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A CONTRIBUTION TO THE OBJECTIVE ASSESSMENT OF AREAL RAINFALL AMOUNTS

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ABSTRACT

The application of trend surface analysis to the assessment of the mean basin rainfall is discussed and results obtained by that method are compared with those determined by the more traditional approach.

PROBLEM

The ease of making 'accurate' point measurements of rainfall and the simplicity of methods for determining the mean depth of rainfall over a river basin are both very deceptive. For the conventional standard rain gauge, with its orifice in a plane at some arbitrary distance above ground level, provides an under-estimate of the amount of water reaching the earth's surface (Struzer *et al.*, 1965; Rodda, 1967a). Then the sampling problems are such that even the most dense network of gauges collects only a minute portion of the total amount of rain falling on a river basin; while for the usual type of network, vast areas are not sampled at all. In addition, the techniques of areal integration still rely very largely on the arithmetic mean or the Thiessen polygon methods, or on the subjective drawing of isohyets or isopercentiles (Whitmore *et al.*, 1961; Rainbird, 1967). These are, of course, simple well-tried methods that are usually adequate in most circumstances, although they tend to be employed without sufficient appreciation of their limitations.

It is true that there is now a greater awareness of the shortcomings of the standard type of rain gauge (Uryvaev *et al.*, 1965; Rodda 1967b). Based upon a better appreciation of the wind field over the gauge, some improvements have been made to design and installation, so that the effects of wind, the greatest source of error, are lessened (Robinson and Rodda, 1969). In the same way, networks are being extended and more rationally planned, while some fresh techniques have been developed for determining the mean from a number of point samples. Use has been made

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of graphical correlation (Spreen, 1947), regression analysis (Rodda, 1962), and variants of these approaches (Dawdy and Langbein, 1960; Sutcliffe and Carpenter, 1967) for a number of areas where relief exercises control over the distribution of rainfall. Mean basin precipitation has even been determined from the sum of measured runoff and estimated evaporation, for regions where it is virtually impossible to establish a satisfactory network of gauges, such as in the mountains of Norway and Sweden (Wallen, 1968). Radar could be employed for assessing the mean depth of rain as well as its distribution, if only the problems of calibration and integration were to be solved. But of course, there is no absolute standard of rainfall measurement and even the ground-level or pit gauge, which produces the most nearly true measurements, has not yet been widely used. Until the magnitude of the systematic error inherent in the standard gauge can be readily determined, for single gauges and for complete networks, there will always be doubt about the validity of assessments of the mean basin rainfall. In fact the true basin mean is not known and this is a point that is all too often overlooked.

Despite these problems, improvements are still needed, both in network design and in the methods employed for calculating the basin mean. In fact, for a number of purposes, including the mathematical modelling of river basins, it could be valuable to assess the mean rainfall by a straightforward method that combined, say, the objectivity of Thiessen polygons, with the verisimilitude of the isohyetal method. Should the technique permit the calculation of rainfalls at ungauged sites, rather than their interpolation subjectively, its value would be further enhanced.

Such advantages are offered by the use of a trend-surface analysis for assessing areal rainfall amounts. This is a method that is often employed in the geological sciences (Krumbein, 1959) and in meteorology (Panofsky, 1949; Bushby and Hackle, 1957) in connection with numerical forecasting, but less frequently in hydrology. A sampling model for rainfall with spatial co-ordinates was proposed by McCulloch (1961) in a study of rainfall for several East African catchments. Amorocho and Brandstetter (1967) used the method to represent precipitation patterns in storms over a basin in California. More recently Unwin (1969) employed the same approach in a study of rainfall variations over Snowdonia, North Wales. Excepting the investigations of precipitation and runoff in Newfoundland that were made by Solomon *et al.* (1968), few studies have been concerned with the actual use of trend-surface analysis for assessing areal rainfall amounts — the subject dealt with here.

