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VARIATION OF MODEL PARAMETER VALUES AND SENSITIVITY WITH TYPE OF OBJECTIVE FUNCTION

F. T. Sefe[1] and W. C. Boughton[2]

[1] *Department of Geography, University of Nigeria, Nsukka, Nigeria*

[2] *School of Australian Environmental Studies, Griffith University, Nathan, Qld 4111, Australia*

ABSTRACT

The derivation of parameter values by optimisation techniques is vital for the use of lumped-parameter conceptual models. Optimisation is dependent on objective function, which may be formed in several ways. The multiplicity of objective functions introduces a high element of subjectivity into the choice of an appropriate function. It is not known what effect any particular objective function would have on model parameter values or their sensitivity.

Ten objective functions selected from the literature were used to optimise the Boughton Model for five catchments in order to determine how parameter values and sensitivity vary with type of objective function.

INTRODUCTION

Optimisation plays a major role in the derivation of parameter values of lumped-parameter models due to the lack of appropriate techniques of evaluation of parameter values from direct field measurements. The optimisation is based on an objective function. There are an infinite number of ways in which an objective function may be formulated, making the choice of an appropriate objective function an arbitrary and subjective task. Furthermore, the optimal set of parameters is optimal only in the context of the objective function selected (Diskin and Simon, 1977).

The need to reduce subjectivity in the selection of an objective function is more apparent in studies where the derived parameters are to be related to relatively accurately-measured catchment characteristics. As the relationships between model parameters and catchment characteristics are essential for the use of catchment models in ungauged catchments, the objective function selected for the optimisation should be one that would cnable the derivation of the best possible combination of parameters.

This paper reports a comparative study of ten objective functions. The ten objective functions were used in optimising an updated version of the Boughton Model (Boughton and Simpson, 1978) so as to determine how the optimised parameter values varied with type of objective function. These objective functions were also used in determining whether model parameter sensitivity varied with type of objective function. Mein and Brown (1978) raised this question, but did not resolve it.

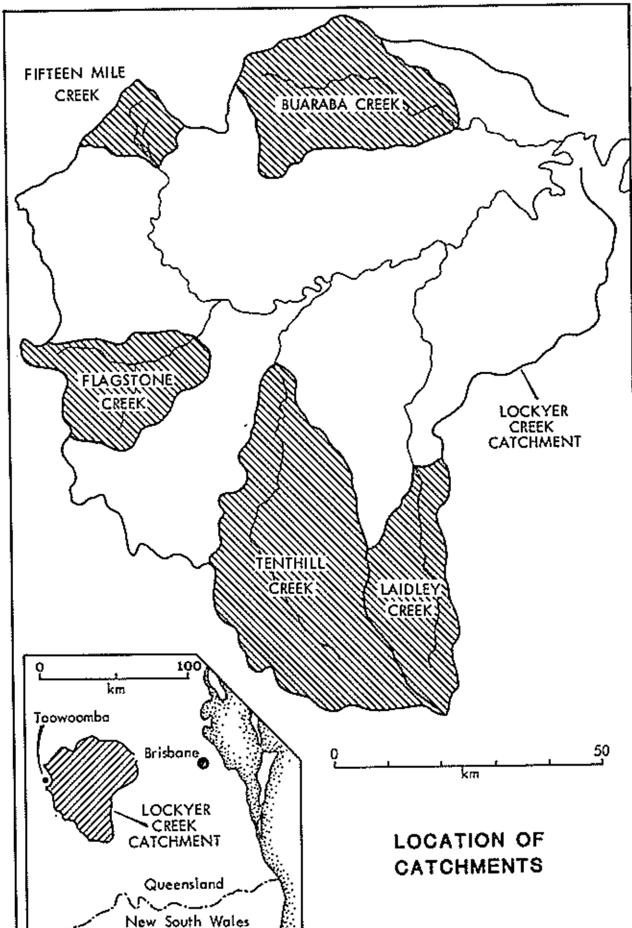

FIG. 1—Location of study catchments.

STUDY AREA

Five catchments, the Buaraba, Laidley, Tenthill, Flagstone and Fifteen-Mile catchments, in Lockyer Valley, south-east Queensland (Figure 1) were used for this study.

