

Toward multi-objective optimisation for Lake Taupō water level management

Earl Bardsley

*School of Science, University of Waikato, Hamilton, New Zealand.
Corresponding author: earl.bardsley@waikato.ac.nz*

Abstract

A multi-goal optimisation strategy may provide a basis for Lake Taupō water management. This involves defining an agreed 'optimal zone' of lake levels which are away from both high and low levels. The aim of water releases to the Waikato River would be to maximise the duration that lake levels are within the optimal zone, while also seeking to meet other, possibly conflicting, goals such as optimal hydro power operation.

A sense of how a multi-goal model could operate is given, by considering a simple dual-goal linear programming model that manages Taupō water levels on a daily time step, optimised toward (i) hydro power income and (ii) water level in the optimal zone. The model serves as an illustration only but the approach is readily capable of being expanded to a full model that includes further goals and constraints.

Keywords

Lake Taupō, multi-objective, lake level, optimisation, linear programming, hydro power, Waikato River

Introduction

Lake Taupō water levels have been controlled since 1941 when the Taupō Gates were constructed to regulate outflow to the Waikato River from the lake. Discharge rates are currently set by the power company

Mercury, to supply a chain of hydro power stations along the Waikato River. Outflows are also sometimes managed to reduce Waikato River flood peaks or to supplement discharge during times of low flow. The consented lake level operating range at present is 1.4 metres, between 355.85 and 357.25 metres above sea level (asl). The maximum possible outflow rate at the Taupō Gates is a function of lake level, with the maximum rate increasing linearly with lake level up to a discharge of around $300 \text{ m}^3\text{s}^{-1}$.

Lake level rise is unavoidable whenever inflows exceed the maximum possible outflow rate for the current lake level. However, this imbalance occurs only rarely, so water level rises mostly reflect management decisions to release water at a lesser rate than lake inflow. Similarly, lake level decreases occur if lake water is released at a greater rate than current inflows.

In the past there has been public concern that lake level management may contribute to lake shore erosion by way of extended durations of higher lake levels. That is, there may be greater shoreline erosion opportunity in the event of significant wind-induced waves coinciding with higher lake levels.

Sustained high Lake Taupō water levels, whatever the cause, offer no obvious environmental advantages. In addition to erosion effects, there is an increased flood risk in the event of major storm inflows into an already full lake. Extended periods of

high lake levels may also be associated with sediment accumulation near the mouths of some of the inflow rivers.

The present management of Lake Taupō requires the natural lake level regime to be maintained. However, this in itself places no constraints on day-to-day lake operations. It would seem, therefore, that there is scope for a more rigorous approach to Lake Taupō water level management that considers the needs of multiple users. In seeking to optimise toward multiple and possibly conflicting goals, it would be necessary to obtain a consensus decision to no longer to maintain the historic elevation-duration curve for lake levels. In fact, some degree of departure is already in place because post-1941 lake discharge control has resulted in extreme high lake levels being now less likely.

Given a permitted departure to a new operating regime, the question arises as to how to construct a lake operating system that reflects multiple user preferences. The operating system would need to be some form of algorithm-based continuous management, as opposed to simply operating within upper and lower consented levels.

The purpose of this brief communication is to give a sense of how alternative Lake Taupō outflow management might be carried out using multi-objective optimisation algorithms. The discussion here is for illustration only and there is no suggestion that the model involved should be used for practical application in its present form.

Two end-goals are considered for illustration: (i) maintain lake levels within a defined optimal range as much as possible, and (ii) water release to best achieve income from the electricity market (while allowing lake levels to fluctuate within the currently permitted range). The level of approximation is quite basic and the Waikato hydro power component is lumped into a single conceptual power station operating on a daily time step. However, the model is readily capable of

being expanded to a more realistic form that includes additional goals and constraints.

