Journal of Hydrology (NZ) 39(1): 19-47, 2000
© New Zealand Hydrological Society (2000)

Evaluation of low-flow frequency analysis methods

Brian S. Caruso
Otago Regional Council, Dunedin, New Zealand.

Abstract

Low flows in 21 rivers in the Otago region of the South Island of New Zealand were estimated using several different frequency analysis methods and compared to assist in setting minimum flows for protection of instream uses. Theoretical frequency distributions, including the log-normal, Weibull, and Extreme Value Type 1 distributions, and the Gringorton plotting position were used with historical discharge data. Goodness-of-fit tests, including L-moment methods for regional data, were used to determine the best distributions. Regional analysis for ungauged locations, such as regression methods using catchment characteristics and low-flow contour maps, were also evaluated.

The specific 7-day mean annual low flows ranged from 0.23 l s^{-1} km^{-2} to 7.76 l s^{-1} km^{-2}. The mean value for all stations was 2.62 l s^{-1} km^{-2}. Based on the log-normal estimate, values for the specific 7-day 10-year low flow ranged from 0.01 l s^{-1} km^{-2} to 3.93 l s^{-1} km^{-2}. The mean for all stations was 1.3 l s^{-1} km^{-2}. The log-normal estimate was usually higher than the Weibull estimate with differences ranging up to 46%. Few stations had differences greater than or equal to 20%, and the mean difference for all stations was 13%. Results for the Extreme Value Type 1 distribution were usually lower than for the other two distributions. There was no clear pattern of the plotting position values being higher or lower than the theoretical estimates.

The L-moments method is good technique for evaluation of goodness-of fit and regional analysis. The Generalised Pareto distribution was best for most individual stations. For the Otago sites, L_{CV} values ranged from 0.1808 to 0.5694, and L_{SK} and L_{KUR} values ranged up to 0.3895 and 0.6186, respectively. Only one station had a discordancy value considerably higher than the others. Heterogeneity tests showed that the group of stations can be considered homogeneous. The best distribution for the group was the Generalised Extreme Value distribution, but the 3-parameter log-normal and Extreme Value Type 1 distributions also fit the data well.

Regression equations can be used for very rough estimates of low flows,

but more detailed data from nearby stations and evaluation of catchment characteristics are needed for ungauged sites, especially where minimum flows are proposed. The detailed evaluation of historical discharge data for stations across the region can improve low-flow contour maps that could be useful for ungauged locations. The techniques used in this study can help to provide a sound scientific basis for delineating minimum flows and protecting instream values.

Introduction

Estimation of low flows is receiving increasing attention in New Zealand and internationally as part of the assessment and management of competing water uses and instream values, such as aquatic life habitat, recreation, and the natural geomorphologic forms of channels (Mosley and Pearson, 1997; MfE, 1998). The 1997-99 drought in many parts of New Zealand, especially the South Island, emphasises the need for reliable estimation of low flows for effective water management. Methods available for assessing low flows and setting minimum flows to maintain and enhance ecological habitat are shown in Table 1 (MfE, 1998).

Table 1 – Methods for assessment of low flows for ecological habitat

Exceedance methods, such as frequency analysis
Tennant (or Montana) Method, typically percentages of mean annual flow
Hydraulic methods, such as river width or wetted perimeter
Habitat methods, including the Incremental Flow Instream Method (IFIM)
Regional methods, including regression analysis

Little guidance is available on which methods are best under varying conditions to protect instream values. However, low flows estimated using various methods can be very different, and even minor differences in minimum flows can have significant implications for resource management and users. Therefore, comparing the performance of alternative methods and evaluating differences are very important.

Exceedance methods, including standard and regional frequency analysis using statistical models, are often used to evaluate the probability or risk of low flows for given durations and return periods. Theoretical frequency distributions that have been used most often in low-flow analysis include the log-normal, Extreme Value Type I (EV1) and Type III (EV3), Gamma, Pearson Type III (P3) and log-Pearson Type III (McMahon, 1980; Stedinger *et al.*, 1993). The log-normal and EV3 distributions have probably been used and recommended most widely (Haan, 1977; McMahon, 1980; Nathan and McMahon, 1990; Pearson and Davies, 1997). The EV3 distribution for low values (also known as the Weibull distribution) arises when the extreme is from a parent distribution that is limited in the direction of interest; in the case of low flows they are bounded by zero flow on the left. This model was recommended by Pearson (1995) for many Otago catchments, and Pearson and Davies (1997) for low-flow frequency analysis in New Zealand in general. The log-normal distribution has been widely used for both high and low flows because it is simple, tables for its evaluation are readily available, and many hydrologic variables are bounded by zero on the left and positively skewed (Haan, 1977; McMahon, 1980). This distribution has been recommended by Snelder *et al.* (1997) for Otago rivers, and by Pearson and Davies (1997) for low-flow frequency analysis in general. Studies in the U.S. have also shown that the Weibull and P3 (McMahon, 1980; Matalas, 1963), and the Gamma (McMahan, 1980; Joseph, 1970) distributions performed best for estimation of low flows.

Goodness-of-fit tests, such as the Chi-Square and the non-parametric Kolmogorov-Smirnov tests, can be used to help determine the most appropriate distribution. However, these tests generally do not address the tails of data which are of primary interest here (McMahon, 1980). Methods based on L-moment (linear moment) ratios were developed in the early 1990's (Hosking, 1990) and have been recommended for hydrological regional frequency analysis, including for determining goodness-of-fit of parent distributions (Hosking and Wallis, 1993; Vogel and Fennessey, 1993; Pearson, 1995). Vogel and Fennessey (1993) concluded that the L-moments method is much better than conventional regional frequency analysis methods. Conventional moment estimators are also biased for extremely large samples ($N \geq 1000$) from highly skewed distributions, but L-moment ratio estimators are nearly unbiased for all underlying distributions. L-moment methods also are not limited to small sample sizes and are more reliable for discerning homogeneous regions and identifying likely parent statistical distributions (Hosking, 1990; Hosking and Wallis, 1993; Pearson, 1991a). Using L-moment ratio methods, Pearson (1995) found that the EV1 distribution was a satisfactory fit for many individual stations in New Zealand, but not for groups of stations in homogeneous regions.

Empirical distributions using plotting positions, such as the Weibull and Gringorten plotting positions, in general should only be used to estimate frequencies less than $N/3$ (McMahon, 1980), where N is equal to the number of years of record. Therefore, 30 years of record would generally be required to use plotting positions to estimate low flows with a return period of 10 years. Most gauging stations have shorter record lengths. Use of theoretical distributions, therefore, has been recommended in most cases (McMahon, 1980). Subjective methods, such as graphical techniques fitting lines to data by eye, have also been recommended, due to shortcomings in the objective fitting of theoretical distributions to limited data (Bardsley, 1989; 1994). These methods, however, are not widely used.