OBJECTIVE FUNCTIONS USED IN THE STUDY

Ten objective functions selected from the literature were used in this study. These are:

$$OF_1 = \Sigma(y_i - x_i)^2$$
$$OF_2 = \Sigma[2(y_i - x_i)^2/(y_i + x_i)]$$

$OF_3 = [\{n\Sigma(y_i - x_i)^2\}^{\frac{1}{2}}]/\Sigma y_i$
$OF_4 = (\Sigma|y_i - x_i|)/\Sigma y_i$
$OF_5 = |\Sigma(y_i - x_i)|/\Sigma y_i$
$OF_6 = (^1/n)\Sigma[2(y_i - x_i)/(y_i + x_i)]^2$
$OF_7 = [\Sigma(y_i - x_i)^2]/[\Sigma(y_i - \bar{y})^2]$
$OF_8 = [\{\Sigma(y_i^{1/3} - x_i^{1/3})^2\}^{3/2}]/n^{\frac{1}{2}}\Sigma y_i$
$OF_9 = [\Sigma(y_i/^{\frac{1}{2}} - x_i^{\frac{1}{2}})^2]/\Sigma y_i$
$OF_{10} = \Sigma[(y_i - x_i)/y_i)^2/(n - 1)]^{\frac{1}{2}}$
where OF = Objective Function, x_i = estimated runoff, y_i = observed runoff and n = length of data.

The characteristics of these functions are as follows:

OF_1: This function is well known in statistics. An early application of this function to hydrological modelling is described by Dawdy and O'Donnell (1965). Certain properties of the statistical distribution of the residuals are implied in this function. Clarke (1973) summarised these as:

i) the residuals have zero mean and constant variance;
ii) the residuals are mutually uncorrelated;
iii) the residuals are distributed normally;
iv) the log-likelihood function defined by
$\log L = \text{constant} - n \log \sigma_\varepsilon - \Sigma\varepsilon^2/2\sigma_\varepsilon^2$
is approximately quadratic in the parameter values
$\theta_1, \theta_2, \ldots, \theta_n$ in the neighbourhood of its maximum,
so that its contours are approximately ellipsoidal.
$\varepsilon_i = y_i - x_i$ = residuals
σ_ε = variance of the residuals

The function gives much more weight to large differences than small differences because the residuals are squared. This normally leads to a better estimate of high flows.

OF_2: This function was first proposed by Boughton (1968). The function tends to decrease the influence of large residuals associated with OF_1. The extent to which this is achieved is doubtful because of the squaring of the residuals involved.

OF_3: The function was used by Ibbitt and O'Donnell (1971). It is a normalised form of OF_1 and is analogous to the coefficient of variation. OF_3 differs from OF_1 in that it is dimensionless and independent of the number of items included in the series, provided the residuals are not correlated.

OF_4: This function is dimensionless and gives equal weight to all residuals. A form of this function was used by Deiniger (1969) and by Claborn and Moore (1970).

OF_5: This was derived from a proposal of Boughton (1968) by Simon and Diskin (1975). Like OF_4 the function is dimensionless and gives equal weight to all residuals. Due to compensating errors, this function may be zero without a good fit of the model to the data necessarily being achieved.

OF_6: This function was derived from OF_2. It differs from OF_2 in that it is independent of the number of items included in the series.

OF_7: This function was first introduced by Nash and Sutcliffe (1970) and used as an objective function by Simon and Diskin (1975). It has the same properties as OF_1 because of the use of OF_1 as a numerator.

OF_8: This function was derived by Diskin and Simon (1977) from a proposal of Chapman (1970), but was not used by them as an objective function. The function attempts to reduce the effect of large residuals and the length of recorded data. It is also dimensionless and gives more weight to low runoff events. The cube root transformation of the function is aimed at achieving the four properties enumerated by Clarke (1973). The use of $n^{\frac{1}{3}}$ in the denominator ensures that undue weight is not given to long records of data as opposed to short records.

OF_9: The function was proposed by Diskin and Simon (1977). It differs from OF_8 in that it uses a square root transformation and is independent of the length of record.

OF_{10}: This function was proposed by Manley (1978). The function gives equal weight to equal proportional residuals. It also gives greater weight to smaller absolute residuals at times of low flow. In this respect, it is similar to OF_8.

TYPE OF OBJECTIVE FUNCTION AND OPTIMISATION

The model was optimised for each of the ten objective functions by the steepest descent (ascent) direction as modified by Johnston and Pilgrim (1973). In this method the direction in which a step is taken in order to arrive at the global minimum is the direction of steepest slope from the current point. Johnston and Pilgrim (1973) reviewed the methods of determining the components (slope of the response surface) of the steepest descent vector and recommended the evaluation of the objective

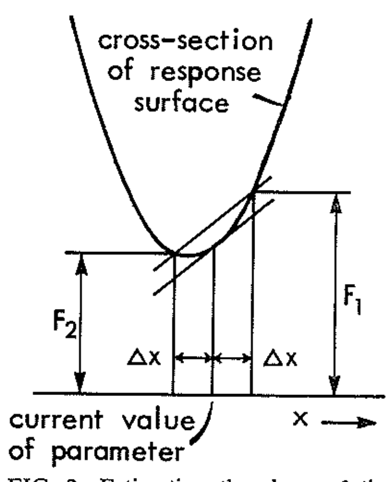

FIG. 2—Estimating the slope of the response surface (Johnston and Pilgrim, 1973).