Methodology

Multi-objective optimisation tools do not appear to be used presently in New Zealand's controlled lakes. There are, however, many examples of the methodology in the international literature. Mohan and Raipure (1992) were early proponents of multi-objective linear programming methods. Other multi-objective applications include Casadei *et al.* (2016), Ghimire and Reddy (2014), Li and Qui (2016), Qi *et al.* (2017) and Malekmohammadi *et al.* (2011). An overview of different methods is given by Fayaed *et al.* (2013).

The optimisation approach used in this paper is linear programming (LP), configured to take the dual goals into account. Linear programming is conceptually simple: finding the maximum or minimum of a linear function subject to linear constraints. Setting up LP models is straightforward with modern software. Even quite large models can be quickly constructed and run using an Excel spreadsheet coupled with an LP Add-in.

With multi-objective LP, the linear functions associated with different goals are combined into a single linear function and the relative importance of the various goals is quantified by assigning different weights. An overview of LP applications is included in the review by Fayaed *et al.* (2013).

Lake systems are not amenable to LP optimisation if there are nonlinearities involved. However, the linear nature of Lake Taupō maximum outflows enables LP model discharges to be conveniently bounded by a linear function of lake water level.

Modelled electricity prices

For input to the LP optimisations for the hydro power income component, electricity prices were obtained initially as half-

hour values for the Hamilton node, from Wholesale Information Trading System (n.d.). These values were then converted to daily mean prices.

As a matter of interest, it is evident from Figure 1 that the lowest electricity prices are strongly associated with high Lake Taupō water levels. An initial interpretation might be that the high water levels reflect reduced water release to the Waikato River when electricity prices are low. While there will always be an element of price-related lake outflow management, the water level extremes appear more a reflection of hydrology than commercial operation. For example, the highest water level in Figure 1 was 357.3 metres on 23 September 2010, which was preceded by 17 consecutive days of lake inflows exceeding $300 \text{ m}^3\text{s}^{-1}$. Similarly, the low lake level associated with the highest daily price in Figure 1 (\$261 per MWh) occurred in June 2008. This was at the end of an extended period of low inflows and high wholesale electricity prices over the country.

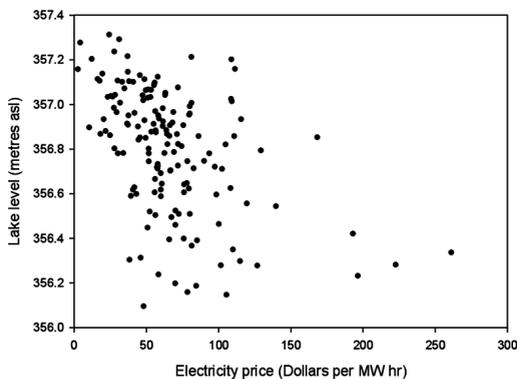


Figure 1 – Lake Taupō monthly maxima of daily mean water levels for 2005–2017, plotted together with the 24-hour average Hamilton electricity price on the day of the maximum.

Dual-objective LP model

The LP model utilised here is based on a 1-day time step. That is, water releases to the Waikato River are represented as daily mean

discharges. In practice, there is considerable within-day variation in water release rates through the Taupō Gates. However, daily means are sufficient for the level of approximation involved.

Development of any multi-objective model for Lake Taupō that incorporates a desirable water level range requires numerical specification of the range concerned. For the model utilised here, it is set as the 0.4-metre range extending from 356.5 to 356.9 metres asl. In practice, any optimal range would be decided as a consensus among lake users.

The hydro power component of the model is represented as a single power station located just below the Taupō Gates. ‘Income’ from this hypothetical station for a given day is set as the product of the day’s mean electricity price and model outflow discharge. The single station is obviously not a physical approximation to the multiple Waikato power stations with interactive operation. However, it serves as an indicative income-generating component for the purposes of the dual-objective illustration.

The selected time period for model application is 2013–18. This includes a period of low inflows in the first half of 2013, a period of elevated lake levels from mid-2016, and some high inflows associated with cyclonic rainfalls in 2017. The model’s hydro power income is quantitatively defined as $\beta = k / 1000$, where k is the sum of the (model discharge \times price) products over all of the days of the model simulation period.