APPROACH

Over the given interval of time (t), the total rainfall (P_1) recorded by a standard gauge at any point ($U_1V_1Z_1$) can be expressed as:

$$P_1 = \mathrm{f}(U_1V_1Z_1i_1) + B + \varepsilon$$

where f is a function of intensity (i_1), B the systematic error inherent in the standard gauge and ε the inevitable random errors of observation distributed about a mean of zero. Obviously B cannot be determined in normal circumstances, but it can be estimated from simultaneous comparisons of ground-level and standard gauges both individually and in networks (Rodda, 1970). Of course, the form of the surface defined by a group of point samples of rainfall would normally be too complex to analyse, especially for short periods. One approach is to assume that this surface (i.e. the trend surface) has the form of a simple polynomial, and to fit it with an expression of the type:

$$P = C_1 + C_2U + C_3V + C_4U^2 + C_5UV + C_6V^2 + C_7U^3 + C_8U^2V + C_9UV^2 + C_{10}V^3$$

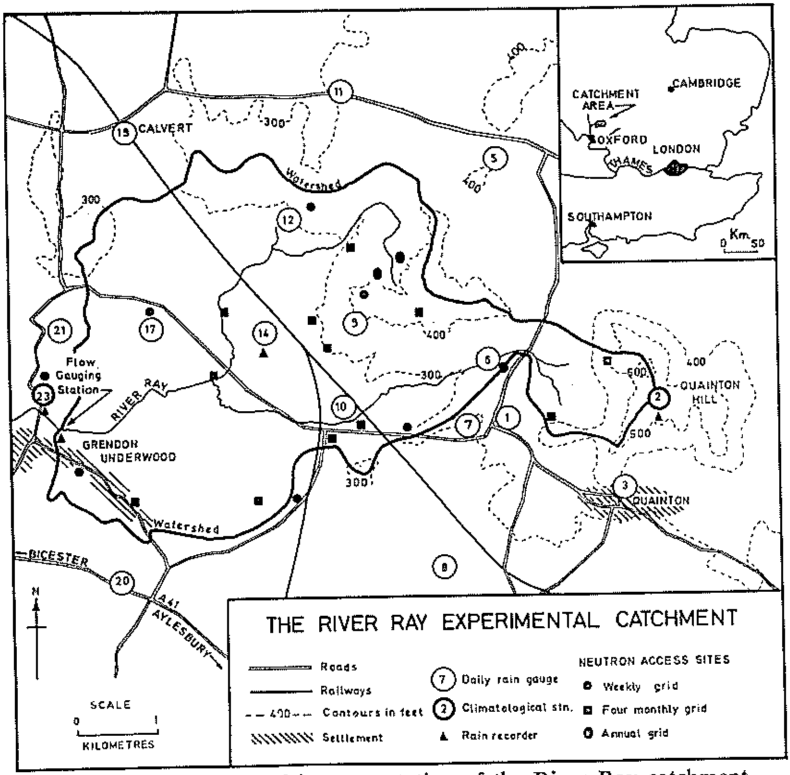

FIG. 1 — Location and instrumentation of the River Ray catchment.

where firstly $C_4, C_5 \ldots\ldots C_{10}=0$ in order to define the linear surface, then $C_7 \ldots\ldots C_{10}=0$ for the quadratic and lastly, for the cubic surface, all coefficients are included. The co-ordinates of the stations in the network (U_1V_1), (U_2V_2) .... (U_nV_n) and the corresponding rainfall records $(P_1)(P_2)$ . . . (P_n) are employed to calculate these coefficients by the method of least squares (Whitten, 1963). The goodness of fit of the surfaces can be assessed by several statistical tests and a study of the distribution of residuals can also reveal