This study evaluates frequency analysis methods that have been widely used or recommended to estimate low flows and to set minimum flows in rivers for instream uses. Representative rivers in the Otago region of the South Island of New Zealand are used as a case study. Theoretical and empirical distributions are applied to historical discharge data, and the methods used are compared and assessed. Several regional frequency analysis methods developed by others are also applied and tested. The information derived from this study is being used by the Otago Regional Council to establish a sound scientific basis for setting minimum flows in rivers to maintain instream values as part of their Proposed Water Plan (Otago Regional Council, 1998). The information should also be applicable and useful in other areas of New Zealand as well as internationally.

Problems associated with low-flow frequency analysis

In addition to choice of evaluation method, problems associated low-flow frequency analysis include water abstractions, inaccurate low-flow measurements, zero flows, short record lengths, missing data, and ungauged rivers.

Abstractions

Because water is abstracted from many rivers so that the flow is "modified" relative to natural flows, frequency analysis for these rivers provides low-flow estimates that might not be appropriate for protection of instream values. There is some debate on whether modified flows or unmodified "natural" flows based on corrections using water usage data should be used. However, water usage data are often not available. At a minimum, any available information on abstractions can be assessed to categorise rivers as relatively modified or unmodified, and low-flow estimates evaluated as such.

Inaccurate low-flow measurements

Most gauging stations in New Zealand were designed to measure typical and flood flows, and some produce inaccurate and unreliable low-flow measurements. Data from these stations can result in unreliable estimates.

Zero flows

A zero in a set of data that is being logarithmically transformed requires special handling. There are three possible solutions to this problem. The first is to add a small constant to all of the observations. Another method is to analyse the non-zero values and then adjust the relation to the full period of record. However, both of these methods bias the results (Haan, 1977). A third, and theoretically more sound, method is to use the theorem of total probability (Haan, 1977; Nathan and McMahon, 1990). Low-flow frequencies for non-zero flows are first analysed. This result is modified by considering the probability of zero flows (*pz*), which is estimated as n_z/N (frequency of zero flows) where n_z is the number of zero flows and N is the total sample size. Thus, the correction factor multiplied by low flows estimated using the non-zero flows is (1-*pz*).

Short record length and missing data

The reliability of low-flow frequency estimates, like other hydrologic variables, depends on record length. For example, the error in estimating the 10-year flood can be up to 25% for records of less than 20 years (Linsley *et al.*, 1982). In practice record lengths are often shorter than this. A rule of thumb is not to extrapolate return periods far beyond twice the sample size. Regional methods or deterministic hydrologic modeling should be considered for sites with 10 or fewer years of record (Pearson and Davies, 1997).

In many cases some data from a station is missing during the low-flow period. Therefore, all the records should be checked prior to statistical analysis. If significant gaps occurred during a given year, data for that year should be excluded.

If the record length is short or data are missing, low-flow regression equations (regional methods) can be developed for a site using a nearby site with a longer or better record to extend the record or fill data gaps. This can be done only if there is enough record overlapping between the two sites. Therefore, the consistency of low flows between the two stations should first be examined, and pairs of corresponding low flows should be used to establish the relationship.

Ungauged rivers

Low-flow estimates are often needed for locations where no gauging station or historical data exist. Regional frequency analysis methods, including regression analysis, can be used in these cases. A "region" is an area with a group of sites, each of which is assumed to have data drawn from the same parent frequency distribution. Regional analysis can involve assigning sites to regions, testing whether the regions are relatively homogeneous, and selecting distributions that fit each region's data. Regression equations can also be developed to estimate low flows at ungauged sites based on correlation of low flows between ungauged and gauged locations within a hydrological region (Grant, 1971; McKerchar and Dymond, 1981; Harrison, 1988). Regression can also be used to estimate low flows at ungauged sites based on catchment characteristics (Hutchinson, 1990; Pearson, 1995; Snelder *et al.*, 1997). Hutchinson (1990) developed regression equations for different regions of New Zealand to estimate the specific 7-day low flow with a return period of five years ($SQ_{7,5}$) based on mean annual precipitation, average slope, and an index for hydrogeologic characteristics. These values can be estimated using data from the New Zealand National Land Resources Inventory, which are generally available for all areas in New Zealand except for some small catchments of less than 0.1 km^2 (Pearson, 1995). Pearson (1995) developed a single regression equation for all of New Zealand using data from over 500 catchments to estimate the mean annual specific 7-day low flow ($SQ_{7,m}$) based on mean annual precipitation, average slope, proportion of bare land, average soil porosity and an index for vegetation type. Using these data Pearson (1995) also developed contour maps of log $SQ_{7,m}$ across New Zealand that can be used to derive a rough estimate of $Q_{7,m}$ at any point in a catchment.

Regional low-flow frequency analysis methods based on L-moment ratios have been recommended in recent years. Data requirements include annual low flows for some monitored sites and catchment characteristics, some of which can be derived from the Land Resources Inventory (Pearson, 1991b; Pearson, 1995; Clausen and Pearson, 1995). Using these methods, Pearson (1995) found that the 3-parameter Generalised Logistic (GLO) distribution was the best fitting distribution for the groups of stations in Otago monitored by the National Institute for Water and Atmospheric Research Ltd (NIWA) Alexandra and Dunedin field teams, but that the Generalised Pareto (GPA) and the 3-parameter Weibull distributions were best for the group of Otago catchments monitored by the Otago Regional Council.

Methods

Twenty-one gauging stations in representative locations throughout Otago were used for frequency analysis (Fig. 1). These stations were selected because they were important from a management perspective and most of their low-flow measurements were considered reliable. Table 2 presents the stations used and the data collection authority (Otago Regional Council or NIWA), length of record, and catchment area. The catchments varied in size from less than 50 km^2 (Lovells Creek at State Highway (SH) 1 and Mill Creek at Fish Trap) to very large areas such as the Taieri River at Outram (4705 km^2). Four other stations had catchment areas greater than 1000 km^2. For each station, two primary low-flow values were considered important for maintaining instream values and were estimated using historic discharge data: the mean annual 7-day low flow ($Q_{7,m}$), and the 7-day low flow with a return period of 10 years ($Q_{7,10}$).

Most of the discharge data were collected by the Otago Regional Council and NIWA. However, data at some stations on the Taieri River from the 1960s to mid-1980s were collected by the former Ministry of Works and Development (MWD). All of the data are stored in the Otago Regional