TABLE 1—Initial parameter values.

Catchment	Parameter and values*														
	H	PCUS	CEPMX	USMAX	DRMAX	SSMAX	SDRMX	AIMP	BIMP	Fo	Fc	AAK	BASEF	AAC	R
Buaraba	35.0	50.0	20.0	108.0	190.0	324.0	100.0	0.1	150.0	105.0	1.5	0.005	0.03	0.03	1.0
Tenthill	35.0	50.0	15.0	113.0	152.0	307.0	100.0	0.1	150.0	130.0	1.0	0.0045	0.03	0.03	1.0
Fifteen-Mile	35.0	50.0	25.0	142.0	190.0	268.0	100.0	0.1	150.0	110.0	1.0	0.005	0.03	0.03	1.0
Flagstone	35.0	50.0	15.0	142.0	190.0	268.0	100.0	0.1	150.0	120.0	1.0	0.0055	0.03	0.03	1.0
Laidley	35.0	50.0	15.0	113.0	160.0	396.0	100.0	0.1	150.0	150.0	2.0	0.0045	0.03	0.03	1.0

* Values for CEPMX, USMAX, DRMAX, SSMAX, SDRMX, BIMP, Fo, Fc are in *points* (1 point = 0.254 mm or 0.01 inch).

AIMP, BASEF, AAC and R are dimensionless.

The values were converted into points to facilitate computation. All results presented in this paper are in points, except where otherwise stated.

function on either side of the current parameter value, as shown in Figure 2.

For this study steps were taken at ±10%, ±5%, ±1%, ±0.5% and ±0.1% of the original parameter values in the computed direction. The initial parameter values used in the optimisation are listed in Table I. The input to the model was daily areal rainfall computed from point rainfall by the Areal Reduction Factor method as suggested by Meija and Rodriguez-Iturbe (1973). Data analysed ranged from 11 years for the Buaraba and Tenthill catchments, 12 years for the Laidley, 13 years for the Flagstone and 17 years for the Fifteen-Mile.

A sample of the results of the modelling are shown in Table II for the least squares objective function. The final values of the objective criteria are listed in Table III.

Table IV shows the values of the optimised parameters for each objective function and for each catchment. Two points are worth noting about Table IV. Firstly, the values in this table differ markedly from the initial estimates in Table I. Secondly, the values in Table IV show little variation from one objective function to the other.

The total runoff produced by each optimum parameter set are shown in Table V. As can be seen from this table, some optimum parameter sets produced the same amount of total runoff. By putting together objective functions that optimised at the same total runoff, the groups shown at the bottom of Table V were arrived at.

TYPE OF OBJECTIVE FUNCTION AND PARAMETER SENSITIVITY

The use of sensitivity in this study follows the definition by Plinston (1972). If it were possible to picture the objective function surface in the n-dimensional space (where $n > 3$) a graphic representation of sensitivity would emerge. As such a picture would be made of n planes defined by any two parameters, any of the planes would, in fact, represent a cross-section through such an n-dimensional space. This cross-section can be drawn as a contour map of the function as shown in Figure 3. The ideal model would give a series of concentric circles, (Figure 3a), indicating equal response on the function surface for an equal change in each parameter direction; that is, the function value would be changed by the same amount following a constant change in either parameter value. Thus the parameters are said to be sensitive, and in this case they are equally sensitive. Consequently, interdependence would be non-existent in such a model. If the parameters have different sensitivities but are not interdependent, the map of the function surface produces a pattern of ellipses, with the orientation of the ellipses being determined by the parameter having the higher sensitvity, (Figure 3b). The line through the centre of the ellipses would be parallel to either axis, but this situation, too, would be unreal.

In the real world, sensitivity goes with interdependence and so the resultant map of the response surface would be as in Figure 3c. It is important to note that the line through the centre of the ellipses is not parallel to either axis. There are an infinite number of positions the

TABLE II—Observed and estimated daily runoff for the Tenthill Creek catchment 1977.