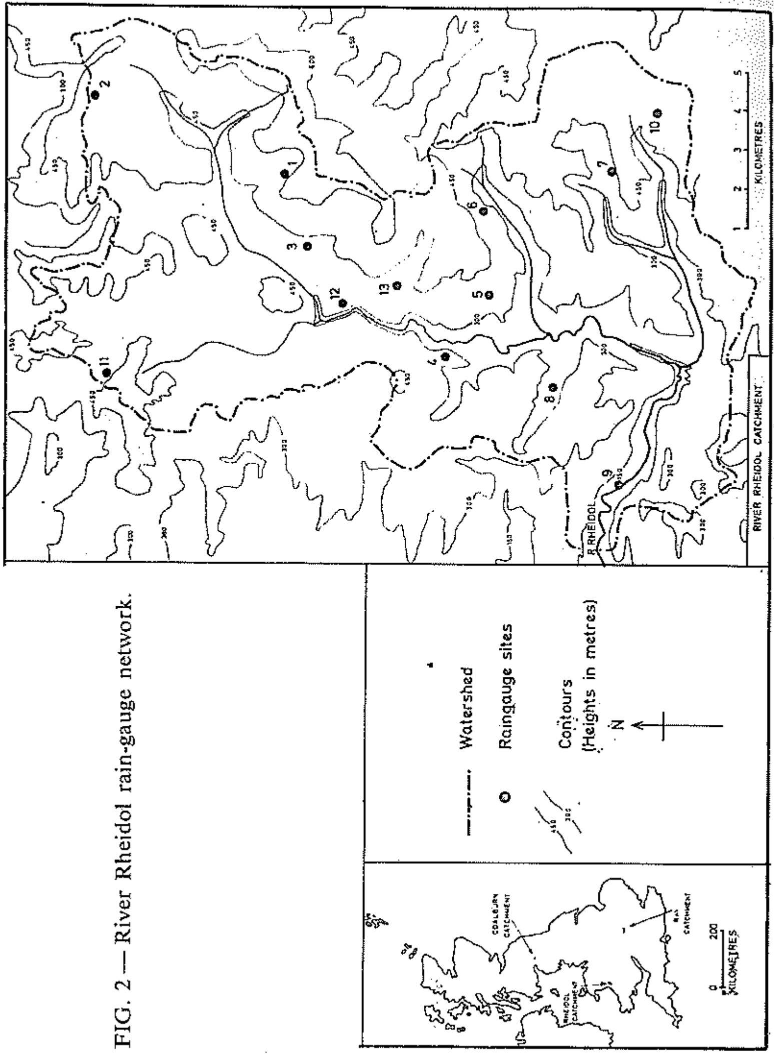

FIG. 2 — River Rheidol rain-gauge network.

anomalous values, discontinuities and like features that need further investigation.

APPLICATION

The method was applied to records from two standard rain-gauge networks established in representative basins in contrasting parts of the United Kingdom (Figs. 1 and 2). The River Ray catchment at Grendon Underwood (Hydrological Research Unit, 1966) is flat and dry by comparison with the basin of the River Rheidol, which is characterized by large amounts of rain and considerable relief differences. The first has an area of 18.6 km^2 and is sampled by 17 daily gauges of the usual British Meteorological Office pattern (30.5 cm tall with an aperture of 123 cm^2). The second basin is much larger (146 km^2) and was equipped with 13 monthly storage gauges of the same dimensions as those in the Ray catchment. For the purposes of this exercise grids were drawn on the catchment maps so that the co-ordinates of the gauges could be determined more easily, the spacing of the grid being 0.1 km for the smaller basin and 0.5 km for the larger one. Rainfall amounts registered by the gauges were recorded to 0.025 mm (0.01 in).

TABLE 1 — Information employed for the trend-surface analysis of River Ray rainfalls.

Gauge			*Annual totals (inches)*				
number	*Gauge*	*co-ordinates*	*1963*	*1964*	*1965*	*1966*	*1967*
2	47.0	80.0	24.39	19.56	29.47	31.13	29.67
3	58.0	78.0	25.71	19.23	28.45	30.60	30.38
4	32.0	69.0	23.76	20.82	27.94	31.22	27.66
5	16.0	62.0	24.74	21.33	29.00	31.44	26.99
6	41.0	61.0	23.27	20.56	27.87	31.16	28.03
7	48.0	68.0	23.40	18.88	26.65	29.81	27.02
8	66.0	55.0	22.58	18.83	27.42	30.11	28.65
9	36.0	44.0	23.05	20.52	26.96	30.73	26.81
10	47.0	43.0	23.77	21.03	27.41	32.00	28.27
11	9.0	44.0	24.73	20.69	28.39	29.69	24.50
12	23.0	18.0	23.25	20.37	27.37	29.93	25.70
14	38.0	34.0	23.27	20.88	27.10	31.32	26.73
17	36.0	22.0	23.93	19.80	28.12	31.41	26.40
20	68.0	18.0	23.37	18.88	26.33	31.03	28.52
21	11.0	38.0	21.68	19.12	27.25	29.63	26.60
23	8.0	44.0	22.72	18.53	27.55	30.36	27.07

