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SEEPAGE CHARACTERISTICS OF COARSE GRANULAR MEDIA

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ABSTRACT

The coefficients of nonlinear seepage flow can be estimated from a reformulation of the Forchheimer equation, using grain diameter, porosity and fluid properties. This avoids the use of permeameter tests in preliminary investigations of seepage in coarse media. The suggested semi-empirical procedure agrees satisfactorily with data from a range of tests on various coarse ganular media.

INTRODUCTION

Seepage flow through coarse porous materials such as river-bed gravels, rockfill dams, slurry separators, and fish incubators is governed by a nonlinear relationship between hydraulic gradient i and discharge velocity v:

$$av + bv^2 = i \qquad (1)$$

in which a is linear hydraulic resistivity and b is nonlinear hydraulic resistivity (Gill, 1977). The relationship is nonlinear, even at small values of i (Dudgeon, 1964).

The values of a and b are usually obtained from permeameter tests on the granular materials involved. Wall effects caused by the relatively large grain-size can be overcome by modifying the apparatus (Dudgeon, 1964; Zagni, 1974). A method is proposed herein by which a and b can be estimated from the grain diameter and porosity of the seepage medium, thus avoiding the difficulty of permeameter tests, at least in the early stages of an investigation. This method is based on data from permeameter tests of clean river gravels, and is tested with published data for a wide range of granular media.

ANALYSIS

Ward (1964) and Ahmed and Sunada (1969) showed that equation (1) can be written in the form

$$\frac{\mu v}{\rho g k} + \frac{C_2}{g k^{\frac{1}{2}}} v^2 = i \qquad (2)$$

where μ = dynamic viscosity, ρ = fluid density, g = gravitational acceleration, k = intrinsic permeability, and C_2 is a dimensionless coefficient. The hydraulic resistivities a and b can then be written as

$$a = \frac{\mu}{\rho g k} \tag{3}$$

$$b = \frac{C_2}{g k^{\frac{1}{2}}} \tag{4}$$

a is the reciprocal of permeability K, and depends on fluid properties (ρ and μ) and medium properties expressed by the intrinsic permeability k (Bear, 1972):

$$k = Cd^2 \tag{5}$$

where d = grain diameter, and C is a dimensionless coefficient involving all properties of the medium that affect the flow except d. If C and C_2 can be estimated from values of d and porosity f, then a and b can be obtained from equations (3), (4) and (5). Bear (1972) showed that

$$C = \frac{f^3}{180\,(1-f)^2} \tag{6}$$

where 180 is an empirical factor: this relationship was confirmed by Beavers *et. al.* (1973) for a bed of randomly packed spheres with f = 0.37 (C = 7.09×10^{-4}).

The nonlinear hydraulic resistivity b depends on the medium properties C_2 and k (equation 4). McWhorter and Sunada (1977) showed that C_2 lies in the range 0.05 – 2.0, with the majority of values between 0.5 and 0.6. Beavers *et. al.* (1973) found empirically that

$$C_2 = 0.55\,(1 - 5.5d/D) \tag{7}$$

where D = permeameter diameter. This implies, however, that the maximum possible value of C_2 is 0.55 which is in conflict with the findings of McWhorter and Sunada (1977).

Joseph *et. al.* (1982) formulated C_2 as

$$C_2 = 0.05\,[r(1-f)]^{\frac{1}{2}} \tag{8}$$

from the equation for drag on a sphere in nonlinear laminar (Oseen) flow, where r is defined by Brinkman (1947) for flow through a swarm of spheres as

$$r = \left(1 + \tfrac{3}{4}\,(1-f)\left[1 - \left(\frac{8}{1-f} - 3\right)^{\frac{1}{2}}\right]\right)^{-1} \tag{9}$$

Equation (8) is said to be valid for spheres which are not too closely packed, i.e. when the porosity is greater than about 0.6; with f = 0.34, however, equation (8) gives C_2 = 7.02 which seems unreasonable in the light of the experimental maximum value of about 2.0.

In principle, equations (6), (7) and (8) provide sufficient information to allow a and b to be calculated from d and f: however, although equation (6) is quite reliable in predicting C, equations (7) and (8) do not seem capable of predicting C_2 at all relevant values of f, and an improved predictor is required for C_2.