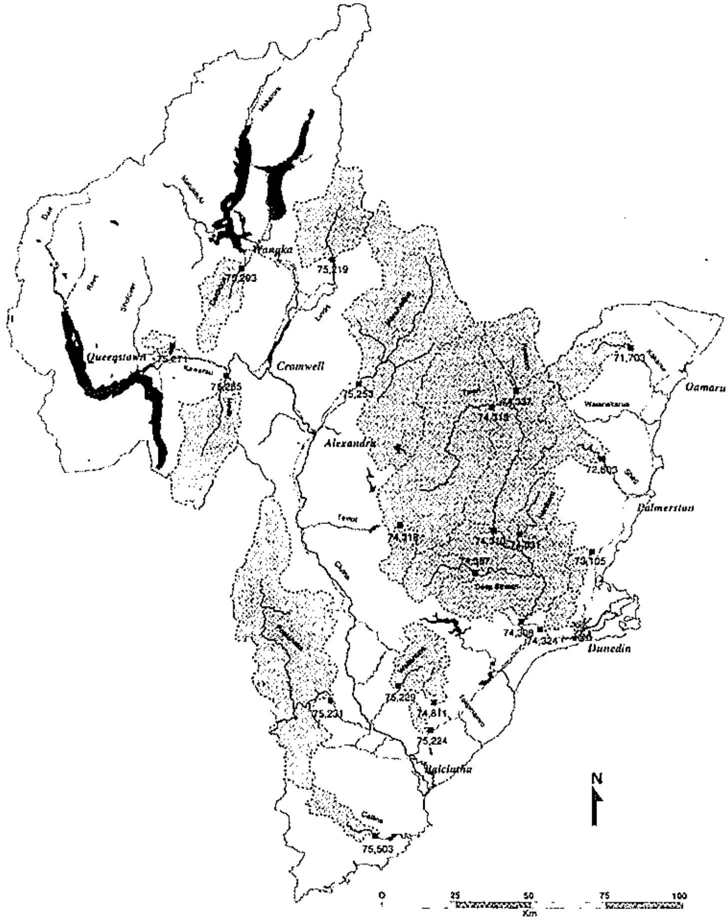

Figure 1 – Locations of gauging stations and catchments used for low-flow analysis.

Council Time Dependent Database (TIDEDA) (Rodgers and Thompson, 1992), which enables the data to be retrieved and manipulated. The data within the last ten years were collected to ISO 9002 accuracy standards, which requires that 95% of instantaneous water level measurements are resolved to within ±3 mm or ±10 mm, depending on site instrumentation. The early records were collected to varying standards of accuracy, depending largely on the purpose for the data at the time.

Table 2 – Gauging stations used for low-flow analysis

Station Name	No.	Authority	Start	End	N	Area (km^2)
Cardrona River at Mt Barker	75293	ORC	1976	1988	9	248
Catlins River at Houipapa	75503	ORC	1993		4	163
Deep Stream at SH87	74357	ORC	1992		9	236
Kakanui River at Clifton Falls	71703	ORC	1981		15	286
Kyeburn at SH85 Br	74337	ORC	1968	1996	23	376
Lindis River at Lindis Peak	75219	NIWA	1976		16	542
Lovells Creek at SH1	75224	ORC	1969	1980	4	39
Manuherikia River at Ophir	75253	NIWA	1971		24	2036
Mill Creek at Fish Trap	75271	ORC	1983		13	44
Nenthorn Stream at Mt Stoker Rd	74331	ORC	1982		13	213
Nevis River at Wentworth	75265	NIWA	1977		11	689
Pomahaka River at Burkes Ford	75231	ORC	1961		35	1924
Shag River at the Grange	72603	ORC	1989		8	319
Silverstream at Taieri Depot	74324	ORC	1987		10	92
Taieri River at Canadian Flat	74318	NIWA	1982		14	158
Taieri River at Waipiata	74313	ORC	1967		9	1865
Taieri River at Sutton	74310	ORC	1960		14	3066
Taieri River at Outram	74308	ORC	1968		19	4705
Tokomariro River at W. Branch	74811	ORC	1981		16	68
Waikouaiti S. Branch at Lawsons	73105	ORC	1991		7	74
Waitahuna River at Tweeds Br	75229	ORC	1992		5	311

ORC - Otago Regional Council
NIWA - National Institute for Water and Atmospheric Research

All the records were first checked manually, and most had some gaps in them. If significant gaps occurred during the low-flow period for a given year, data for that year were excluded from further analysis. Only three stations had a record length of 20 years or more. The longest record was for the Pomahaka River at Burkes Ford (35 years). Several had records as short as four or five years, including the Catlins River at Houipapa and the Waitahuna River at Tweeds Bridge. For Lovells Creek at SH1, data for seven out of 11 years had to be excluded due to significant gaps during low-flow periods, so data for only four years were used. Therefore, results for these three sites are merely indicative. However, these stations were used to gain insight and provide useful information on problems associated with use of short records.

For the Kyeburn at SH85 Bridge, records collected by MWD from 1968 to 1986 were combined with records collected by the Otago Regional Council from 1986 until 1996. Data for a 5-year period were excluded due to significant gaps during low-flow periods. Therefore, data for a total of 23 years were used.

Data were collected by MWD from 1967 to 1987 for the Taieri River at Waipiata, and the station was reopened by the Otago Regional Council in 1992. The two sets of data were combined to produce a full record. However, only the data recorded since the introduction of the Maniototo Irrigation and Hydroelectric Power Scheme (since 1984) were used. No data were collected during five of those years, so data for nine years were used for this station. For the Taieri River at Sutton, data from 1984 were used for the same reason as for the Taieri River at Waipiata (14-year record). Data collected prior to July 1978 for the Taieri River at Outram were excluded from analysis because the station was operated as a flood warning station during the 1960s and most of the 1970s, and there were significant data gaps during low-flow periods. Therefore, the record length was 19 years.

Nenthorn Stream at Mt Stoker Rd typically stops flowing under very dry conditions. Therefore, two zero flow values were present in the data set. This was the only station with any zero flows. These values were removed from the data set, and low-flow values were estimated using the theorem of total probability with N=13.

Five years of data from 1993 to 1997 were available for Deep Stream at SH87. Linear regression was used to establish a relationship between flows at SH87 and an upstream station at the Dunedin City Council 60 Weir. The consistency of low flows between the two stations was first examined, and pairs of annual 7-day low flows and monthly 7-day low flows during summer and autumn were then used in regression (R^2 of 0.97) to extend the record at SH87 back to 1989 for a total record of nine years.

For Mill Creek at Fish Trap, data for one year were excluded due to gaps during the low-flow period. Therefore, a flow record of 13 years was used. Data for three years were excluded for the Cardrona River at Mt Barker due to gaps, and a flow record of nine years was used. For the Kakanui River at Clifton Falls, data for one year were excluded due to gaps. Therefore, data for 15 years were used.

Records were collected by NIWA on behalf of Central Electric Ltd. for the Nevis River at Wentworth. Data for nine years were excluded due to significant gaps during low-flow periods and other times during the year. Therefore, a record length of only 11 years was used.

A TIDEDA process called "pmove" was used to produce a consecutive 7-day low flow for each year and station. A hydrological or water year was used, divided so that low flows occurred during the middle part of the year. This ensured that the low flow for each year was used only once in the calculation. The water year starts from July 1 for most rivers in Otago, and from October 1 for some rivers in the Central Otago and Queenstown-Lakes districts. The $Q_{7,m}$ was computed as the mean of the 7-day low flows for each year and station. Summary statistics, including the standard deviation, variance, coefficient of variation and skewness for each low-flow series, were also computed. Pearson (1995) found that serial correlation between successive years for New Zealand annual low-flow series was negligible, so these were not examined in this study.