Day	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	0	0	1	1	0	1	0	0	0	0	0	0
	0	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
2	0	0	1	1	0	1	0	0	0	0	0	0
	0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
3	0	0	1	1	0	1	0	0	0	0	0	0
	0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
4	0	0	1	1	0	1	0	0	0	0	0	0
	0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*1*	*0*
5	0	0	2	1	0	1	0	0	0	0	0	0
	0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*1*	*0*
6	0	0	1	1	0	1	0	0	0	0	0	0
	0	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
7	0	0	1	1	0	0	0	0	0	0	0	0
	0	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
8	0	0	1	1	0	0	0	0	0	0	0	0
	0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
9	0	0	2	1	0	0	0	0	0	0	0	0
	0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
10	0	0	2	1	0	0	0	0	0	0	0	0
	0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
11	0	0	35	1	0	0	0	0	0	0	0	0
	0	*0*	*31*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
12	0	0	14	0	0	0	0	0	0	0	0	0
	0	*0*	*12*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
13	0	0	7	0	0	0	0	0	0	0	0	0
	0	*0*	*11*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
14	0	0	4	0	0	0	0	0	0	0	0	0
	0	*0*	*5*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
15	0	0	3	0	0	0	0	0	0	0	0	0
	0	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
16	0	0	2	0	1	0	0	0	0	0	0	0
	0	*0*	*1*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
17	**	0	2	0	1	0	0	0	0	0	0	0
	20	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
18	1	0	2	0	2	0	0	0	0	0	0	0
	7	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
19	0	0	1	0	2	0	0	0	0	0	0	0
	3	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
20	0	0	1	0	1	0	0	0	0	0	0	0
	1	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
21	0	0	1	0	1	0	0	0	0	0	0	0
	1	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
22	0	1	1	0	1	0	0	0	0	0	0	0
	0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
23	0	1	1	0	1	0	0	0	0	0	0	0
	0	*3*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
24	0	1	1	0	1	0	0	0	0	0	0	0
	0	*2*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*

TABLE II—Continued.

Day	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug.	Sep	Oct	Nov	Dec
25	0	1	1	0	1	0	0	0	0	0	0	0
	0	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
26	0	1	1	0	1	0	0	0	0	0	0	0
	0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
27	0	1	1	0	1	0	0	0	0	0	0	0
	0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
28	0	1	1	0	1	0	0	0	0	0	0	0
	0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
29	0		1	0	1	0	0	0	0	0	0	0
	0		*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
30	0		1	0	1	0	0	0	0	0	0	0
	0		*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
31	0		1		1		0	0		0		0
	0		*0*		*0*		*0*	*0*		*0*		*0*

Note: Top figure for each data is observed runoff; lower figure is estimated runoff.

** denotes no data available.

TABLE III—Optimal values of objective criteria.

OF	Buaraba	Laidley	Tenthill	Flagstone	Fifteen-Mile
1	559748	693263	141431	54032	2638318
2	251.27	206.54	257.18	95.86	350.44
3	2.11	1.21	2.38	1.47	2.71
4	1.18	0.64	1.16	0.81	0.85
5	0.0	0.0	0.01	0.0	0.0
6	7.29	5.99	7.46	2.78	10.16
7	1.10	0.20	0.88	0.35	0.50
8	0.26	0.06	0.25	0.20	0.09
9	0.66	0.22	0.63	0.40	0.33
10	43	14	15	29	28

ellipses can take without the lines through their centres being parallel to either axis. Thus it is virtually impossible to say when a parameter is sensitive: sensitivity can only be viewed in relative terms, (Plinston, 1972; Dawdy and O'Donnell, 1965).

Parameter sensitivity was evaluated by the method suggested by Mein and Brown (1978). The method involved evaluating the covariance matrix of the model parameters. The derived sensitivity values are shown in Table VI. The same groupings of objective functions as in Table V are retained in this table.

TABLE IV—Variation of individual parameter values with type of objective function.

