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CHARACTERIZATION OF SIMPLE EXPONENTIAL BASEFLOW RECESSIONS

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ABSTRACT

The simple exponential baseflow recession equation is usually characterized by the baseflow recession constant, k. The use of a more sensitive and meaningful measure of low-flow stream behaviour is advocated, and an alternative means of characterization is derived from consideration of the time taken for the stream discharge to halve.

INTRODUCTION

The baseflow recession curve is an important characteristic of the catchment and drainage system from which it is derived. As the demand for water has increased and water resource management techniques have been applied more widely, greater interest has been shown in baseflow recession curves as a means of portraying baseflow stream behaviour and determining minimum water yields and rates of depletion. In particular, baseflow recession curves may be used to assist with the allocation of water rights, the design of water supply schemes, the rostering of water users in water-deficient areas, and for testing the concept of representativeness before extrapolation of low-flow data from one catchment to another.

THE k VALUE

A large number of streams and rivers have baseflow recessions of the simple exponential type, and these recede according to the equation

$$q_t = q_0 e^{-at} \qquad (1)$$

where q_t is the discharge of the river at time t, q_0 is the intial discharge ($t=0$), t is the time lapse (usually in days) between q_t and q_0, and e^{-a} is a constant defining the rate of recession. It is common practice to replace this baseflow recession constant e^{-a}

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with k. Thus, equation (1) becomes

$$q_t = q_0 k^t \tag{2}$$

Equation (2) is the more usual form of the simple exponential baseflow recession equation and is characterized by the baseflow recession constant k. The value of k defines the rate of the baseflow recession but may only be regarded as constant for a particular recession, although in some 'well behaved' catchments all baseflow recessions may have the same value of k; only for such streams is it possible to derive a master baseflow recession curve and a master k value.

If a plot of log q_t against time is drawn for a stream whose baseflow recession is of the simple exponential type, a straight line results. The gradient of this straight line is readily evaluated by reading the graph, and can be seen to have the value of log k. Thus, by simple graphical means, the value of the baseflow recession constant can be obtained. No doubt it is for this reason that k values have come to be the commonly accepted means of characterizing recessions of the simple exponential form.

The range of values k may take is from zero to unity inclusive, but the values usually lie between 0.500 and 1.000, with a very distinct 'bunching' as k approaches unity (see Table 1). Because of this bunching effect and the compressed range of possible values, the baseflow recession constant is a relatively insensitive measure of recession rates of streams. This lack of sensitivity is compounded at higher k values in particular, by the fact that k can generally be evaluated only to two or three significant figures.

TABLE 1 — Some k values with their corresponding $t_{0.5}$ values ($t_{0.5}$ is the time taken, in days, for the baseflow to halve).

k	$t_{0.5}$ *(days)*	k	$t_{0.5}$ *(days)*
0.500	1.0	0.960	17
0.600	1.4	0.970	23
0.700	1.9	0.980	34
0.800	3.1	0.985	45
0.900	6.6	0.990	68
0.910	7.3	0.995	137
0.920	8.3	0.996	177
0.930	9.6	0.997	232
0.940	11	0.998	334
0.950	13		

A further disadvantage of the k value is that it has no real physical meaning. Most people (many hydrologists included) do not have an adequate concept of the rate of recession giving rise to any particular k value. Confusion generated in this way is added to by the bunched and compressed nature of the scale of k values.

AN ALTERNATIVE

Simple exponential decay is a common form of decay in nature. In nuclear physics the decay of elements by emission is governed by the law:

$$N_t = N_0 e^{-bt} \tag{3}$$

where N_t is the sample activity at time t, N_0 is the activity at time 0, and b is the probability of decay.

It can be seen that equation (3) is of the same form as the baseflow recession equation (1). However, the nuclear physicist does not substitute for the e^{-b} term as the hydrologist does, but profitably considers the time taken for the activity of the sample to decay to one half of its former activity. The same manipulative technique can be used on the baseflow recession equations (1) and (2) as follows.

If the 'half-flow period', $t_{0.5}$, is defined as the time required for the baseflow of a stream to halve, then by substitution into equation (2)

$$q_{t0.5} = q_0 k^{t0.5} \tag{4}$$

where $q_{t0.5}$ is the discharge at time $t_{0.5}$.
By definition of $t_{0.5}$

$$2q_{t0.5} = q_0$$

Substituting into equation (4)

$$q_{t0.5} = 2q_{t0.5} k^{t0.5}$$

Therefore $\quad \frac{1}{2} = k^{t0.5}$

Taking logarithms of both sides

$$\log \tfrac{1}{2} = t_{0.5} \log k \tag{5}$$

or $\quad t_{0.5} = \text{constant} / \log k.$

From equation (5) it can be seen that, provided the conditions for simple exponential recession are satisfied, the parameter $t_{0.5}$ has the following properties:

(a) It is independent of q_0 and q_t. Given k, $t_{0.5}$ can be calculated.

(b) It is independent of the time lapse between q_0 and q_t.