Trend Surfaces

Trend-surface analyses were conducted with the different periods of record as follows:

(a) annually, for 5 successive years for the River Ray (Table 1);

TABLE 2 — Mean percentage sums of squares accounted for by different surfaces.

	Surface		
Analysis	*Linear*	*Quadratic*	*Cubic*
(a) R. Ray, annual	30	58	74
(b) R. Rheidol, monthly	52	70	80
(c) R. Ray, daily convective	29	44	69
(d) R. Ray, daily frontal	28	46	73

(b) monthly, for 21 successive months for the River Rheidol;

(c) daily, for 8 days with rain of a convective origin for the River Ray;

(d) daily, for 8 days with rain of a frontal origin for the River Ray.

The results of these sets of analyses are shown (Table 2) as the mean of the percentage variance accounted for by each of the three surfaces. As might be expected, the best fit was obtained with polynomials of the highest degree. Results for some individual months for the Rheidol showed that a remarkably high percentage of the variance was being accounted for; the figure of 95% being exceeded in three cases. In contrast, the fit of the surfaces was less successful in the Ray, even when the daily falls were grouped according to the origin of the rain. The reverse might have been expected in view of the wider variation in rainfall across the Rheidol basin and the greater likelihood of much larger gauge errors. On the other hand, this result could also be taken to indicate the existence of an easily defined pattern of rainfall distribution in the Rheidol as opposed to its absence from the Ray — a pattern which would be largely governed by relief.

To calculate the areal rainfall (mean basin rainfall) for each catchment, the first step was to determine the sets of co-ordinates corresponding to the two watersheds by means of the appropriate grid. Next, the rainfall predicted by the three surfaces was calculated for each grid intersection point within the watersheds. Then the

TABLE 3 — Areal rainfall estimates by different methods.

(a) River Ray, annual totals (in.)

	Linear Surface	*Quadratic Surface*	*Cubic Surface*	*Arithmetic Mean*	*Thiessen Polygon*	*Isohyets*
1963	23.48	23.41	23.69	23.70	23.44	24.63
1964	19.76	20.32	20.29	19.95	20.10	20.09
1965	27.23	27.29	27.51	27.59	27.45	27.27
1966	30.63	31.06	31.21	30.65	30.93	31.13
1967	27.14	27.03	27.07	27.16	27.10	26.94
5-year mean	25.65	25.82	25.95	25.81	25.80	26.01

TABLE 3 (continued)

(b) River Rheidol, monthly totals (in.)

Month	*Linear Surface*	*Quadratic Surface*	*Cubic Surface*	*Arithmetic Mean*	*Thiessen Polygon*	*Isohyetal Method*
0266	6.05	6.18	7.36	6.12	6.29	6.57
0366	4.53	4.53	4.16	4.58	4.65	4.36
0466	4.68	5.02	3.94	4.73	4.75	4.45
0566	5.70	5.63	5.22	5.74	5.75	5.66
0666	8.88	8.91	8.63	8.96	8.99	8.56
0766	6.43	6.32	6.31	6.47	6.56	5.98
0866	3.77	3.89	3.23	3.79	3.67	3.90
0966	4.59	4.73	4.31	4.65	4.75	4.39
1066	6.79	6.52	6.60	6.79	6.69	6.44
1166	7.58	7.87	8.03	7.65	7.83	7.07
1266	15.96	15.51	14.58	16.08	16.01	16.07
0167	5.65	5.55	4.74	5.71	5.63	5.34
0267	6.45	6.43	5.59	6.52	6.49	6.17
0367	4.35	4.56	4.12	4.36	4.37	4.30
0467	3.49	3.37	2.65	3.52	3.39	3.05
0567	8.39	8.35	7.84	8.44	8.28	7.98
0667	3.25	3.23	2.46	3.26	3.14	3.12
0767	8.38	8.84	9.98	8.44	8.85	8.45
0867	7.02	6.77	7.41	7.09	7.01	6.97
0967	10.38	10.44	11.32	10.46	10.81	10.43
1067	15.42	15.31	15.10	15.57	15.78	15.52
21-mth total	147.74	147.96	143.58	148.93	149.69	144.78

