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Surface tension in small hydraulic river models - the significance of the Weber number

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Abstract

This short paper reviews critical conditions for surface tension effects in small hydraulic river models. The relationship between inertia and surface tension forces is evaluated by the Weber number (We) (We = $V^2 \rho \, l \, / \, \sigma$ where V is the average velocity of the fluid system, ρ is fluid density, l is a length usually taken as average depth (d) and σ is surface tension). Weber numbers for model studies vary over almost three orders of magnitude and there is no consensus on a single critical condition (critical values vary over 10-100). Care should be taken in interpreting these values because they represent a variety of hydraulic conditions and Weber numbers are commonly expressed in two forms, as a ratio (as above) or as the square root of the ratio. Theoretical attempts to define a critical Weber number based on the energy equation and surface capillary waves yield spurious results. The potential influence of surface tension in small river models should be assessed in all experiments.

Introduction

Surface tension is the tensile force per unit length (N m^{-1}) acting perpendicular to a line in the plane of a fluid surface. Surface tension results from intermolecular forces which attract molecules to one another. At the surface, forces are less than those within the fluid and as a result there is net tension over the surface. In water, due to intermolecular hydrogen bonding, this force is very strong and is the basis of capillarity and the reason for the formation of familiar drops, bubbles and meniscuses. However, in most natural fluid flows, surface tension is very small when compared to other forces and in the majority of cases can be ignored (Chadwick and Morfett, 1986). Free surface flows are most commonly a gravity phenomenon and surface tension forces are negligible. However where flow depths and flow velocities are small, such as in hydraulic models with a large vertical scale ratio, surface tension and viscosity become important (Sharp, 1981). In small hydraulic models of rivers, surface tension forces do not scale in the same way as other variables and under certain conditions may have an important effect on fluid motion. An extreme example of the influence of surface tension in small flows are the *surface tension meanders* studied in the experiments of Davies and Tinker (1984).

Dingman (1984) estimates the importance of surface tension forces by computing the relative forces acting in open channel flow for a variety of channels. Table 1 reproduces two of these examples for a *model* and *medium river* and calculates similar forces for channels in a small 1:50 braided river hydraulic model (Warburton and Davies, 1994). Gravitational and turbulent forces predominate in all the examples. Surface tension and viscous forces are negligible in the so-called *model river* and *medium river* but in the braided river model are only an order of magnitude less than gravitational and turbulent forces. These estimates are for *average conditions* so locally the surface tension force may be equal to the other open channel forces and may therefore affect flow processes and sediment transfer. For example, in hydraulic models which use low density sediment, at the start-up of the experiment when sheetflow is common, surface tension may support sediment by flotation or rafting on the fluid surface.

The Weber Number

The relationship between inertia (mass (m) × acceleration (a)) and surface tension forces is usually expressed by a dimensionless parameter termed the *Weber number* (We) where,

$$\mathrm{We} = \frac{\mathrm{m\,a}}{\sigma\,\mathrm{l}} = \frac{\mathrm{V}^2\rho\;\mathrm{l}^2}{\sigma\,\mathrm{l}} = \frac{\mathrm{V}^2\rho\;\;\mathrm{l}}{\sigma} \qquad (1)$$

Occasionally the parameter is also expressed as the square root

$$\mathrm{We} = \sqrt{\left(\frac{\mathrm{V}^2\rho l}{\sigma}\right)} \qquad (2)$$

where V is the average velocity of the fluid system, ρ is fluid density, l is a length usually taken as average depth (d) and σ is surface tension. The number is occasionally expressed as the square root or even reciprocal of

Table 1 Comparison of the force / unit mass of the main forces acting in open channel flow (modified from Dingman, 1984, p.85).