The $Q_{7,10}$ values were estimated using the log-normal, Weibull and EV1 distributions. The computer programme "Statgraphics" was used to estimate the frequency distributions based on the annual 7-day low-flow series for each station. The $Q_{7,10}$ was also estimated with empirical frequency curves using the Gringorton plotting position in "Excel":

$$T = (N + 0.12) / (I - 0.44)$$

where: T = return period (years)
N = number of years of record
I = rank of event in order of magnitude, the largest event having $I = 1$

The Chi-Square test and nonparametric Kolmogorov-Smirnov test were used to evaluate the goodness-of-fit for each distribution and station at the 90% and 95% confidence levels. L-moment ratios were also used to evaluate the goodness-of-fit for the individual sites and for the group of sites within the region (Pearson, 1995; Hosking and Wallis, 1993; Hosking, 1990), using a Fortran programme developed by Hosking and provided by NIWA. Values for L_{CV}, L_{SK}, L_{KUR} and discordancy ($D(I)$) were calculated for each site. The record length-weighted mean values for L_{SK} and L_{KUR} for the group of all stations were also estimated. Hosking's heterogeneity tests using V_1, V_2 and V_3 were computed for the group of stations (Hosking and Wallis, 1993). Values for L_{KUR} for the group of stations were compared to standard Z values for the 3-parameter GLO, Generalized Extreme Value (GEV), P3, and GPA distributions, and the 2-parameter EV1 distribution to determine the best fitting distribution for the group. The L_{SK} and L_{KUR} values for each station (except Lovells Creek and the Catlins), and the record length-weighted mean values for the group of all stations, were also plotted relative to the theoretical curves for the 3-parameter log-normal (LN3), GLO, GEV, GPA, P3, and Weibull distributions, and the 2-parameter EV1 distribution. The 3-parameter distributions plot as curves, and the 2-parameter distribution plots as a single point. The shortest distance from the point for each station to the nearest curve (or point) was used to determine the best fitting distribution.

Regression equations developed by Hutchinson (1990) and based on catchment characteristics were used to estimate low flows ($SQ_{7,5}$) for five stations: Kakanui River at Clifton Falls, Shag River at the Grange, Pomahaka River at Burkes Ford, Catlins River at Houipapa and Mill Creek at Fish Trap. These estimates were compared to estimates by Snelder *et al.* (1997) for the Kakanui, Shag and Pomahaka rivers using the historical discharge data and frequency analysis based on the log-normal distribution. This was done to assess the accuracy and usefulness of Hutchinson's regression method for ungauged stations.

The Hutchinson equation (equation I) used to estimate $SQ_{7,5}$ for the Kakanui and Shag rivers was:

$$SQ_{7,5}^{0.22} = 1.115\,R - 1.9\,H_4 - 0.186\,H_{5678} - 0.083$$

The equation (equation J) used for the Pomahaka and Catlins rivers and Mill Creek was:

$$SQ_{7,5}^{-0.16} = 0.18\,R^{-1.7} - 0.0195\,S_m + 0.265\,S_{defg} + 0.911$$

where: R = mean annual precipitation (m)
H = hydrogeology index (see Hutchinson (1990) for details)
S = slope index (see Hutchinson (1990) for details)

Precipitation values were estimated using the methods recommended by Snelder *et al.* (1997) using the mean annual discharge plus 700 mm for evapotransipiration for each catchment (Table 3). The values for slope and hydrogeology were estimated using Land Resources Inventory data.

The regression equation developed by Pearson (1995) and based on catchment characteristics was also used to estimate low flows ($SQ_{7,m}$) for the same five stations. These estimates were compared to values estimated as part of this study using frequency analysis with the log-normal distribution, to assess the accuracy and usefulness of the regression method. The equation used to estimate log $SQ_{7,m}$ (LQA) was:

$$\mathrm{LQA} = -5.2 + 1.64 \log(P) + 0.03\, DWP + 0.02\, BL - 0.02\, S + 0.34\, VEG$$

where: P = mean annual precipitation (mm)
DWP = soil depth-weighted porosity
BL = proportion of bare land
S = average slope (degrees)
VEG = vegetation index (values ranging from 1 to 2 for bush and forest)

Precipitation values were estimated using the methods recommended by Snelder *et al.* (1997) and other values were estimated using using Land Resources Inventory data (Table 3).

Contour maps of log $SQ_{7,m}$ across Otago developed by Pearson (1995) were also used to estimate $Q_{7,m}$ at each of the 21 stations. These values were compared to the values estimated using frequency analysis with the log-normal distribution to evaluate the accuracy and usefulness of the contour maps, and to revise the contour maps using more detailed discharge data.

Table 3a. – Data used for Hutchinson's equations

Station Name	R (m/yr)	H_4	H_{5678}	S_m	S_{defg}
Kakanui	1.06	0.093	0	N/A	N/A
Shag	0.84	0.067	0.044	N/A	N/A
Mill Creek	1.01	N/A	N/A	15.0	0.443
Pomahaka	1.15	N/A	N/A	15.3	0.499
Catlins	1.50	N/A	N/A	23.5	0.906

Table 3b – Data used for Pearson's equation

Station Name	P (mm/yr)	DWP	BL	S	VEG
Kakanui	1064	0.485	0	24.3	1.00
Shag	835	0.459	0	19.8	1.00
Mill Creek	1008	0.485	0	15.0	1.00
Pomahaka	1147	0.474	0	15.3	1.12
Catlins	1501	0.485	0	23.5	1.46

Parameters are defined in text.

Results and Discussion

Summary statistics

Values for $Q_{7,m}$ ranged from 9 l s^{-1} for Lovells Creek to 5492 l s^{-1} for the Taieri at Outram (Table 4 and Figure 2). The Lovells Creek value, however, was based on only four years of record and is merely indicative. Coefficients of variation (*CV*) ranged from 0.32 to 1.16, but most values were less than 0.7. The greatest skewness was 1.56, with five other stations having values greater than one. The Nevis River had a negative skewness of –0.57, and only one other site had a slightly negative value.

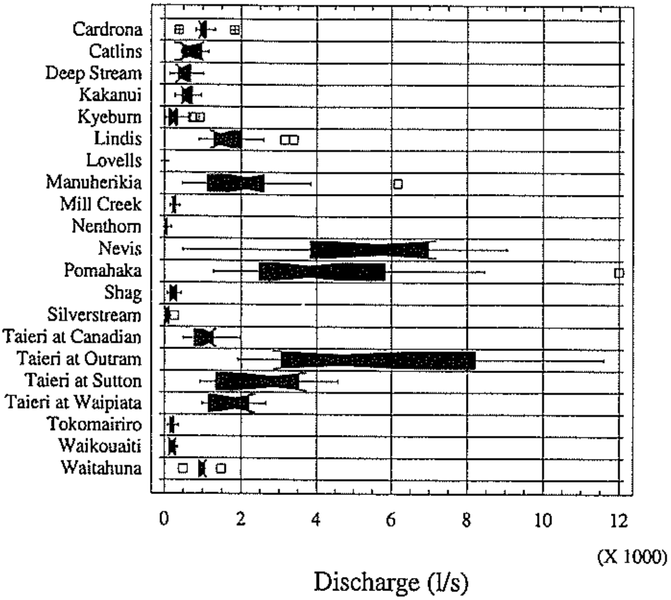

Figure 2 – Box plots of annual low-flow series by station.