Rumer (1969) derived Darcy's Law using channel flow resistance as an analogy, and obtained

$$v = \frac{Cd^2\rho gi}{\mu} \tag{10}$$

$$\text{and } C = \frac{\beta f^2}{m(1-f)} \tag{11}$$

where β is a proportionality constant relating particle volume to diameter (volume $= \beta d^3$, i.e. for a sphere $\beta = \pi/6$); m is a constant depending on the physical properties of the particle, including the effect of surrounding particles (for a sphere in an infinite fluid, $m = 3\pi$). Note that equation (11) is different from equation (6) because the latter was derived using hydraulic radius (Bear, 1972 p. 166).

Equation (11) was derived using the particle drag coefficient C_D in the Stokes' range. The following development extends the argument to the non-linear laminar (Oseen) range of flows past a particle.

In a volume element of flow through a porous medium, balancing the pressure, body, and resistance forces (Fig. 1) gives

$$pfdA - (p + \frac{dp}{dl}dl)\, fdA = \rho gfdAdl\frac{dz}{dl} + F \tag{12}$$

where p = pressure, dA = cross-sectional area of element, dl = element length, dz = vertical distance and F = resistance force. If the hydraulic head h is defined by

$$h = \frac{p}{\rho g} + z \tag{13}$$

then with ρ and g assumed constant, equation (12) can be simplified to

$$\frac{F}{\rho gfdAdl} = -\frac{dh}{dl} \tag{14}$$

The drag equation for flow past a single particle can be written as

$$F_D = \tfrac{1}{2}C_D U^2 \rho A_p \tag{15}$$

where F_p = resistance force of a single particle, C_D = drag coefficient, U = fluid velocity, and A_p = frontal projected area of particle. For flow past a particle, just beyond the valid range of Stokes' Law ($0.1 \leqslant R \leqslant 1$) where inertia can no longer be neglected, Bear (1972, pp. 167–168) shows that

$$C_D = \frac{2md^2}{A_p}\left(\frac{1}{R} + C_4\right) \tag{16}$$

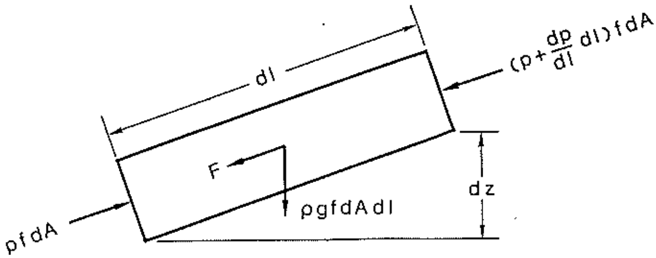

FIG. 1—Forces on a volume element of flow through a porous medium.

where R = Reynolds' number and C_4 = a constant including the effect of surrounding particles (for a sphere in an infinite fluid $C_4 = 0.1875$).

By substituting C_D from equation (16) into equation (15) we obtain

$$F_p = md^2U^2\rho \left(\frac{1}{R} + C_4\right) \tag{17}$$

The resultant force, F, exerted by all particles, N, in the element is NF_p. The number of particles N can be calculated from

$$N = \frac{(1-f)\, dAdl}{\beta d^3} \tag{18}$$

Hence, from equation (17)

$$F = \frac{mU^2\rho}{\beta d}(1-f)\left(\frac{1}{R} + C_4\right) dAdl \tag{19}$$

From equation (19), by substituting F into equation (14), replacing R by $Ud\rho/\mu$ and U by v/f, we obtain

$$m\frac{(1-f)}{\beta f^2}\frac{v^2}{gd}\left(\frac{\mu}{\rho dv} + \frac{C_4}{f}\right) = -\frac{dh}{dl} \tag{20}$$