	Width (m)	Depth (m)	Velocity ($m\ s^{-1}$)	Force/unit mass ($m\ s^{-2}$)			
				Gravitational	Turbulent	Viscous	Surface Tension
Model River	0.5	0.05	0.1	9.8	0.2	5.2×10^{-5}	6×10^{-3}
Model Braided River	0.03	0.009	0.247	9.8	6.8	5.4×10^{-3}	5.4×10^{-1}
Medium River	25	2	1	9.8	0.5	3.3×10^{-7}	3×10^{-6}

Note: Calculated as an average of 14 cross-sections in a 1:50 braided river model.

the ratio but general convention, and that adopted here, uses equation (1). Nevertheless care should be taken when interpreting Weber numbers because in some popular texts the square root version is used (e.g. Chadwick and Morfett, 1986) and some papers quote values without indicating the particular form of equation used. In this paper all values are calculated and reported with reference to equation (1). The Weber number is significant when the value is small, indicating that the forces of surface tension are large relative to the inertia forces. Given a constant surface tension, as the forces of inertia increase the Weber number also increases and surface tension forces are less important. As Rouse (1946) explains, just as the Froude number in open channel flow indicates the ratio of the velocity of flow to the celerity of a gravity wave, the Weber number represents the ratio of the velocity of flow to the celerity of a capillary wave.

This paper reviews the extent to which surface tension effects have been considered in small hydraulic models and whether it is possible to define critical conditions for which surface tension become important. The emphasis will be placed on small river models with mobile beds.

Surface Tension in Small Hydraulic River Models

Surface tension forces are often important at low flow velocities or in small hydrostatic systems where the surface area to volume ratio is large and the linear dimensions of the system are small (Fig. 1). These conditions could apply to small models of rivers or harbours and when operating conditions prevent the scaling-up of measurements (Massey, 1989). As Henderson (1966) states, surface tension effects will be appreciable only when the radii of curvature of the liquid surface, and the distances from solid boundaries, are very small. This means surface tension effects are negligible in rivers and similarly must be kept negligible in any hydraulic model. Maxwell and Weggel (1969) suggest the smallest scale of a Froude model is determined by a combination of fluid viscosity, surface tension, model roughness, accuracy of model construction, and the ability to collect accurate data.

Table 2 summarises various estimates of critical conditions, for when surface tension is significant, reported in the literature for small hydraulic models operating under a variety of hydraulic conditions. Defining a critical Weber number for small hydraulic models has to date been based on actual operating experience. For example Henderson (1966) advises that, as long as channel depths and widths are an inch or two (0.025-0.05 m), capillarity can be ignored. Novak *et al.* (1990) state *'For models of intakes* [where flows are shallow] *the Weber number should be greater than 11* [121, equation (1)] *and the Reynolds Number (Vd/u) greater than* 3×10^4 *to avoid surface tension and viscous effects.'* Novak and Cábelka (1981) suggest three minimum operating conditions to reduce surface tension effects:

(i) The length of a surface wave on the model is ≥ 0.017 m.
(ii) The velocity at the water surface of the model is ≥ 0.23 m s^{-1} so that gravity waves are free to develop.
(iii) The depth of flow in the model is > 0.015 m.

Using these conditions to calculate a Weber number results in a critical value of approximately 11 (Fig. 1). This approach was used in a recent paper by Warburton and Davies (1994) to examine surface tension effects in a 1:50 braided river model. Separate Weber numbers were calculated for the main and subsidiary channels in the model. For the main channels Weber numbers were approximately 16.9 (s.d. 5.4) and for the subsidiary channels values were 2.5 (s.d. 1.2), indicating that surface tension effects were likely to be important in the subsidiary channels (Table 2). The large range in calculated Weber numbers and the range of critical conditions described in Table 2 indicate there is considerable scope for improving the definition of the critical Weber number.

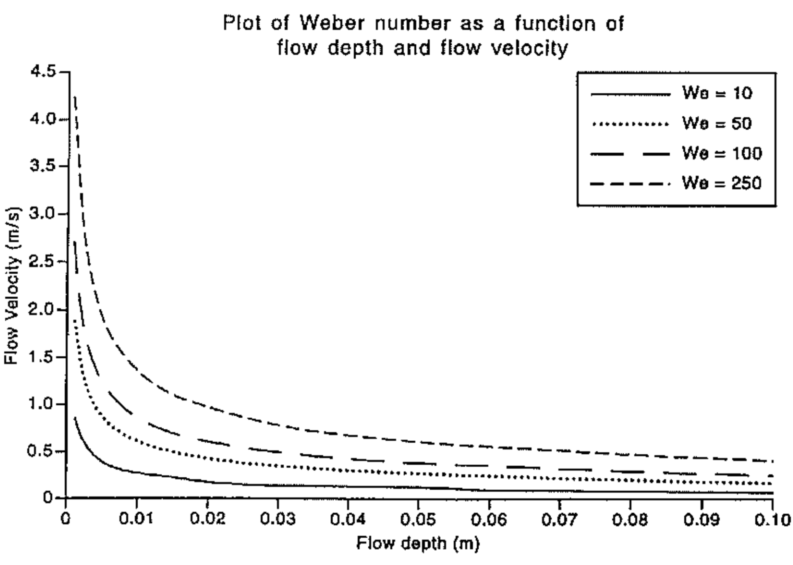

Figure 1 Plot of Weber number as a function of flow depth and flow velocity.