Comparison among distributions

Values for $SQ_{7,m}$ ranged from 0.23 l s^{-1} km^{-2} for Lovells Creek and Nenthorn Stream to 7.76 l s^{-1} km^{-2} for the Nevis River (Table 5). The mean $SQ_{7,m}$ for all stations was 2.62 l s^{-1} km^{-2}. Based on the log-normal estimate, $Q_{7,10}$ values ranged from 2.1 l s^{-1} for Nenthorn Stream to 2397 l s^{-1} for the Taieri at Outram. Values for $SQ_{7,10}$ ranged from 0.01 s^{-1} km^{-2} for Nenthorn Stream to 3.93 l s^{-1} km^{-2} for the Taieri at Canadian Flat. The mean for all stations was 1.3 l s^{-1} km^{-2}.

Table 4 – Data and summary statistics for 7-day low-flow series

Year	Station																				
														Taiere							
	Cardrona	Catlins	Deep Str	Kakanui	Kyeburn	Lindis	Lovells	Manuherikia	Mill Creek	Nenthorn	Nevis	Pomahaka	Shag	Silverstream	Canadian	Waipiata	Sutton	Outram	Tokomairiro	Waikouaiti	Waitahuna
1962												2628									
1963												2149									
1964												1823									
1965												8322									
1966												2407									
1967												3911									
1968												2269									
1969					244							3527									
1970					111		16					5844									
1971												1278									
1972					227		10	2747				3089									
1973					43			835				1773									
1974					249		4	857													
1975					205		4	2465				2671									
1976					127			718				2382									
1977	960				298			2860				3101									
1978	383				74			1211			3079	2155									
1979					340			2140			5721	4861						4380			
1980					930	3385		6170			9051	5881						11600			
1981					268	1361		1584			487	3272						2769			
1982	830			454	67	1308		479				4178						2019	154		
1983	1321			740	736	1636		3464		183		12000			1971			10000	326		
1984	1827			653	516	3143		3842	293	88	7776	8462			1350	2068	3479	8838	378		
1985	1061			280	126	1443		1019	196	*	4827	4436			1231	988	1335	3496	142		
1986	923			610	655	2409		2265	151	48	5085	3243			1338	991	2790	5753	115		
1987	1093				231	1499		2361	223	3	6980	4672			1105	1148	1625	4515	185		
1988	952			542	238	1588		2286	220	29	6147	5979		88	784		3527	5649	178		
1989			353	345	105	1257		1761	264	4	5850	3797		31	787		1138	1918	146		
1990			193	544		1367		2072	210	1		2478	108	11	824		1171	2516	110		
1991			375	738	84	1528		1669	192	23		5477	186	29	1215		2125	3807	213	271	
1992			449	393		910		1031	228	2		5276	76	46	1257		2883	4669	169	112	
1993			616	604		1277		2183		43		6294	289	146	1150	2608	3643	5803	286	294	1480
1994		797	689	546		2595		3494	401	72		5996	345	123	1273	1845	3435	5657	237	199	1016
1995		495	142	471	37	1101		2166	384	*	3826	3918	168	37	497	1399	935	3047	90	120	493
1996		442	681	963	792				409	115		3514	283	78	501	2652	4577	8211	179	200	905
1997		1156	1021	966					268	119		6581	443	263		2209	4361	9739	260	343	1068
N	9	4	9	15	23	16	4	24	13	13	11	35	8	10	14	9	14	19	16	7	5
Mean ($Q_{7,m}$)	1039	723	502	590	291	1738	9	2153	265	49	5348	4276	237	85	1092	1768	2645	5494	198	220	992
St Deviation	388	329	277	201	257	735	6	1250	84	57	2340	2257	125	76	388	662	1248	2899	81	87	354
CV	0.37	0.45	0.55	0.34	0.88	0.42	0.68	0.58	0.32	1.16	0.44	0.53	0.53	0.89	0.36	0.37	0.47	0.53	0.41	0.40	0.36
Skewness	0.60	0.90	0.54	0.57	1.31	1.33	0.85	1.42	0.78	1.01	-0.57	1.42	0.33	1.56	0.36	0.09	0.00	0.74	0.86	0.05	-0.08

* zero flows excluded, theorem of total probability used with N=13.

Table 5 – Results of low-flow estimation

Station Name	Area (km²)	$Q_{7,m}$ (l/s)	$SQ_{7,m}$ (l/s km²)	Log $SQ_{7,m}$ (l/s km²)	$Q_{7,10}$ Log-Normal	Weibull	EV1	PP	Diff.* (%)	$\frac{Q_{7,10}}{Q_{7,m}}$**	$SQ_{7,10}$** (l/s km²)
Cardrona	248	1039	4.19	0.62	566	550	532	652	3	0.54	2.28
Catlins	163	723	4.44	0.65	379	362	294	385	5	0.52	2.33
Deep Stream	236	502	2.13	0.33	189	188	141	173	1	0.38	0.80
Kakanui	286	590	2.06	0.31	357	331	329	344	8	0.61	1.25
Kyeburn	376	291	0.77	-0.11	63	50	**-43**	60	**26**	0.22	0.17
Lindis	542	1738	3.21	0.51	1049	818	781	1114	**28**	0.60	1.94
Lovells	39	**9**	**0.23**	-0.64	2.9	2.8	2	**0.2**	4	0.32	0.07
Manuhirikia	2036	2153	1.06	0.02	849	732	522	811	16	0.39	0.42
Mill Creek	44	265	6.02	0.78	171	154	154	183	11	**0.65**	3.89
Nenthorn	213	49	**0.23**	-0.64	**2.1**	**2.6**	-15	0.98	**20**	**0.04**	**0.01**
Nevis	689	5348	**7.76**	0.89	1611	**2352**	**2296**	**2494**	**46**	0.30	2.34
Pomahaka	1924	4276	2.22	0.35	1990	1621	1333	2103	**23**	0.47	1.03
Shag	319	237	0.74	-0.13	95	96	75	91	1	0.40	0.30
Silverstream	92	85	0.92	-0.03	18	15	-14	24	**20**	0.21	0.20
Taieri at Canadian	158	1092	6.91	0.84	621	598	584	501	4	0.57	**3.93**
Taieri at Waipiata	1865	1768	0.95	-0.02	992	980	905	990	1	0.56	0.53
Taieri at Sutton	3066	2645	0.86	-0.06	1158	1177	1016	1126	2	0.44	0.38
Taieri at Outram	4705	**5494**	1.17	0.07	**2397**	2128	1714	2253	13	0.44	0.51
Tokomariro	68	198	2.91	0.46	110	97	92	109	13	0.56	1.62
Waikouaiti	74	220	2.97	0.47	117	118	106	115	1	0.53	1.58
Waitahuna	311	992	3.19	0.50	559	584	531	433	4	0.56	1.80
		Mean =	**2.62**	**0.42**					**12**	**0.44**	**1.30**

Notes:

Discharge values defined in text.