With input for the simulation period being the recorded daily inflows and mean daily electricity prices, application of the dual-objective LP model determines a sequence of daily outflow values which give the maximum of:

$$a\beta + bP \quad (1)$$

where P is the proportion of time lake levels are within the optimal range and a and b are

positive weighting constants. The relative magnitudes of the parameters a and b in Eq. (1) determine weighting toward income generation and time spent within the desired water level range, respectively. The various a/b ratios as numerical values are not of importance for the simple approximate model used and are not listed here. In practice, multi-use model numerical weightings in an extended Taupō model will arise from consensus about the relative importance of various multi-use options.

The dual-objective model was further constrained such that all simulated outflows must be between 75 and $250 \text{ m}^3\text{s}^{-1}$. In addition, no model discharge can exceed the maximum discharge possible for a given lake level.

Application

Figure 2 shows the lake level output of the dual-objective LP model with weighting toward income generation. The weighting was chosen to give approximate similarity between the model and the actual Lake Taupō water level sequence over the simulation period, with the simulations constrained so

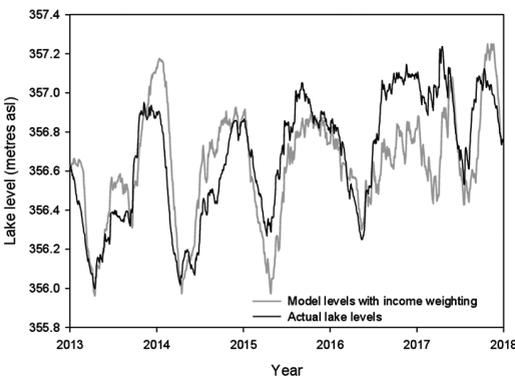


Figure 2 – Lake Taupō daily mean water levels 2013–2018, and lake levels as simulated from dual-objective optimisation with weighting toward income maximisation. Simulated levels here and in Figure 3 were constrained to match the actual levels at the end of 2017, but the two time series are otherwise independent.

that simulated water level equalled actual water level at the end of the simulation period. Such end-period constraints are required because otherwise the income optimisation component would tend to maximise income by unrealistically having minimal water storage at the end of the simulation period. That is, all possible stored water would have been utilised for electricity generation without regard to the future.

The similarities between actual and simulated lake levels in Figure 2 do not imply that the dual-objective LP model is accurately simulating the Waikato hydro system over 2013–18. However, it does suggest that the past record of Lake Taupō water levels reflects some degree of income maximisation, as would be expected from commercial hydro power operation.

If the LP model had instead been weighted to its full extent toward income maximisation, this would result in water levels often held at the upper or lower limits of the permitted range. This is because the LP model has perfect foresight of all coming inflows and electricity prices, so it is able to maximise income by sometimes forcing the lake toward extreme levels in a way that would never be possible or desirable in reality.

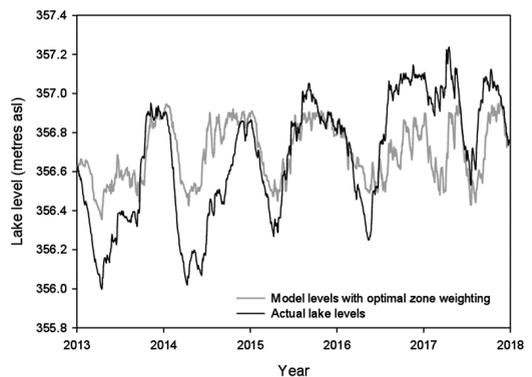


Figure 3 – Lake Taupō daily mean water levels 2013–2018, and lake levels as simulated from dual-objective optimisation with weighting toward achieving lake levels in the optimal zone (356.5 to 356.9 metres asl).