(c) River Ray, daily totals (in.)

Date	*Linear Surface*	*Quadratic Surface*	*Cubic Surface*	*Arithmetic Mean*	*Thiessen Polygon*	*Isohyetal Method*
	Convective					
21/ 7/64	2.31	2.68	2.46	2.31	2.48	2.60
14/ 7/65	0.41	0.40	0.46	0.39	0.41	0.37
20/ 2/66	0.37	0.37	0.38	0.37	0.37	0.37
16/ 6/66	0.34	0.32	0.37	0.31	0.33	0.33
4/10/66	0.25	0.25	0.23	0.24	0.25	0.25
21/ 3/68	0.15	0.15	0.15	0.16	0.15	0.14
13/ 5/68	0.21	0.22	0.23	0.22	0.22	0.21
9/ 8/68	0.29	0.30	0.31	0.32	0.30	0.30
	Frontal					
1/ 6/64	0.50	0.51	0.51	0.53	0.51	0.50
26/ 4/65	0.34	0.33	0.34	0.32	0.33	0.33
17/ 9/65	0.26	0.25	0.26	0.26	0.26	0.25
21/ 1/65	0.26	0.27	0.29	0.25	0.27	0.28
18/ 4/66	0.49	0.49	0.51	0.49	0.49	0.48
2/10/66	0.51	0.52	0.50	0.51	0.52	0.52
9/12/66	0.66	0.67	6.68	0.65	0.66	0.68
14/ 5/67	0.91	0.90	0.92	0.89	0.91	0.91

mean rainfall was obtained, by summing these calculated values, weighting the ones located within a grid square of the watershed to take into account the smaller area that these points represented. The results are shown in Table 3 for comparison with those obtained by the more traditional methods.

Plotting Isohyets

The isohyets predicted by the various trend surfaces can be produced automatically by a contour plotting programme (Gilbert, 1965). Such a programme was modified and applied to the trend surfaces that had been computed for the River Ray annual totals. The results for one year are shown (Fig. 3), along with the isohyetal map drawn by the traditional method. The three predicted surfaces display the same gradient from north-west to south-east that appears in the hand plot, the isohyetal pattern in that map being most closely reproduced by the cubic surface.