PP = Gringorton plotting position.

Shaded numbers are highest values between log-normal and Weibull distributions.

*Difference is between log-normal and Weibull, calcluated as absolute difference/smaller value.

** $Q_{7,10}$ and $SQ_{7,10}$ are based on log-normal estimate.

Bold values are highest and lowest values for each column.

Bold values for differences are those greater than or equal to 20%.

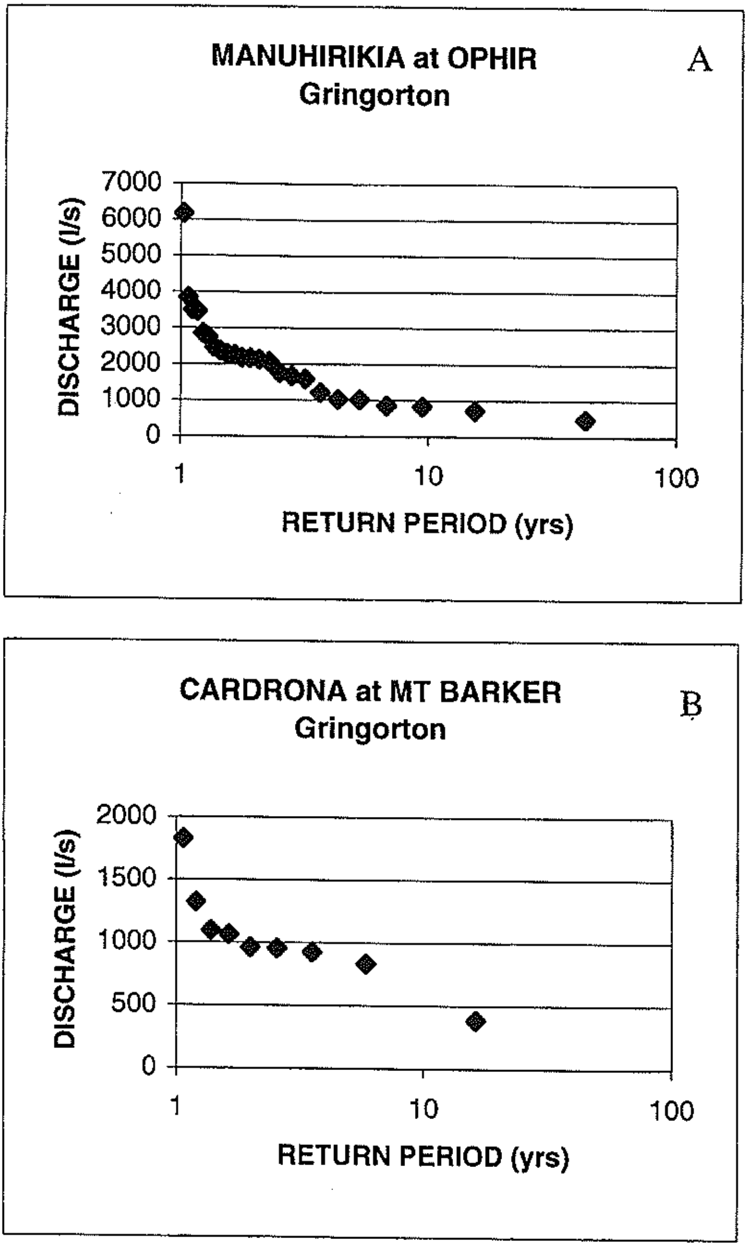

Figure 3 – Example graphs of Gringorton plotting positions for (A) Manuhirikia at Ophir and (B) Cardrona at Mt Barker.

In most cases (15 out of 21 times), the log-normal estimate of $Q_{7,10}$ was higher than the Weibull estimate. Differences between estimates using the two distributions ranged from about 1% for four stations to 46% for the Nevis (the Weibull estimate was higher here). The mean difference for all stations was 12%. Only six stations out of 21 had differences greater than or equal to 20%. The highest estimate using the Weibull distribution (2352 $l\ s^{-1}$) was for the Nevis (not the Taieri at Outram as estimated using the log-normal distribution).

Flow estimates predicted by the EV1 distribution were always lower than for the other two theoretical distributions. The only exception was for the Nevis, where the value was higher than that for the log-normal, but lower than for the Weibull distribution. This value was the highest for the EV1 distribution. The lowest value was –43 $l\ s^{-1}$ for the Kyeburn; two other sites also had negative values, all implying zero flows. These values indicate that the EV1 distribution tends to underestimate low flows more than the other distributions, at least for some stations with small flows.

Figures 3a and 3b are sample graphs using the Gringorton plotting position and data from the Pomahaka and Cardrona, respectively. The empirical low-flow estimates using the plotting position varied in relation to the theoretical estimates (there was no clear pattern of the empirical values being higher or lower than the theoretical values). However, the estimates using the plotting position were much closer to those using the log-normal and Weibull distributions, compared to the estimates using the EV1 distribution, which were negative.

Goodness-of-fit

The Kolmogorov-Smirnov test showed that all three distributions fit the data from each station reasonably well at the 90% confidence level. However, the test rejected the EV1 distribution for the Kyeburn and Lindis at the 95% confidence level. With the exception of the Pomahaka, the Chi-Square test could not be used because the record lengths were too short. The best fitting Weibull distribution for the Pomahaka data (Fig. 4a) was rejected at the 95% confidence level, and the EV1 distribution was rejected at the 99% confidence level. On the other hand, the best fitting log-normal distribution (Figure 4b) was not rejected by the test and appears to fit the data relatively well.

L-moments could not be calculated for Lovells Creek or the Catlins because of the small sample sizes (N=4). The highest L_{CV} value was 0.5694 for the Nenthorn Stream (Table 6). The greatest L_{SK} and L_{KUR} values were 0.3895 and 0.6186, respectively. Nenthorn Stream also had the highest $D(I)$ value (2.71). One other station had a high $D(I)$ value of 2.01, but all other values were less than or equal to 1.67. The high L_{CV} and L_{SK} values for Nenthorn