The quantity $\frac{\beta f^2}{m(1-f)}$ depends on only the geometrical structure of the porous medium and can be replaced by C as in equation (11). Likewise, C_4/f can be replaced by C_1 giving

$$\frac{\mu}{\rho g C d^2}v + \frac{C_1}{gCd}v^2 = -\frac{dh}{dl} = i \tag{21}$$

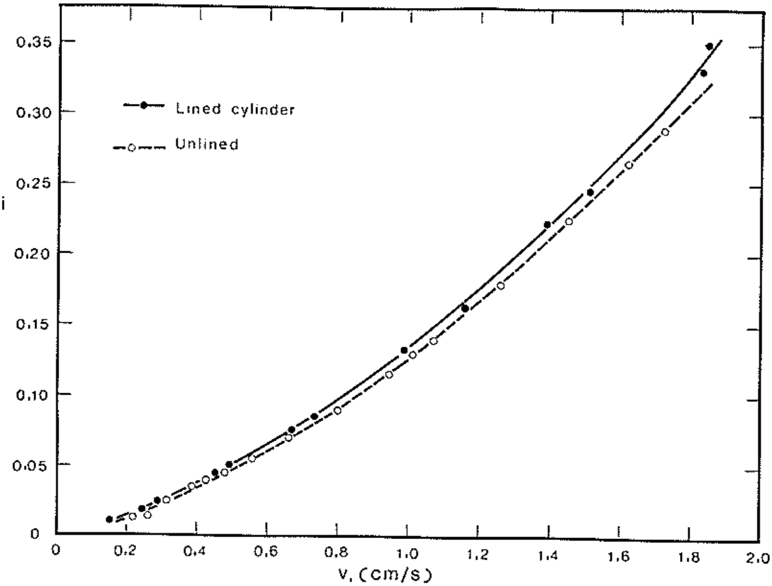

FIG. 2—Effect of lined and unlined walls on hydraulic gradient (i) and discharge velocity (v).

and using equation (5)

$$\frac{\mu}{\rho g k} v + \frac{C_1}{g(Ck)^{\frac{1}{2}}} v^2 = i \qquad (22)$$

The nonlinear hydraulic resistivity b can be written from equations (1) and (22) as

$$b = \frac{C_1}{g(Ck)^{\frac{1}{2}}} \qquad (23)$$

C_2 is now obtained from equations (4) and (23)

$$C_2 = C_1/(C)^{\frac{1}{2}} \qquad (24)$$

Thus C and C_2 can be calculated from equations (11) and (24) respectively if the values of β/m and C_1 are known.

Experiments were therefore performed to study the variation of β/m and C_1 with d and f for clean river gravels.

EXPERIMENTS

A standard Darcy-type permeameter was used, with inside diameter 43.6 mm, vertical distance between manometer tappings 200 mm, and sample length 295 mm. The samples comprised five samples of closely-graded gravel of different diameters, and five samples of widely-graded gravels of similar mean diameters; the ranges of mean diameter d_m and standard deviation σ were chosen to be as wide as possible with the available apparatus, i.e.

$$3.4 \text{ mm} \leqslant d_m \leqslant 10.4 \text{ mm}$$

and

$$0.38 \leqslant \sigma \leqslant 3.12 \quad .$$

The effect of increased porosity at the cylinder walls was overcome by lining the walls with thin sponge-rubber sheet: the correction to i and v was about 3% to 5% (Fig. 2), which can be compared to the error of 5% to 10% found by Dudgeon (1967) with a special permeameter using larger particles. The flow through the sponge sheet was measured to be negligibly small. The problem of air coming out of solution and causing a progressive decrease in sample permeability was overcome by using water which had been allowed to stand at atmospheric pressure and room temperature for 24 hours; this was found to be just as satisfactory as de-airing the water by heating it or subjecting it to reduced pressure (Bouska, 1973).

CALCULATION OF HYDRAULIC RESISTIVITIES

Hydraulic resistivities a and b are calculated from a series of hydraulic gradient-discharge velocity relationships This can be done from equation (1) using a quadratic least-squares method (Gill, 1977). Sunada (1965) and Ward (1966), however, preferred to recast equation (1) in the form

$$a + bv = i/v \qquad (25)$$

from which a and b can be calculated by linear least-squares regression. Subsequently Cox (1977) has suggested that this latter procedure is inferior to the algebraic solution of equation (1) using the two most suitable sets of i and v. In the present study, all three methods were used and the best result was chosen on the basis of least standard error, using, in addition, a temperature correction. Since a depends mainly on fluid viscosity μ, it is sensitive to temperature; the value of a at any temperature t can be corrected to the corresponding value at a standard temperature s by

$$a_s = \left(\frac{\mu_s}{\mu_t}\right) a_t \qquad (26)$$

where the subscripts indicate temperatures. The effect of temperature on ρ is insignificant; inaccuracies could result, however, if water temperature changes which occur during a test (due, for example, to changes in air temperature or heating by a pump) are not specifically accounted for.

