simple cross section and assuming steady, uniform flow some parabolic type equation may be fitted by graphical (Morrissey,1956), regression, or multivariate, (Snyder 1962), analysis. The graphical method does not take errors into account and the regression method presumes the dependent variable to be without error. All three methods fail when the initial assumption, that a parabolic type equation applies, is incorrect. The latter condition is common, e.g. where effects of back water or variable slopes occur or a change in the uniform shape of the cross section takes place.

However, it is better to resort to graphical methods; a very suitable one is the one used in the polyfactor analysis, (Toebes, 1955). Here a mean line is fitted through the points - this line accurately portrays the trend and takes into account the relative errors of the dependent and independent variables i.e. stage and discharge.

The method assumes that each plotted point has an error in the x axis direction (S_x) and an error in the y axis direction (S_y). Normally these errors will not be equal and the plotted point may be portrayed as the centre of an ellipse with axes proportional to the errors S_x and S_y (Fig.1).

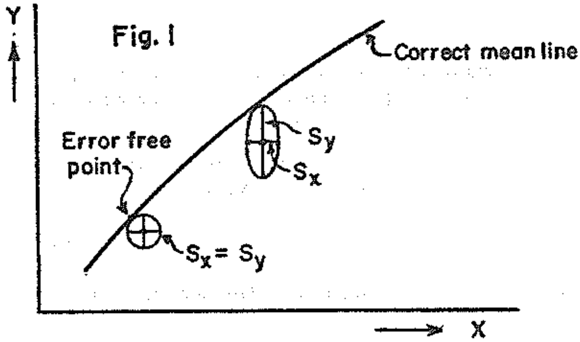


Fig.1 - Equal and unequal errors in X axis (discharge) and Y axis (stage) and the positions of the error-free points.

By transformation of the scales the errors may be made equal and the ellipse becomes a circle. The error free point lies on the circumference and the point cut by the axis perpendicular to the error free point has the greatest probability of being the correct error free point.

The method is applicable to an approximate minimum of 15 points. Three cases may be considered:

(i) the errors in discharge and stage are not known;

(ii) the errors in discharge and stage may be estimated and the scales can be adjusted;

(iii) the errors in discharge and stage may be estimated and the scales cannot be adjusted.

In case (i) the errors in both stage and discharge are assumed equal, or the errors in stage are assumed zero. When assuming that the errors S_x and S_y are equal the error free point lies on the circumference of a circle. The method is to plot the data and to draw a free hand mean curve (first trial curve). The data are now grouped in strips, five being a minimum, with at least three points in each strip. Strip boundary lines are now drawn perpendicular to the first trial curve as shown in Fig.2.

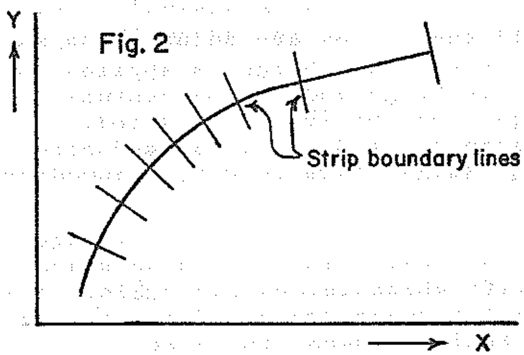


Fig.2 - Strip boundary lines perpendicular to the first trial curve.

It is not strictly necessary to have an equal number of points in each strip; also it is important that strips are narrow at bends of the curve.

The points are now averaged in each strip by algebraically adding distances from the points perpendicular to the first trial curve and dividing the algebraic sum by the number of points in the strip, the points above the curve have a positive distance; those below, a negative distance. Plotting this new point halfway between the strip boundary lines gives a representative point for each strip (Fig.3.)

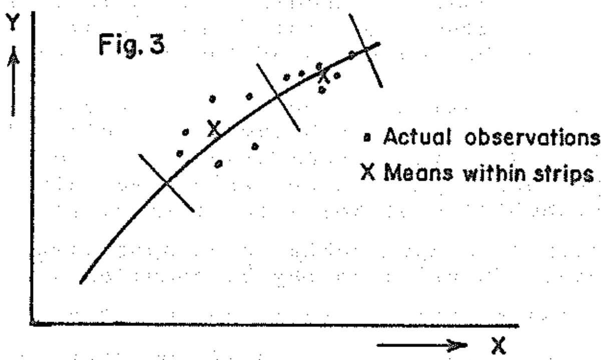


Fig.3 - Strip means as a basis for a second trial curve.

The new averaged points are now used to draw a second trial curve. The process is repeated until the curve does not alter. Normally, when the first trial curve has been drawn with proper care, the second trial curve will be the correct mean curve.

If the assumption is made that the error in stage (S_y) is zero, the strip boundary lines and the measured distances should portray the error S_x only. Therefore the strip boundary lines and average points should be drawn parallel to the x axis.

In case (ii) the scales are adjusted to make $S_y = S_x$. The percentual error in discharge is applied to the mean of the range of discharge and the percentual error in stage to the mean stage, and the scales adjusted.
Example: Discharge range 0-1000, assumed error 10%, scale -100 cusecs/1in.; stage range 0-10ft, assumed error 2%, scale 1ft/1in.

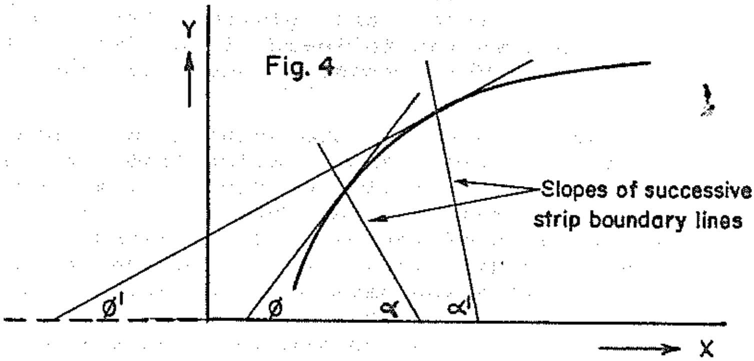
The mean discharge is 500 cusecs, average error 50 cusecs which is, on scale, $\frac{1}{2}$in. The mean stage is 5ft, average error 0.1ft which is, on scale, $\frac{1}{10}$in. Adjust the scales by making the stage scale, 0.2ft/1in. so that the 2% error on the scale becomes $\frac{1}{2}$in. also.

The error S_x now equals S_y and the procedure for drawing the curve is as for case (i).

In case (iii) the scales, if adjusted, may be awkward and the correct points are then found by drawing strip boundary lines and measuring distances, not perpendicular to the last curve, but at an angle α to the x axis given by:

$$\tan \alpha \tan \phi = -(S_{y}/S_{x})^2$$

Where ϕ is the angle made with the x axis by the last approximate curve as shown by Fig.4.



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The application of the above method to stage/discharge relationship problems appears to be new, and should be welcomed by many.

ED.