Table 2 Estimates of critical conditions for which surface tension becomes significant in small hydraulic models and calculated values for selected river model studies. Critical conditions are estimated from models of differing flow conditions and scales.

AUTHOR and YEAR	LIMITING VALUES			
	Surface velocity m s^{-1}	Surface wave wavelength m	Flow depth m	Weber number α
Critical Conditions				
Henderson, 1966			0.025-0.05	
Maxwell and Weggel, 1969				> 100 +
Novak and Cábelka, 1981	0.23	0.017	0.015	> 11
Dake, 1983				> 100
French, 1985			0.05	
Novak *et al.*, 1990				121 (> 11)
Estimates from Model Studies				
Schumm and Khan, 1972	0.238 - 0.732		0.0164 - 0.0518	29-160 Δ
Hoey and Sutherland, 1989	0.189 - 0.464		0.0060 - 0.0157	3.8-18 Δ
Ashmore, 1991	0.310 - 0.529		0.008 - 0.018	10.5-64.8 Δ
Warburton and Davies, 1994	0.134 - 0.362		0.0060 - 0.0170	2.5-17 δ

Notes: α All values calculated using equation (1), brackets indicate original values calculated using equation (2).

Δ No water temperature quoted. Weber numbers calculated assuming a value of 15°C

δ Mean values

\+ Maxwell and Weggel (1969, p.680)

Defining a Critical Weber Number

Surface tension is important when the solid boundaries of a liquid surface are in close proximity, or the surface separating two immiscible fluids has a very small radius of curvature (Massey, 1989). Therefore, surface tension affects the stability of liquid droplets, the formation of small (capillary) waves, the shape of small jets issuing from an orifice, the flow of thin sheets or jets over weirs, the break-up of a jet into droplets, capillary movement, and the hydrostatic equilibrium of liquids in porous media. Surface tension forces, which may be entirely negligible in prototype rivers, may be important in hydraulic river models. In calculating critical conditions for surface tension effects the Weber law has rarely been seen as a decisive criteria (Novak and Cábelka, 1981). However, two general approaches have been adopted based on the energy equation and capillary wave theory.

Energy Equation Approach

Maxwell and Weggel (1969) used a form of the energy equation that includes an additional surface energy term to demonstrate that the minimum energy flux (H) may be given by

$$H = d + \frac{q^2}{2gd^2} + \beta \frac{\sigma}{\rho g d} \qquad (3)$$

where g is acceleration due to gravity, q is the fluid discharge per unit width and β is the surface energy correction factor given by the ratio of the surface velocity to the mean velocity. Maxwell and Weggel (1969) make the simplifying assumption that β is equal to unity rather than to a lower value, and differentiating with respect to d gives a minimum energy flux per unit weight flux (H) for given q

$$\frac{q^2}{gd^3} = 1 - \frac{\sigma}{\rho g d^2} \qquad (4)$$

which can be restated in terms of the more familiar Froude (Fr) and Weber numbers

$$Fr^2 = 1 - \frac{1}{We} \qquad (5)$$

In order to determine a critical value at which surface tension effects become important the critical flow is assumed to be that for which the energy flux per unit weight flux is a minimum i.e. when the Weber number is unity