In the present tests, discharge velocities measured at some temperature t (v_t) were converted to a standard temperature s before calculation of a and b. From equation (1) and (26)

$$v_s = \left(\frac{e^2 \mu^2_s}{\mu_t^2} + \frac{i}{b}\right)^{\frac{1}{2}} - e \frac{\mu_s}{\mu_t} \qquad (27)$$

and

$$e = (i - b v^2_t) / 2 b v_t \quad . \qquad (28)$$

TABLE 1—Some properties of gravel samples.

Mean diameter d_m (mm)	Standard deviation, σ	Drainable porosity, f	Linear hydraulic resistivity, a, (s/cm)	Nonlinear hydraulic resistivity, b, (s^2/cm^2)	Intrinsic permeability, k, (cm^2)	Coefficient C	Constant C_1
Closely-graded gravels							
3.4	0.60	0.384	0.0957	0.0674	1.07×10^{-4}	9.26×10^{-4}	0.0208
4.4	0.38	0.375	0.0698	0.0634	1.47×10^{-4}	7.65×10^{-4}	0.0208
5.4	0.62	0.377	0.0492	0.0466	2.08×10^{-4}	7.20×10^{-4}	0.0177
8.8	0.77	0.395	0.0148	0.0259	6.90×10^{-4}	8.98×10^{-4}	0.0200
10.4	0.84	0.400	0.0087	0.0160	11.82×10^{-4}	11.00×10^{-4}	0.0179
Widely-graded gravels							
6.7	1.82	0.353	0.0437	0.0471	2.35×10^{-4}	5.29×10^{-4}	0.0163
6.4	2.13	0.351	0.0491	0.0455	2.08×10^{-4}	5.15×10^{-4}	0.0146
6.8	2.63	0.347	0.0513	0.0490	2.00×10^{-4}	4.32×10^{-4}	0.0141
6.4	2.95	0.337	0.0768	0.0734	1.33×10^{-4}	3.24×10^{-4}	0.0150
6.8	3.12	0.336	0.0836	0.0888	1.23×10^{-4}	2.66×10^{-4}	0.0157

TABLE 2—Average values and standard deviations of f, C and C_1.

	Drainable porosity, f	Coefficient, C	Constant, C_1
	Closely-graded gravels		
Average value	0.386	8.82×10^{-4}	0.0194
Standard deviation	±3%	±15%	±7%
	Widely-graded gravels		
Average value	0.345	4.13×10^{-4}	0.0151
Standard deviation	±2%	±25%	±5%

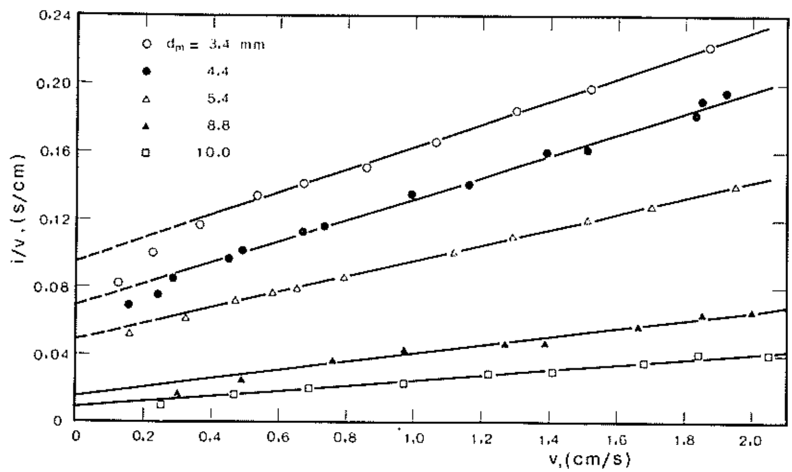

FIG. 3—Relationship of i/v to v for closely-graded gravels.

Equation (27) has to be applied after a value of b (which is not sensitive to temperature) has been approximated from the uncorrected i-v set. The computation steps in the present programme are:

(i) a and b are computed from the raw data by all three methods and the best value of b is selected using the least standard error;

(ii) values of v_t are converted to v_s using equation (27), and

(iii) improved values of a and b are computed from the refined data using all three methods.