Catchments	Optimised parameters										
	USMAX	DRMAX	SSMAX	SDRMX	AIMP	BIMP	Fo	Fc	AAK	BASEF	AAC
Buaraba	51	34	191	102	0.089	25	508	15	0.0045	0.0197	0.0356
Laidley	35	21	291	75	0.244	54	545	15	0.0036	0.0526	0.1885
Tenthill	46	34	647	60	0.149	126	503	25	0.0049	0.097	0.4545
Flagstone	72	45	285	100	0.1	150	756	10	0.004	0.05	0.075
Fifteen-Mile	35	27	198	61	0.289	56	398	25	0.0035	0.065	0.0441
Objective Function 1: $\Sigma(y_i - x_i)^2$											
Buaraba	51	34	191	102	0.089	25	508	15	0.045	0.0197	0.0356
Laidley	35	21	288	75	0.239	53	545	15	0.0036	0.0516	0.1889
Tenthill	46	34	647	60	0.149	126	603	25	0.0049	0.097	0.4545
Flagstone	72	45	291	100	0.1	150	720	10	0.004	0.05	0.075
Fifteen-Mile	35	27	197	61	0.288	56	400	25	0.0035	0.065	0.0441
Objective Function 2: $\Sigma[\{2(y_i - x_i)2\}/(y_i + x_i)]$											
Buaraba	51	34	191	102	0.089	25	508	15	0.0045	0.0197	0.0356
Laidley	35	21	291	75	0.244	54	545	15	0.0036	0.0526	0.1885
Tenthill	46	31	647	60	0.149	126	603	25	0.0049	0.098	0.4545
Flagstone	73	46	288	101	0.099	152	727	10	0.004	0.0495	0.0758
Fifteen-Mile	35	27	198	61	0.289	56	398	25	0.0035	0.065	0.0441
Objective Function 3: $[\{n\Sigma(y_i - x_i)^2\}^{\frac{1}{2}}]/\Sigma y_i$											
Buaraba	51	34	191	102	0.089	25	508	15	0.0045	0.0197	0.0356
Laidley	35	21	291	75	0.24	54	545	15	0.0036	0.0518	0.1889
Tenthill	46	34	647	60	0.149	126	603	25	0.0049	0.097	0.4545
Flagstone	73	46	288	101	0.099	152	727	10	0.004	0.0495	0.0758
Fifteen-Mile	35	27	197	61	0.288	56	400	25	0.0035	0.065	0.0441
Objective Function 4: $(\Sigma\|y_i - x_i\|)/\Sigma y_i$											
Buaraba	51	34	191	102	0.089	25	508	15	0.0045	0.0197	0.0356
Laidley	35	21	291	75	0.244	54	545	15	0.0036	0.0526	0.1885
Tenthill	46	31	647	60	0.149	126	603	25	0.0049	0.098	0.4545
Flagstone	72	45	285	100	0.1	150	720	10	0.004	0.05	0.075
Fifteen-Mile	35	27	198	61	0.289	56	398	25	0.0035	0.065	0.0441
Objective Function 5: $\|\Sigma(y_i - x_i)\|/\Sigma y_i$											
Buaraba	51	34	191	102	0.089	25	508	15	0.0045	0.0197	0.0356
Laidley	35	21	288	75	0.239	53	545	15	0.0036	0.0516	0.1889
Tenthill	46	34	647	60	0.149	126	603	25	0.0049	0.097	0.4545
Flagstone	72	45	291	100	0.1	150	720	10	0.004	0.05	0.075
Fifteen-Mile	35	27	197	61	0.288	56	400	25	0.0035	0.065	0.0441
Objective Function 6: $(\frac{1}{n})\Sigma[2(y_i - x_i)/(y_i + x_i)]^2$											
Buaraba	51	34	191	102	0.089	25	508	15	0.0045	0.0197	0.0356
Laidley	35	21	291	75	0.244	54	545	15	0.0036	0.0526	0.1885
Tenthill	46	31	647	60	0.149	126	603	25	0.0049	0.098	0.4545
Flagstone	73	46	288	101	0.099	152	727	10	0.004	0.0495	0.0758
Fifteen-Mile	35	27	198	61	0.289	56	398	25	0.0035	0.065	0.0441
Objective Function 7: $[\Sigma(y_i - x_i)^2]/[\Sigma(y_i - \bar{y})^2]$											

TABLE IV—Continued.

Catchments	Optimised parameters										
	USMAX	DRMAX	SSMAX	SDRMX	AIMP	BIMP	Fo	Fc	AAK	BASEF	AAC
Buaraba	51	34	191	102	0.089	25	508	15	0.0045	0.0197	0.0356
Laidley	35	21	288	75	0.239	53	545	15	0.0036	0.0516	0.1889
Tenthill	46	34	647	60	0.149	126	603	25	0.0049	0.097	0.4545
Flagstone	72	45	289	100	0.1	151	723	10	0.004	0.0493	0.0754
Fifteen-Mile	35	27	197	61	0.288	56	400	25	0.0035	0.065	0.0441
	Objective Function 8: $[\{\Sigma(y_i^{1/3} - x_i^{1/3})^2\}^{3/2}]n^{\frac{1}{2}}\Sigma y_i$										
Buaraba	50	34	194	100	0.09	25	500	15	0.0045	0.02	0.035
Laidley	35	21	288	75	0.239	53	545	15	0.0036	0.0516	0.1889
Tenthill	46	34	647	60	0.149	126	603	25	0.0049	0.097	0.4545
Flagstone	72	45	285	100	0.1	150	756	10	0.004	0.05	0.075
Fifteen-Mile	35	27	197	61	0.288	56	400	25	0.0035	0.065	0.0441
	Objective Function 9: $[\Sigma y_i^{\frac{1}{2}} - x_i^{\frac{1}{2}})^2]/\Sigma y_i$										
Buaraba	51	34	191	102	0.089	25	508	15	0.0045	0.0197	0.0356
Laidley	35	21	291	75	0.244	54	545	15	0.0036	0.0526	0.1885
Tenthill	46	34	647	60	0.149	126	603	25	0.0049	0.097	0.4545
Flagstone	72	45	285	100	0.1	150	756	10	0.004	0.05	0.075
Fifteen-Mile	35	27	197	61	0.288	56	400	25	0.0035	0.065	0.0441
	Objective Function 10: $\Sigma[((y_i - x_i)/y)^2/(n-1)]^{\frac{1}{2}}$										

USMAX, DRMAX, SSMAX, SDRMX, BIMP, Fo and F_c are in points (1 point = 0.01 inch = 0.254 mm).