(Equation 5). Because g is a constant and r and s can be treated as constants, the critical depth (d_c) can be calculated as

$$d_c = \left(\frac{\sigma}{\rho g}\right) \tag{6}$$

The critical depth is therefore the value for which the unit discharge (q) would be zero, implying ponding of the fluid. In a Froude scale model the minimum value of q corresponds to a unit discharge high enough to maintain fully turbulent flow. Adopting a value for the Reynolds number of 2000, treating viscosity as a constant and assuming the flow depth equals the critical depth, a critical unit discharge can be approximated using the continuity equation for flow

$$q_c = Vd_c \tag{7}$$

By substituting q back into the energy equation the normal critical depth of open channel flow d_c can be calculated

$$d_c = \left(\frac{q^2}{g}\right)^{1/3} \tag{8}$$

The value of d_c can then be substituted back into the Weber number to yield a minimum value for the flow e.g. for water at 20°C and Re = 2000, d_c = 7.41 and We = 7.38. This is dramatically different from the value of 50 mm quoted by French (1985) as a critical depth to be free of surface tension effects. To a large extent this value depends on the choice of Reynolds number for fully turbulent flow. For example French (1985) uses a value of 12500 which equates with a critical depth of 25 mm and a critical Weber number of 85, which is a reasonably close approximation to the estimate of Dake (1983) that surface tension is insignificant if We > 100. These critical values can be compared with a plot of Weber numbers as a function of depth and flow velocity (see Figure 1).

However, there are a number of assumptions in the approach of Maxwell and Weggel (1969) which can be questioned. If equation (7) is substituted into equation (4) it gives

$$\frac{V^2}{gd} = 1 - \frac{\sigma}{\rho g d^2} \tag{9}$$

One variable (depth) is common to both sides of the equation. On the right hand side all parameters are constant except for depth, however altering depth on the right hand side will not change the left hand side from zero as long as velocity remains at zero. Substituting the Froude law $V^2 = gd$ into equation (9) so that the Weber number is expressed in its standard form, yields

$$\frac{V^2}{gd} = 1 - \frac{\sigma}{\rho V^2 d} \qquad (10)$$

There are now two expressions (in Equation 10) which are common to both sides and for the Weber number to be unity, both V and d must be greater than zero, and as a consequence the left-hand side cannot equal zero. Even if velocity is taken to be vanishingly small the form of the expression is changed.

Surface Tension Waves

The characteristic wavelength for the limiting condition when surface tension effects dominate is approximately 17 mm (Novak and Cábelka, 1981). The importance of surface tension can be determined by the parameter ζ (Acheson, 1990)

$$\zeta = \frac{\sigma k^2}{\rho g} \qquad (11)$$

where k (wave number) and λ (wavelength) are given by

$$k = \left(\frac{\rho g}{\sigma}\right)^{0.5} \qquad \lambda = 2\frac{\pi}{k} \qquad (12)$$

With λ much greater than 17 mm surface tension effects can be neglected. With λ less than 17 mm waves will be effectively surface tension waves in that the tensile force is the principal force involved in surface wave motion (Acheson, 1990). Using this approach it is unlikely a specific critical value can be determined, other than for the condition when waves move upstream, in a situation analogous to the Froude number.

Values given by Novak and Cábelka (1981) appear to be for the limiting condition between gravity (deep water) and capillary (surface tension) waves in open channel flow, which is defined by a wavelength of approximately 17 mm, the minimum speed of a gravity wave which is 0.23 m s^{-1} (Lighthill, 1978; Acheson, 1990), and a flow depth of 15 mm. However, justification for these guidelines is rather vague. For example, the velocity used is the minimum speed of both gravity and capillary wave and is incapable of distinguishing between the two. In other words if surface velocity is less than 0.23 m s^{-1}, waves will propagate upstream regardless of whether they are gravity or capillary waves. Choice of a critical depth of 15 mm is apparently arbitrary, although it is somewhat less than the condition where deep water waves cease to propagate e.g. wavelength = depth (17 mm).

Similarity in Models

In small models the equality of Weber numbers in the model and prototype is given by (Novak and Cábelka, 1981)

$$\frac{\rho_p V_p^2 l_p}{\sigma_p} = \frac{\rho_m V_m^2 l_m}{\sigma_m} \tag{13}$$