From the ten sets of data in Figs. 3 and 4, at a standard temperature of 20°C, the algebraic method gives the least standard error in six tests and the linear squares method in four tests.

EXPERIMENTAL RESULTS AND DISCUSSION

The results of the present tests are presented in Tables 1 and 2 and are shown in Figs. 3 and 4. The standard deviation of C_1 (Table 2) is small (7%

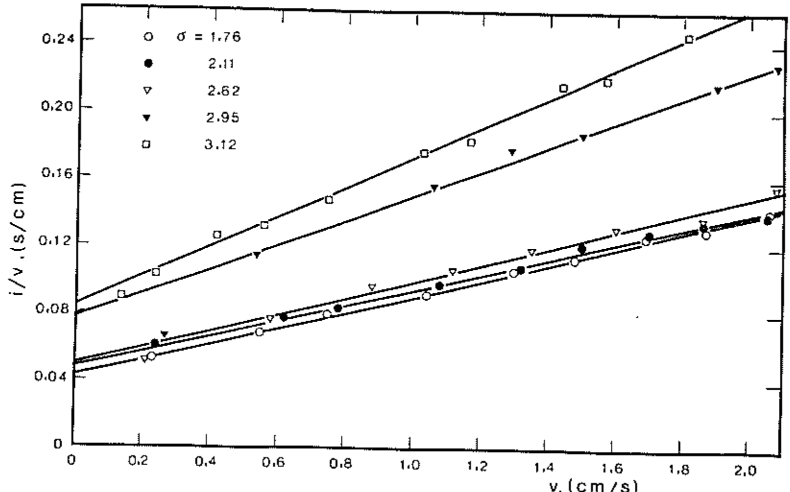

FIG. 4—Relationship of i/v to v for widely-graded gravels.

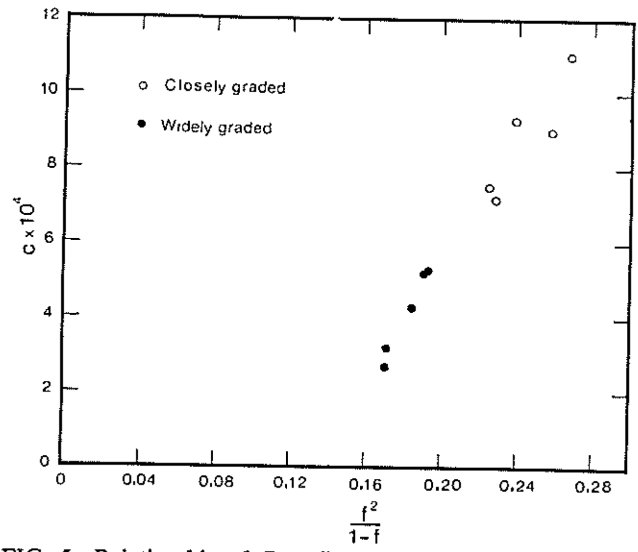

FIG. 5—Relationship of C to $f^2/(1\text{-}f)$.

for closely-graded gravels and 5% for widely-graded gravels); it may thus be assumed that C_1 is a constant for clean river gravels within the experimental range of this study at

$$C_1 = 0.017 \pm 0.002 \qquad (29)$$

The experimental relationship between C and $f^2/(1-f)$ is linear (Fig. 5), and is described by

TABLE 3—Comparisons of values of C from equation (30) with Bear (1972: equation (6)), and of C_2 from equation (31) with Joseph *et. al.* (1982: equation (8)).

Porosity	$C \times 10^4$		C_2	
	Equation (30)	Equation (6)	Equation (31)	Equation (8)
0.34	3.612	5.013	0.894	7.024
0.37	6.842	7.090	0.650	1.238
0.40	10.625	9.877	0.522	0.659

$$C = 10^{-4}\left(76.63\frac{f^2}{(1-f)} - 9.81\right), \quad 0.336<f<0.400 \qquad (30)$$

Since the porosities of clean gravels lie within the range 0.3 to 0.4 (Walton, 1970; Halek and Svec, 1979), equation (30) may apply to such gravels generally.