(c) It may take values in the range zero to infinity, and has increased sensitivity to change as it increases in value.

(d) It is easily evaluated. Log k can be obtained graphically, and it is then a simple matter to calcuate $t_{0.5}$ from equation (5).

(e) It is a direct measure of the rate of recession and can therefore be used as a means of characterizing the simple exponential baseflow recession equation.

(f) It is simply related to the k value by equation (5).

COMPARISON OF METHODS OF CHARACTERIZATION

The simple exponential baseflow recession may be characterized by either the baseflow recession constant or the half-flow period as defined above. Table 2 shows a comparison of the properties and characteristics of the two methods, and Table 1 shows their sensitivity with respect to one another.

TABLE 2 — Comparison of methods of characterization.

	Baseflow recession constant, k	*Half-flow period, $t_{0.5}$*
Range of values:	$0 \leqslant k \leqslant 1.000$	$0 \leqslant t_{0.5} \leqslant \infty$
Physical meaning:	Not readily understood. $k = q_{t+1}/q_t$	The time taken for streamflow to halve.
Tendency to bunch: (See also Table 1)	Severe bunching of values occurs as k approaches unity.	No bunching; values become more divergent as $t_{0.5}$ increases.
Ease of computation:	Log k determined, then antilog taken to give value of k.	Log k determined and then divided into log $\frac{1}{2}$ to give value of $t_{0.5}$.
Ease of computing q_t:	Substitute values in equation $q_t = q_0 k^t$.	Evaluate k from $\log k = (\log \frac{1}{2})/t_{0.5}$, then use equation $q_t = q_0 k^t$.

TABLE 3 — Values of k and $t_{0.5}$ for some New Zealand rivers. The k values are those of Waugh (1970).

River	k	$t_{0.5}$ *(days)*
Waipoua hydrological region 1968:		
Mangamuka	0.998	334
Kaihu	0.997	232
Mangakahia	0.996	177
Whangarei hydrological region 1968:		
Waihoihoi	0.997	232
Mangahahuru	0.996	177
Waipapa	0.989	62.7
Tirohanga	0.984	43.0

A comparison of methods is best made by considering some examples of possible usage of both characterizing parameters. Waugh (1970), in considering low-flow representativeness as indicated by the rate of baseflow recession, quotes k values for rivers in the Waipoua and Whangarei hydrological regions (Toebes and Palmer, 1969) as shown in Table 3. The corresponding $t_{0.5}$ values are also shown.

From the $t_{0.5}$ values in Table 3 it is apparent that a significant difference in recession rates exists between the three rivers in the Waipoua region. The extent of this difference in recession rates is not at all apparent from the k values. The same difficulty is evident with the four rivers in the Whangarei region. With these four rivers, it may seem reasonable to group them as a unit on the basis of their k values, but such a grouping does not appear to be nearly as suitable if the $t_{0.5}$ values are considered. Martin and Waugh, in an unpublished paper, showed that water supply and irrigation schemes in Northland should be able to outlast a 3-month drought. In the Whangarei region it is clear that the Waipapa and Tirohanga Rivers would not be particularly suitable for such schemes unless their initial discharges are well in excess of requirements, as their discharges would more than halve in a 90-day drought. This is, however, not apparent from the k values of these rivers.

If for a given stream under baseflow conditions the discharge q_0 is known, then to determine the discharge t days later, the value of k needs to be known for equation (2) to be used. However, if only $t_{0.5}$ and q_0 are known and q_t is required, then k must be determined from the relationship

$$k = (\log \tfrac{1}{2}) / t_{0.5}$$

in order to evaluate q_t.

Alternatively, by manipulation of equations (2) and (5), equation (6) below may be derived. Values of q_0, $t_{0.5}$ and t can be substituted directly into this equation to yield q_t.

$$\log q_t = \log q_0 + (t / t_{0.5}) \log \tfrac{1}{2} \qquad (6)$$

Singh and Stall (1971), in a brief review of baseflow research, mention the work of Barnes (1939) who from considerations of basin storage derived a characterizing factor similar to the half-flow period. This factor is termed the "storage delay factor", which is in effect the "time required for the discharge of a stream to decrease by a factor of 10 or one log cycle". Barnes' storage delay factor can be simply related to the half-flow period.

CONCLUSIONS

The simple exponential baseflow recession curve is a common form of recession curve and can be a powerful tool in low-flow analysis and representativeness studies. It is important that these baseflow recession curves be characterized in some manner to enable comparisons of river low-flow characteristics to be made. An attempt has been made in this paper to show that the *k* value is not a satisfactory means of characterization as it masks rather than reveals differences in recession rates, particularly for high *k* values.

An attempt has been made to show that the half-flow period is a satisfactory parameter for characterizing simple exponential baseflow recession curves and that it accentuates rather than masks differences in recession rates, particularly for slowly receding rivers. The half-flow period has the added advantage that it has a real meaning and is a readily understood concept.

For these reasons, the use of the half-flow period as a method of characterizing the simple exponential baseflow recession curve is strongly advocated.

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