In contrast, Figure 3 shows the lake level sequence simulated by the same model but with weighting toward keeping water levels within the optimal zone, all other aspects being the same. This reduction of emphasis on β maximisation effectively removes both the high and low lake levels over the model period. Even greater weighting toward optimal zone water levels would actually make little difference to the final outcome. That is, almost all the simulated water levels are already within the optimal zone so there is little scope for further change. It might be argued that there is already too much weighting toward the optimal zone in this case, because extrapolating into the future implies a lake level elevation-duration curve almost entirely within the optimal zone.

The associated relative change in β values are of interest. The LP models of Figure 2 and 3 yielded β values of 20,457 and 19,898, respectively, so the weighting toward the optimal zone caused a 4.1% reduction in simulated hydro power income. Therefore, a lesser weighting might yield only a small reduction in hydro power income.

The present Lake Taupō management and the dual-objective LP models used here operate with very different available information. As mentioned, the simulation models have total foresight of daily inflows and electricity prices. Practical management, in contrast, must be based on dynamic, ongoing updates as information comes to hand, with only poorly defined estimates of future electricity prices and inflows.

Zero-foresight model

It might be thought that the dual-objective LP models provide minimal insight into possible real-world, multi-objective operation because these LP models have the advantage of all future information. To demonstrate that this is not the case, a simple dual-objective, 'zero-foresight' model was created from a few lines of code.

The same optimal zone and discharge constraints were applied as for the LP models. However, because the zero-foresight model operates only on the basis of current water level and current price, it was not possible to incorporate an end-period water level. A crude dual-goal optimisation procedure for the model was applied as follows: if a simulated lake level is within the optimal zone then lake water releases are proportional to the current electricity price for the day; and when simulated levels are above or below the optimal zone, water is released at a rate proportional to water level (high release rates for high water levels and vice versa).

The net effect is lowering the highest lake levels and raising the lowest. This simple model has no knowledge of current or forecast lake inflows, with its lake management water release decisions based only on current lake level and current electricity price.

The resulting β value is somewhat lower at 18,556, as would be expected from the basic mode of optimisation and no information of the future. However, the model is still able to reduce both low and high lake level extremes, including the 2016-17 period of extended high levels (Fig. 4).

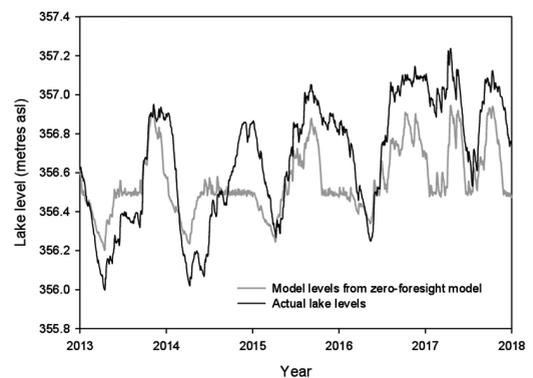


Figure 4 – Lake Taupō daily mean water levels 2013–2018, and lake levels as simulated from the zero-foresight dual-objective optimisation model, using only current lake levels and current electricity prices.

Discussion and conclusion

There is a long data record available to aid development of a more detailed model extending over a decade or more. It would be particularly interesting if on-going multi-objective algorithmic operation of Lake Taupō could be shown in hindsight to have resulted in less damage from the 1998 flood event.

It might be thought that changing the Taupō seasonal hydro storage regime could have negative implications in terms of national security of electricity supply. However, this is unlikely because there would be no reduction in mean power output from the Waikato stations. Also, there would be corresponding adjustments made in hydro storage elsewhere. For example, there seems no national hydro storage advantage in holding back winter river inflows into Lake Taupō while at the same time water is being released from South Island hydro lakes to supply power for North Island use.

Before any change to lake operation could be implemented there would need to be further studies on ecological impact, and ecology-related requirements would need to be incorporated into the operations. This might involve, for example, water management so that the lake is above certain levels for specified durations. There would also need to be consensus about moving the lake level elevation-duration curve away from the long-term situation. This would require a change in current consent conditions. Subject to such provisos, multi-objective optimisation for on-going Lake Taupō operation has potential to provide a helpful, formalised compromise between the values of different lake users.

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