C_2 can now be obtained from equations (24), (29) and (30) as

$$C_2 = 1.7\left(76.63\frac{f^2}{(1-f)} - 9.81\right)^{-\frac{1}{2}}, \quad 0.336<f<0.400 \qquad (31)$$

Equations (30) and (31) are semi-empirical and relate C and C_2 respectively to f. These are compared with the previous equations of Bear (1972: equation (6)) and Joseph *et. al*; (1982: equation (8)) in Table 3. Although equation (30) is derived from the restricted set of data obtained in the present study, the agreement with equation (6) is good especially at the value of f (0.37) at which Beavers *et. al.* (1973) stated equation (6) to be reliable. The agreement of C_2 (Table3) becomes better at large values of f, as is to be expected since equation (8) is valid for loosely-packed granular material ($f \geqslant 0.6$).

These comparisons suggest that equations (30) and (31) may be applicable to coarse granular media other than clean river gravels. This suggestion is now tested with the data of other investigators, as shown in Table 4, and Figs. 6 and 7; the granular media used include glass beads, lead shot and river gravels. About half the media samples have porosities higher than 0.4, the upper range of derivation of equations (30) and (31).

The comparisons of equations (6) and (30) with experimental data in Fig. 6 show good agreement in the range $0.35<f\leqslant 0.55$ which covers most experiments; when $f<0.5$, equation (30) is preferable to equation (6). The relationships of C_2 with f, from equations (8) and (31) and from experiments (Fig. 7), indicate that equation (31) is preferable to equation (8). The degree of deviation and scatter of experimental results is not unexpected since errors can arise from many sources — in experiments (wall effects, temperature variation, air bubbles), in grain diameter measurements and in the calculation of hydraulic resistivity. In fact, C and C_2 are also functions of many other medium properties besides porosity and grain diameter, such as grain roughness, pore structure, and size distribution. For practical purposes, equations

TABLE 4—Comparisons of values of C and C_2 from equations (30) and (31) respectively to the experiment results of various workers.

Investigators	Media	Mean diam. (cm)	Average Porosity	C x 10^4 (Experiment)	C x 10^4 Equation (30)	C_2 (Experiment)	C_2 Equation (31)
Gupte +	Spherical glass beads	0.068	0.640	163.486	77.378	0.191	0.193
"	"	0.068	0.612	120.668	64.162	0.200	0.212
"	"	0.068	0.566	76.212	46.754	0.265	0.249
"	"	0.068	0.501	50.055	28.736	0.290	0.317
"	"	0.068	0.480	30.033	24.143	0.387	0.346
"	"	0.068	0.436	19.252	16.018	0.387	0.425
"	"	0.068	0.409	12.962	11.880	0.489	0.493
"	"	0.068	0.366	7.553	6.381	0.530	0.673
Ranganadha Rao &Suresh (1970)	River gravels	0.101-0.286	0.418	6.352	13.220	0.621	0.468
Dudgeon (1964)	"	0.101-0.550	0.368	4.650	6.610	0.626	0.661
"	"	0.26-11.38	0.368	1.930	6.610	0.758	0.616
Zagni, (1974)	Marbles	1.60-2.90	0.380	5.770	8.037	0.424	0.600
Raimondi, et al. *	Lead shot & steel balls	0.20-1.70	0.385	4.661	8.659	0.653	0.578
Banks, et al. *	-	0.011	0.33	8.889	2.645		
Beavers, et al. (1973)	-	0.546	0.33	6.125	2.645		
"	Glass spheres	0.3	0.368	6.456	6.610		
	"	0.6	0.370	4.361	6.842		
Virto, et al. (1981)	Styropor-P balls	0.26	0.35	6.863	4.632		
Harleman, et al. *	Spherical glass beads	0.14-0.20	0.37	6.54	6.842		
Schneebeli **	Spheres	2.7	0.39	8.409	9.297		
Ward (1964)	-	-	0.442			0.55	0.412
Linquid (1965)	Lead shot	0.105-0.492	0.383			0.566	0.586
Shwartz, et al. (1969)	Polyethylene particles	0.008-0.094	0.6			0.26	0.221

\+ Reported by Macdonald et al. (1979)

* Reported by Rumer (1969)