AIMP, AAK, BASEF and AAC are dimensionless.

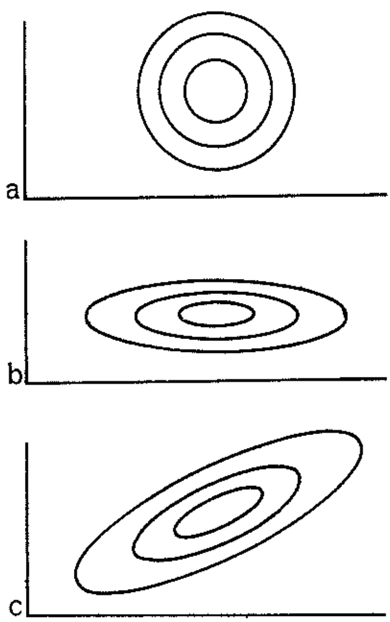

FIG. 3—Cross-sectional contour map of the response surface (Plinston, 1972).

TABLE V—Type of Objective Function (OF) and estimated runoff using optimum parameter set.

OF*	LAIDLEY		TENTHILL		FLAGSTONE		BUARABA		FIFTEEN-MILE	
	Actual runoff	Estimated runoff**	Actual runoff	Estimated runoff**	Actual runoff	Estimated runoff**	Actual runoff	Estimated runoff**	Actual runoff	Estimated runoff**
1	7393	7393	1667	1648	923	919	3614	3611	7366	7365
2		7343		1648		953		3611		7351
3		7393		1666		910		3611		7365
4		7317		1648		910		3611		7351
5		7393		1666		928		3611		7365
6		7343		1648		933		3611		7351
7		7393		1666		910		3611		7365
8		7343		1648		922		3611		7351
9		7343		1648		919		3695		7351
10		7393		1648		919		3611		7351

* OF groupings:

I	1, 3, 5, 7, 10	1, 2, 4, 6, 8. 9, 10	1, 9, 10	1, 2, 3, 4, 5, 6, 7, 8, 10	1, 3, 5, 7
II	2, 6, 8, 9	3, 5, 7	2, 6	9	2, 4, 6, 8, 9, 10
III	4		3, 4, 7		
IV			5		
V			8		

** Estimated runoff at optimum parameter set.

All runoff values are in points.

DISCUSSION

I. *Type of objectve function and optimisation.*

Reference has already been made to the dichotomy between the initial parameter values (Table I) and the optimised values (Table IV). Generally huge reductions, and, in some cases, small increases in the value of the stores were needed to arrive at the optimum parameter values for each objective function. The two upper soil stores, USMAX and DRMAX experienced the largest reduction. The reason for these large differences between the initial and optimum parameter values may be attributed to the inadequacy of the methods of parameter evaluation for lumped-parameter models.

While the optimised parameters varied widely from the initial values for every objective function, they showed almost no variation from one objective function to the other (Table IV). This unique situation resulted despite the fact that at least some of the objective functions optimised at different levels of total runoff. Perhaps this lack of significant variation may be due to the nature of the model rather than the objective functions. As the model parameters are interdependent and compensatory, perhaps a change in only one or a few parameters may be needed to optimise the various objective functions. However, as the parameters are normally considered in combination, it may be said that each parameter set is different from the other. Thus for the Buaraba catchment, for example, the parameter set of OF_9 is different from all the others because the parameters USMAX, SSMAX, AIMP, FO, BASEF and AAC optimised at different values. Objective functions with the same parameter values optimised at same total runoff output, giving the groupings at the bottom of Table V.

It can be seen from Table V that there are two groups for the Tenthill, Fifteen-Mile and Buaraba catchments; three groups for the Laidley and five for the Flagstone catchment. Reference to the description of the objective functions shows clearly that the composition of a group is due to both the characteristics of the objective function and, perhaps, chance. For example, OF_3 and OF_7 always appear together in the same group whereas OF_9 is never grouped with this pair and always results in a different set of parameter values. This suggests that there is an essential difference between the characteristics of OF_9 and either of OF_3 and OF_7. By contrast, the other objective functions show no uniformity or pattern in the groups in which they occur, and this lack of uniformity in the composition of groups in Table VI is noteworthy.