Assuming water to be the same fluid in the model and prototype ($\rho_p = \rho_m$; $\sigma_p = \sigma_m$; $M_\rho = 1$; $M_\sigma = 1$) the relation between the velocity and length scale is

$$M_v = M_l^{-1/2} \tag{14}$$

This has important implications for hydraulic similarity in models. For compliance with the Weber relation, Webber (1971) shows that the model velocities must be $x^{1/2}$ times the prototype velocities:

$$\frac{V_p}{V_m} = \frac{\sigma^{1/2}{}_p}{\sigma^{1/2}{}_m}\frac{\rho^{1/2}{}_p}{\rho^{1/2}{}_m}\frac{l^{1/2}{}_p}{l^{1/2}{}_m} = \frac{\sigma^{1/2}{}_p}{\sigma^{1/2}{}_m}\frac{\rho^{1/2}{}_p}{\rho^{1/2}{}_m}\frac{1}{x^{1/2}} \tag{15}$$

where subscripts p and m represent the prototype and model respectively. For example, using a model scale of 1:50 (common in many braided river models) a prototype velocity of 0.5 m s^{-1} would correspond to a velocity of 3.54 m s^{-1} in the model (0.5 x $50^{1/2}$). In hydraulic models where the fluid in the model and prototype is the same (e.g. water) normal practice is to scale the model in accordance with the Froude number and relax the Reynolds and Weber numbers. The Reynolds number is commonly relaxed because a critical threshold value can be approximated, however, for the Weber number, where no critical value has been defined, the effects of relaxing the Weber number are uncertain.

Discussion

Theoretical works on free surface vortex formation (Hughes, 1975; Yildirim and Jain, 1981) suggest surface tension effects are significant in model studies. However, experimental work has not investigated low enough Weber numbers for this to be observed (Daggett and Keulegan, 1974; Jain *et al.*, 1978). Therefore, there is no generally agreed value for the critical Weber number. In hydraulic modelling a pragmatic approach based on the similarity of dimensionless Froude, Reynolds and Weber numbers is usually adopted. Limits imposed by the physical properties of fluids mean models have only approximate mechanical similarity and this limits the scale of the model and determines the boundary conditions. Previous definitions of the critical Weber number for open channel flows have been based either on empirically derived 'rules of thumb' or on theoretical work which has significant limitations. Most small river experiments have Weber numbers that fall within or below the suggested range of critical values (10-120) (see Table 2) which suggests that a degree of surface-tension induced distortion may have been added to the models. In experimental channels of braided systems where the flow is shared between multiple channels of varying size, if hydraulic conditions are measured the tendency is to focus on the larger channels (e.g. Ashworth *et al.*, 1994) and few studies have made the distinction between channels (e.g. Warburton and Davies, 1994). Concentration on the larger channels stems from difficulties of measuring flow in shallow low-velocity channels. However, this is unfortunate because it is in these smaller channels that scale effects will have their greatest influence on flow i.e. Reynolds, Froude and Weber numbers would all be low.

In attempting to adjust surface tension in a model the options are few. Although surface tension decreases as temperature rises, the change is small and surface tension variation between common liquids is limited. Adding detergent is more effective than attempting to vary these other properties. Maxwell and Weggel (1969) and Karcz and Kersey (1980) have used additives in their experiments to reduce surface tension forces and this approach has been widely used in hydraulic models of weirs where flow depths and velocities are very small; although additives are generally of limited value (Maxwell and Weggel, 1969). The simplest means of minimising the effects of surface energy is to increase the scale of the fluid system so that inertia effects are greater than surface tension forces.

The reporting of Weber numbers from model experiments should therefore be more universally adopted, and for braided river models the hydraulics should be estimated for both the major and subsidiary channels in order that Weber numbers can be calculated and compared. A series of experiments must be carried out to determine the critical Weber number. The basis of this could be a flume study of surface wave propagation for various combinations of velocity and depth. A second flume study is also required to examine the changes in planform characteristics and sediment transport rates of a simple model reach, above and below the critical Weber number. Given these results, more robust criteria for the critical Weber number could be established and some estimate of the degree of surface-tension induced distortion made, for models or parts of models that have values below the critical. Recent advances in modelling small rivers to examine more specific properties, such as bedload transport rates, sediment pulses and three-dimensional alluvial architecture, make the study of such a fundamental parameter as surface tension all the more imperative.

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Notation

a	acceleration
β	surface energy correction factor (surface velocity / mean velocity)
d	average depth
d_c	critical depth
Fr	Froude number
g	acceleration due to gravity
ζ	surface tension parameter of Acheson (1990)
H	minimum energy flux
k	wave number
l	length
λ	wavelength
m	mass
M	length scale
q	fluid discharge per unit width
ρ	fluid density
σ	surface tension
s.d.	standard deviation
V	average velocity of the fluid system
We	Weber number
x	model scale

Subscripts

m denotes the model

p denotes the prototype

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