** Reported by Davis and DeWiest (1966)

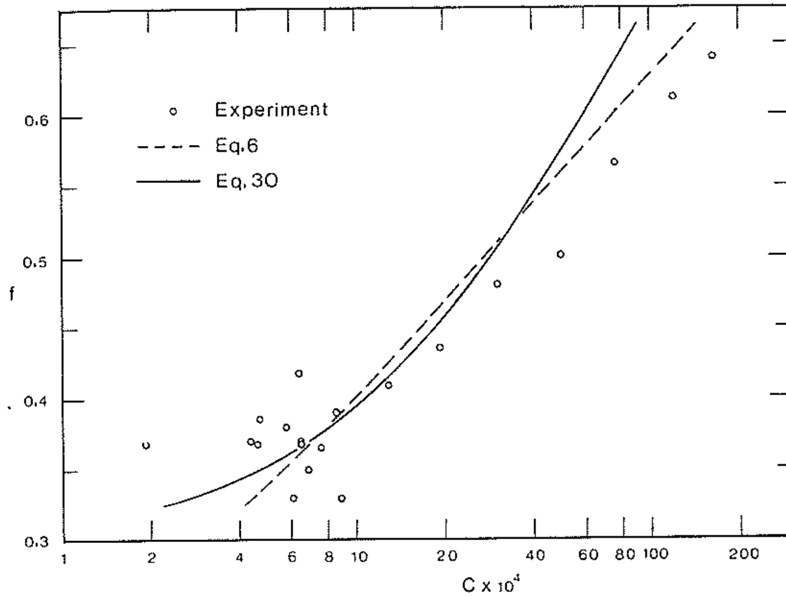

FIG. 6—Comparison of equations (6) and (30) with experimental results.

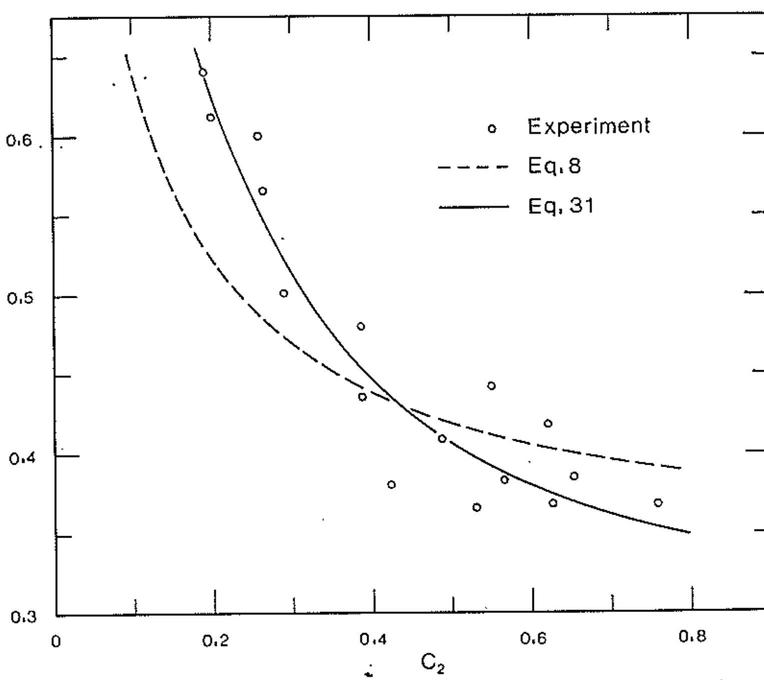

FIG. 7—Comparison of equations (8) and (31) with experimental results.

(30) and (31) appear able to predict *a* and *b* to within the degree of variation to be expected in a natural situation in which grains are of varying shapes and alignments, and packing conditions are likely to be heterogeneous.

Measurement of grain diameter and grading can be accomplished for bulk field samples by simple sieve analysis. The number and size of samples required to obtain representative values will depend on the variability of the medium; M. P. Mosley (M.W.D. Christchurch, pers. comm.) has shown that of the order of twenty 30 kg bulk samples are needed to define grain diameter for a braided gravel river bed, which is probably the least homogeneous situation likely to be encountered. Porosity can be determined from standard methods for field measurement of bulk density (e.g. Great Britain DSIR, 1961), using samples intended for later sieve analysis and assuming that the specific gravity of the solids is 2.65.

SUMMARY

The hydraulic resistivities *a* and *b* of coarse granular media can be estimated from grain diameter and porosity values, using equation (3), (4) and (5), based on equations (30) and (31). Thus seepage characteristics of a coarse granular medium can be estimated using only parameters which can be measured by simple tests; the use of permeameter tests at preliminary stages can be avoided.

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