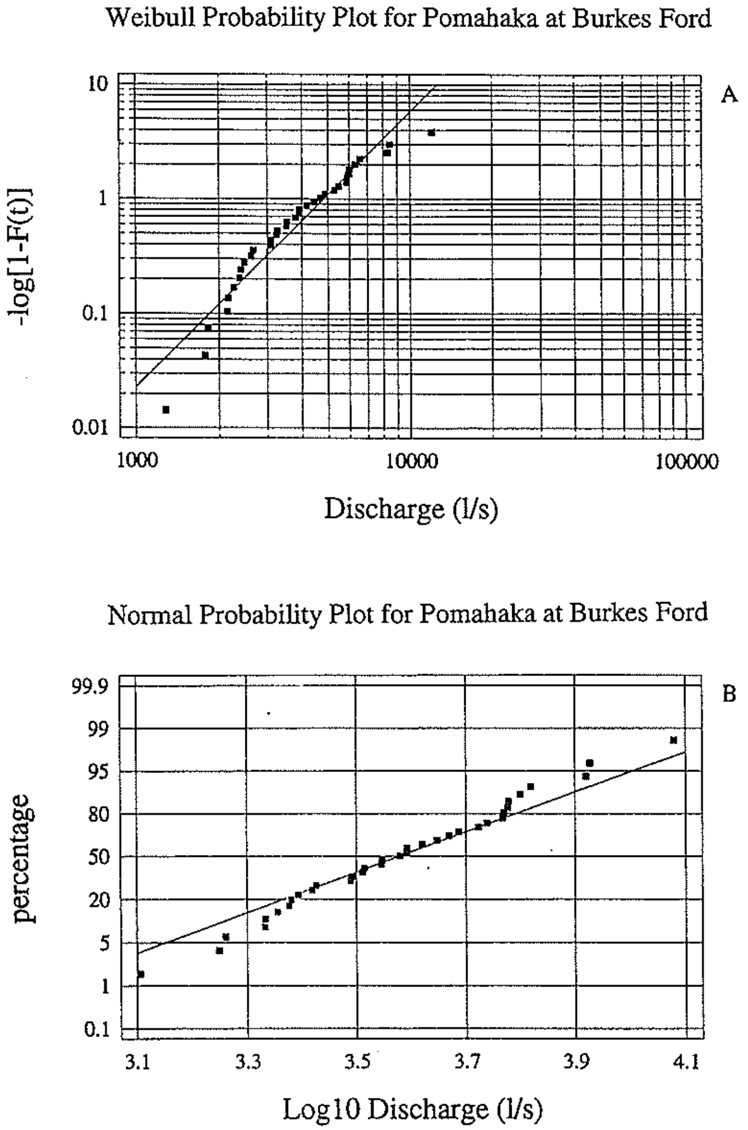

Figure 4 – Pomahaka at Burkes Ford (A) Weibull probability plot and (B) log-normal probability plot.

Stream caused the large discordancy, which might indicate that this station is an outlier and shouldn't be included in the possible group of homogeneous stations within Otago. The reason for the discordancy of this station is not clear. Although there appear to be no discrepancies in the data, it is the only station with records of zero flows. However, Nenthorn Stream had the highest CV (1.16; the only one above 1) and the second smallest $Q_{7,m}$ (49 l s^{-1}). It also had the smallest $SQ_{7,m}$ (0.23 l s^{-1} km^{-2} along with Lovells Creek) and $Q_{7,10}$ and $SQ_{7,10}$ based on the log-normal estimate (2.1 l s^{-1} and 0.01 l s^{-1} km^{-2}, respectively). All of these characteristics may have contributed to the high discordancy.

Figure 5 is a graph of the theoretical distribution curves for L_{SK} and L_{KUR} and the values for each station (except Lovells Creek and the Catlins). The GPA distribution was the best fitting for eight stations, and the GLO was best for six stations (Table 6). The Weibull and the P3 distributions were each the best fitting for two stations. The GPA and LN3 distributions were almost equally best fitting for one station.

With regard to the log-normal, Weibull, and EV1 distributions that were evaluated for differences in estimated values (Table 5), the Weibull distribution fit the data best for nine stations, the log-normal was the best fitting for seven stations, and both fit approximately equally as well for three stations.

The record length-weighted mean values for all stations for L_{SK} and L_{KUR} were 0.1817 and 0.1567, respectively (Table 6). These values are very close to those of the 2-parameter EV1 distribution (0.17 and 0.15, respectively). The value of V_1 for the group of stations was very high (4.47). Values for V_2 and V_3 were 0.59 and –0.64, respectively. This indicates that the mean and standard deviation of the group of stations are variable (possibly due to the influence of the high $D(I)$ of Nenthorn Stream), but that the higher moments can be considered constant for the region. Therefore, a 2-parameter distribution (in theory) should be all that is necessary for the region.

Results of the goodness-of-fit measures (L_{KUR} vs Z values) for the GLO, GEV, P3, GPA and EV1 distributions showed that the data fit most of them adequately, with the exception of the GPA distribution (Z value of –3.15). The best fitting distribution for the group of Otago sites was the GEV distribution with a Z value of –0.23.

The record length-weighted mean values for all stations for L_{SK} and L_{KUR} plotted with curves (and point for the EV1 distribution) for the various distributions showed that the GEV distribution was best for the group of stations. The LN3 and 2-parameter EV1 distributions also fit the data well (Fig. 5).

The influence of the one potential outlier (Nenthorn Stream) on the rest of the stations' discordancy values, and on the homogeneity and best fitting

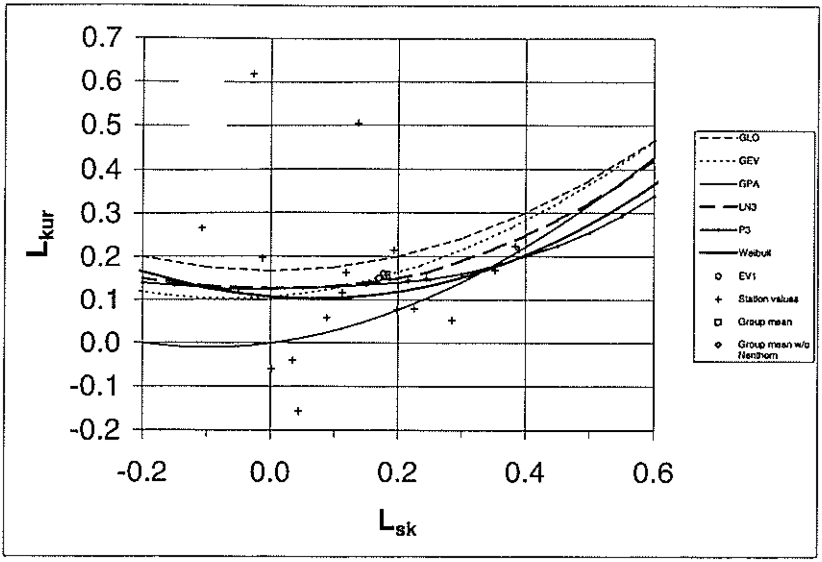

Figure 5 – L_{KUR} vs L_{SK} for standard distributions, station values, and group mean values.

distribution for the group as a whole, was evaluated. The station was removed from the group and all L-moments were re-computed along with the heterogeneity measures. In this case, the highest *D(I)* was 2.61 for the Silverstream, and only one other station had a value above 2 (2.05 for the Kyeburn). Overall, therefore, *D(I)* values decreased. V_1 for the group decreased considerably to 2.61, and V_2 and V_3 were 0.02 and –0.67, respectively. Although V_1 may still be considered somewhat high, the overall homogeneity of the group increased. In this case, the GEV distribution was still the best fitting distribution, but not as good as for the original group. The 2-parameter EV1 distribution was the second best fitting distribution.