The results in Table VI show that optimised values of model parameters do vary with type of objective function; however, the results in Table V show that the variation in estimated runoff due to variation in parameter sets is not large. For the practical purpose of estimating runoff from rainfall for design of reservoirs or similar aims, the choice of objective function may not be critical. However, for studies relating model parameters to catchment characteristics, the choice of objective function can affect the optimised values of model parameters and conse-

TABLE VI: Summary results of sensitivity analysis.

OBJECTIVE FUNCTION	USMAX	DRMAX	SSMAX	SDRMX	AIMP	BIMP	Fo	Fc	AAK	BASEF	AAC
i) *Tenthill creek catchment*											
1 $\Sigma(y_i - x_i)^2$ 2 $\Sigma[\{2(y_i - x_i)^2\}/(y_i + x_i)]$ 4 $(\Sigma\lvert y_i - x_i\rvert)/\Sigma y_i$ 6 $(1/n)\Sigma[2(y_i - x_i)/(y_i + x_i)]^2$ 8 $[\{\Sigma(y_i^{1/3} - x_i^{1/3})^2\}^{3/2}]/n^{1/2}\Sigma y_i$ 9 $[\Sigma(y_i^{1/2} - x_i^{1/2})^2]/\Sigma y_i$ 10 $\Sigma[((y_i - x_i)/y_i)^2/(n-1)]^{1/2}$	4.6127	0.5427	0.9638×10^{-5}	0.5386	0.4951×10^{-9}	4.5614	3.0484	0.0244	4.5323	2.5889	8.8489
3 $[\{n\Sigma(y_i - x_i)^2\}^{1/2}]/\Sigma y_i$ 5 $\lvert\Sigma(y_i - x_i)\rvert/\Sigma y_i$ 7 $[\Sigma(y_i - x_i)^2]/[\Sigma(y_i - \bar{y})^2]$	3.5919	0.4492	0.2492	9.5860	2.7582	6.1154	3.5906	2.1697	3.01402	2.4231	9.6573
ii) *Fifteen-Mile Creek catchment*											
1 $\Sigma(y_i - x_i)^2$ 3 $[\{n\Sigma(y_i - x_i)^2\}^{1/2}]/\Sigma y_i$ 5 $\lvert\Sigma(y_i - x_i)\rvert/\Sigma y_i$ 7 $[\Sigma(y_i - x_i)^2]/[\Sigma(y_i - \bar{y})^2]$	0.2051	0.2914	0.3019	0.4152	.0002	0.2256	0.2661	0.0041	0.2259×10^{-14}	0.1058×10^{-6}	0.2182×10^{-6}
2 $\Sigma[\{2(y_i - x_i)^2\}/(y_i + x_i)]$ 4 $(\Sigma\lvert y_i - x_i\rvert)/\Sigma y_i$ 6 $(1/n)\Sigma[2(y_i - x_i)/(y_i + x_i)]^2$ 8 $[\{\Sigma(y_i^{1/3} - x_i^{1/3})^2\}^{3/2}]/n^{1/2}\Sigma y_i$ 9 $[\Sigma(y_i^{1/2} - x_i^{1/2})^2]/\Sigma y_i$ 10 $\Sigma[((y_i - x_i)/y_i)^2/(n-1)]^{1/2}$	0.9544	0.0922	0.1045	0.0004	0.4244	0.3497	0.4166	0.2576	0.3525×10^{-18}	0.2424×10^{-18}	0.2424×10^{-18}
iii) *Buaraba creek catchment*											
1 $\Sigma(y_i - x_i)^2$ 2 $\Sigma[\{2(y_i - x_i)^2\}/(y_i + x_i)]$ 3 $[\{n\Sigma(y_i - x_i)^2\}^{1/2}]/\Sigma y_i$ 4 $(\Sigma\lvert y_i - x_i\rvert)/\Sigma y_i$ 5 $\lvert\Sigma(y_i - x_i)\rvert/\Sigma y_i$ 6 $(1/n)\Sigma[2(y_i - x_i)/(y_i + x_i)]^2$ 7 $[\Sigma(y_i - x_i)^2]/[\Sigma(y_i - \bar{y})^2]$ 8 $[\{\Sigma(y_i^{1/3} - x_i^{1/3})^2\}^{3/2}]/n^{1/2}\Sigma y_i$ 10 $\Sigma[((y_i - x_i)/y_i)^2/(n-1)]^{1/2}$	3.8836	3.2284	6.9086	3.0918	2.7657	4.7173	4.9706	2.4249	3.3961	2.5505	3.1376
9 $[\Sigma(y_i^{1/2} - x_i^{1/2})^2]/\Sigma y_i$	3.7991	0.5572	5.3421	3.7267	2.4200	4.1402	2.7850	5.2474	3.6553	4.5925	3.5742