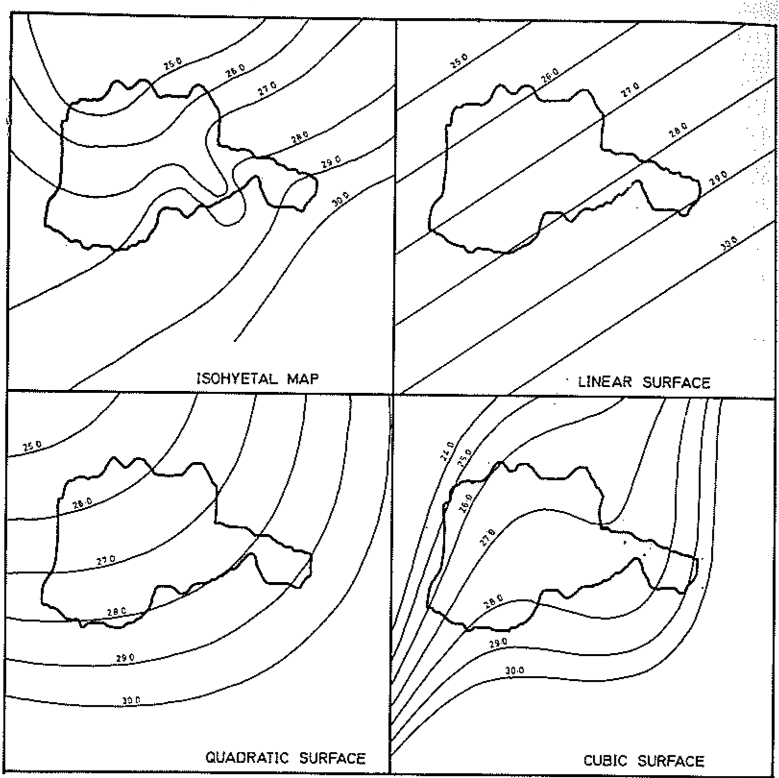

FIG. 3 — Isohyets predicted by three trend surfaces compared with isohyetal map produced by traditional method.

DISCUSSION

Some established methods of estimating the mean basin rainfall are extremely simple; others demand the use of a considerable degree of skill and judgement on the part of the user. In fact, the plotting of isopleths of one sort or another requires several assumptions to be made (Institute of Water Engineers, 1958), for instance about the effects of relief on the distribution of rainfall, assumptions that are different from those involved when the arithmetic mean and Thiessen polygon methods are employed. In these cases the juxtaposition of similar amounts of rainfall is implied. Hence their use is largely confined to the flatter areas where rainfall gradients are not marked. Assumptions like these are far less vital in the case of trend-surface analysis, although of course attention has to be given to certain statistical matters. For example, for a small number of gauges, a low-order polynomial must be used. This is because a surface of sufficiently high order could be made to fit completely to the observed point values, leaving no degrees of freedom for determination of goodness of fit. On the other hand, this method does provide a measure of this goodness of fit, to the observed values — something that the isohyetal method lacks. Its other main advantage is, of course, the objective plotting of isohyets, an extremely valuable facility in a number of circumstances.

The errors inherent in the rain gauge are usually ignored in the application of techniques for assessing mean basin rainfall. Corrections for these errors could cause substantial alteration to patterns of rainfall distribution, especially in areas like the Rheidol. In fact, records from certain gauges for some of the months suggested that highly anomalous readings were responsible for the poor fit of the trend surface. So several trials were made omitting the suspect gauges (Table 4); the accountable variance was increased and the mean basin rainfall estimate was made to agree more closely with the other estimates. The improvement was most

TABLE 4 — Cubic surfaces fitted to Rheidol records with anomalous gauge readings omitted.

	Percentage sums of squares accounted for					
Month	*All gauges*	*Ex. 6*	*Ex. 11*	*Ex. 13*	*Ex. 13+11*	*Ex. 13+6*
8/66	39.6	48.7	55.0	94.7	94.5	94.8
1/67	72.9	73.5	76.0	96.8	98.3	98.7
2/67	81.0	82.1	83.8	97.6	98.1	98.3
4/67	62.1	62.3	66.8	92.8	96.0	96.4
6/67	20.8	26.1	32.0	69.6	94.3	97.2

	Rheidol mean basin rainfall (in.)				
Month	*By isohyets*	*By cubic surface*	*Cubic ex. 13*	*Cubic ex. 13+11*	*Cubic ex. 13+6*
8/66	3.90	3.23	3.57	3.72	3.67
1/67	5.34	4.74	5.06	7.59	5.76
2/67	6.17	5.59	5.88	7.54	6.38
4/67	3.05	2.65	2.89	4.72	3.39
6/67	3.12	2.46	2.88	5.98	3.65

marked when gauge 13 was left out, probably because its position on the edge of the highest area facing the prevailing south-westerly winds would cause its reading to be appreciably larger than those of its neighbours.

CONCLUSION

Trend-surface analysis offers a useful objective alternative to the other more commonly used methods for determining areal rainfall amounts. In fact, for research in representative and experimental basins it appears to be the most expedient method to use, and certainly preferable to the Thiessen polygon method which has been favoured in the past (Corbett, 1967). This is an approach that is worthy of greater attention for general use in hydrological studies, but it should not be taken as the panacea for all the perplexities involved in assessing mean basin rainfall.

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