TABLE VI: Continued

	OBJECTIVE FUNCTION	USMAX	DRMAX	SSMAX	SDRMX	AIMP	BIMP	Fo	Fc	AAK	BASEF	AAC
	iv) Laidley creek catchment											
1 3 5 7 10	$\Sigma(y_i - x_i)^2$ $[\{n\Sigma(y_i - x_i)^2\}^{1/2}]/\Sigma y_i$ $\lvert\Sigma(y_i - x_i)\rvert/\Sigma y_i$ $[\Sigma(y_i - x_i)^2]/[\Sigma(y_i - \bar{y})^2]$ $\Sigma[(y_i - x_i)/y_i)^2/(n-1)]^{1/2}$	2.4194	2.2431	13.3423	3.9983	4.1091	2.8110	4.3575	8.2753	3.5743	4.8225	6.4435
2 6 8 9	$\Sigma[\{2(y_i - x_i)^2\}/(y_i + x_i)]$ $(1/n)\Sigma[2(y_i - x_i)/(y_i + x_i)]^2$ $[\{\Sigma(y_i^{1/3} - x_i^{1/3})^2\}^{3/2}]/n^{1/2}\Sigma y_i$ $[\Sigma(y_i^{1/2} - x_i^{1/2})^2]/\Sigma y_i$	3.6405	6.0291	3.5359	14.2166	4.3611	6.7064	7.0822	2.7235	3.0660	3.9186	2.9461
4	$(\Sigma\lvert y_i - x_i\rvert)/\Sigma y_i$	2.0178	19.1161	3.5569	4.2394	3.1034	4.2280	4.8568	4.0207	4.3169	3.0156	3.0081
	v) Flagstone Creek catchment											
1 9 10	$\Sigma(y_i - x_i)^2$ $[\Sigma(y_i^{1/2} - x_i^{1/2})^2]/\Sigma y_i$ $\Sigma[(y_i - x_i)/y_i)^2/(n-1)]^{1/2}$	4.5652	1.5696	6.3253	3.5112	0.1058	4.4474	2.4498	7.3735	0.1356×10^{-12}	4.2471	3.7416
2 6	$\Sigma[\{2(y_i - x_i)^2\}/(y_i + x_i)]$ $(1/n)\Sigma[2(y_i - x_i)/(y_i + x_i)]^2$	5.5767	6.0874	3.2611	4.1876	6.2517	2.6745	3.5329	4.7837	3.5551	3.5362	11.8847
3 7 4	$[\{n\Sigma(y_i - x_i)^2\}^{1/2}]/\Sigma y_i$ $[\Sigma(y_i - x_i)^2]/[\Sigma(y_i - \bar{y})^2]$ $(\Sigma\lvert y_i - x_i\rvert)/\Sigma y_i$	19.7151	3.6020	8.3339	11.5215	1.8374	3.1979	2.5985	4.7837	0.3398×10^{-8}	2.8829	0.5718×10^{-6}
5	$\lvert\Sigma(y_i - x_i)\rvert/\Sigma y_i$	5.7799	9.0868	2.4915	5.8213	2.7910	3.4910	2.3100	6.7245	4.8807	4.1866	4.8985

quently may give spurious correlations between those values and catchment characteristics.

For studies relating model parameters to catchment characteristics, some other criterion has to be adopted for the choice of an appropriate objective function. The variation in parameter value associated with each of the ten objective functions used in this study is not distinctive enough to be used as a basis for the selection of an appropriate function.

II. *Type of objective function and parameter sensitivity*:

Two observations can be made about the results shown in Table VI. These are:

i) some parameters have identical results for all the groups: taking the Tenthill catchment as an example, for the first group of objective function BASEF has a sensitivity value of 2.5889 as compared to 2.4231 for the second group;

ii) others have different results from one group to the other; for example, for the Fifteen-Mile catchment, the first group of objective function SDRMX has a sensitivity value of 0.4152 as compared to 0.0004 for the second group.

However, taking all the parameters together, it is obvious from Table VI that the differences are not statistically significant. Some objective functions quickly converge to the optimum level, (some of the functions optimised in four iterations), but they do not necessarily lead to significantly different sensitivity values for the model parameters. Perhaps this is to be expected considering the method of sensitivity analysis used in this study. As long as the optimum parameter sets provide identical changes in predicted flow per unit change in parameter value there cannot be any significant difference among the objective functions with respect to parameter sensitivity.

CONCLUSION

This study has demonstrated that optimum parameter sets obtained for a lumped-parameter model do vary with the type of objective function used in the optimisation. Moreover, all the parameters need not vary.

Owing to interdependence and compensatory nature of the model parameters, only a few parameters need vary to produce a distinct optimum parameter set.

Similarly, parameter sensitivity was found to vary with type of objective function if the parameters were considered individually. However, when the parameters were considered in groups according to type of objective functions, the variation in sensitivity values proved insignficant.

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