Comparison to estimates using regression equations

Pearson's (1995) regression equation using catchment characteristics underestimated all $SQ_{7,m}$ values for the five stations by a large margin (up to 794%, Table 7). Hutchinson's (1990) equations underestimated the $SQ_{7,5}$ values relative to those estimated by Snelder *et al.* (1997), based on the log-normal distribution for two stations (Table 7). For the Pomahaka, Hutchinson's equation resulted in a slightly negative value, implying a zero flow (the estimate by Snelder *et al.* (1997) was 1.34 l s^{-1}). The value using the regression was overestimated for the Shag by 28%. Snelder *et al.* (1997)

Table 6 – L-moments and best fitting distributions

Station Name	L_{CV}	L_{SK}	L_{KUR}	D(I)	Best Dist.*
Cardrona	0.2058	0.1371	0.5043	1.02	GLO
Deep Stream	0.3265	0.1123	0.1163	0.18	Weibull
Kakanui	0.1957	0.1186	0.1636	0.28	GLO
Kyeburn	0.4662	0.3527	0.1678	1.24	Weibull
Lindis	0.2219	0.3840	0.2234	1.55	GPA/LN3
Manuherikia	0.3107	0.1934	0.2143	0.05	GPA
Mill Creek	0.1808	0.2255	0.0798	0.88	GLO
Nenthorn	0.5694	0.2845	0.0523	**2.71**	GPA
Nevis	0.2532	-0.1078	0.2647	1.19	GLO
Pomahaka	0.2831	0.2444	0.1496	0.16	P3
Shag	0.3172	0.0878	0.0584	0.32	GPA
Silverstream	0.4867	0.3895	0.2160	1.67	GPA
Taieri at Canadian	0.1988	-0.0130	0.1952	0.48	GLO
Taieri at Waipiata	0.2247	0.0441	-0.1566	1.48	GPA
Taieri at Sutton	0.2777	0.0017	-0.0597	1.04	GPA
Taieri at Outram	0.3007	0.1983	0.0767	0.12	GPA
Tokomariro	0.2326	0.2159	0.1455	0.28	P3
Waikouaiti	0.2411	0.0350	-0.0404	0.79	GPA
Waitahuna	0.2153	-0.0276	0.6186	2.01	GLO
Record-length weighted mean =		**0.1817**	**0.1567**		

Standardised test values for group of stations	
V_1	**4.47**
V_2	0.59
V_3	-0.64

Distribution for group of stations

	L_{KUR}	Z
GLO	0.194	1.09
GEV	0.155	**-0.23**
P3	0.133	-0.94
GPA	0.067	**-3.15**
EV1	0.150	-0.37

Parameters defined in text.

*Best fitting distribution for each station was determined by the shortest distance from the station point to the nearest curve or point (for EV1 distribution) in Figure 5.

Table 7 – Summary of Specific Q estimates for five catchments in the Otago region

River	$SQ_{7,M}$ (Pearson)	$SQ_{7,M}$ (ORC)	Difference* (%)	$SQ_{7,5}$ (Hutchinson)	$SQ_{7,5}$ (Snelder *et al.*)	Difference* (%)
Kakanui	0.43	2.08	384	0.848	1.42	67
Shag	0.35	0.77	120	0.255	0.20	28
Mill Creek	0.60	5.39	794	-0.567		
Pomahaka	0.81	2.46	205	-0.476	1.34	382
Catlins	1.12	4.57	306	-0.218		

* Difference is calculated as absolute difference in SQ values divided by smaller value.

did not evaluate low flows for Mill Creek or the Catlins, but the Hutchinson value for Mill Creek was –0.567 l s^{-1}, and for the Catlins was –0.218 l s^{-1}, both values implying zero flows.

Precipitation is the largest determinant of the low flows estimated from both regression equations. For comparison, therefore, the equations were also used with only precipitation as the independent variable. For the Hutchinson equations, all values increased using only precipitation. This decreased the differences in the estimates based on Hutchinson's equation and by Snelder *et al.* (1997) for the Kakanui and Pomahaka, but increased the difference somewhat for the Shag. For the Pearson equation, values for the Kakanui and Shag increased somewhat, while values for Mill Creek and the Pomahaka and Catlins decreased. Therefore, the estimates for the Kakanui and Shag were improved relative to the estimates in this study, but the values for the three other stations became worse. This may reflect the influence of the vegetation term in the equation, where high values for southwest Otago catchments contribute more to the low-flow values than in the north Otago catchments. The term is large enough for these catchments that when it is removed, the low-flow estimates are reduced.

Comparison to estimates using contour maps

Most $Q_{7,m}$ values estimated using Pearson's (1995) contour map differed considerably from those measured using actual data for the stations (Table 8). Values were underestimated using the contour map for 13 of the 21 stations, with differences ranging from approximately 6% to over 600%. Values were overestimated using the contour map for eight stations. Only nine stations had differences of 50% or less, and the mean difference for all stations was 137%.

Table 8 – Comparison of $Q_{M,7}$ estimated from Pearson's contour map and measured

Station Name	Log10	$SQ_{M,7}$ (m^3/s)	Area (km^2)	Estimated Q (m^3/s)	Measured Q (m^3/s)	Difference* %
Cardrona	0.95	8.91	248	**2210**	1039	113
Catlins	-0.20	0.63	163	103	**723**	603
Deep Stream	0.30	2.00	236	472	**502**	6
Kakanui	-0.10	0.79	286	227	**590**	160
Kyeburn	-0.20	0.63	376	237	**291**	23
Lindis	0.70	5.01	542	**2716**	1738	56
Lovells	0.00	1.00	39	**39**	9	333
Manuherikia	0.48	3.02	2036	**6149**	2153	186
Mill Creek	0.90	7.94	44	**350**	265	32
Nenthorn	-0.10	0.79	213	**169**	49	245
Nevis	0.80	6.31	689	4347	**5348**	23
Pomahaka	0.00	1.00	1924	1924	**4276**	122
Shag	-0.40	0.40	319	127	**237**	87
Silverstream	-0.20	0.63	92	58	**85**	46
Taieri at Canadian	0.60	3.98	158	629	**1092**	74
Taieri at Waipiata	-0.20	0.63	1865	1177	**1768**	50
Taieri at Sutton	0.00	1.00	3066	3066	2645	16
Taieri at Outram	-0.10	0.79	4705	3737	**5494**	47
Tokomairiro	0.50	3.16	68	215	198	9
Waikouaiti	-0.35	0.45	74	33	**220**	566
Waitahuna	0.25	1.78	311	553	**992**	79
					Mean =	**137**

Bold values are the highest values.
*Difference is calculated as absolute difference/smaller value.

The $Q_{7,m}$ values estimated from the actual discharge data were also used to re-plot low-flow contours across the region in more detail (Fig. 6b). Figure 6a is the original contour map developed by Pearson (1995). A comparison of the two maps showed that the overall patterns of high and low values of $SQ_{7,m}$ across the region were generally similar. On both maps, the highest values occur in the mountainous catchments of the Southern Alps, and directly east of the mountains. However, data gaps exist along some of the boundaries of the region where gauging stations do not exist or were not analysed, particularly along the Southern Alps and western boundary. Dashed contour lines are shown on the map in this area to indicate greater uncertainty. This problem could be minimised by analysing data for all stations, possibly including those in neighboring regions. The driest areas

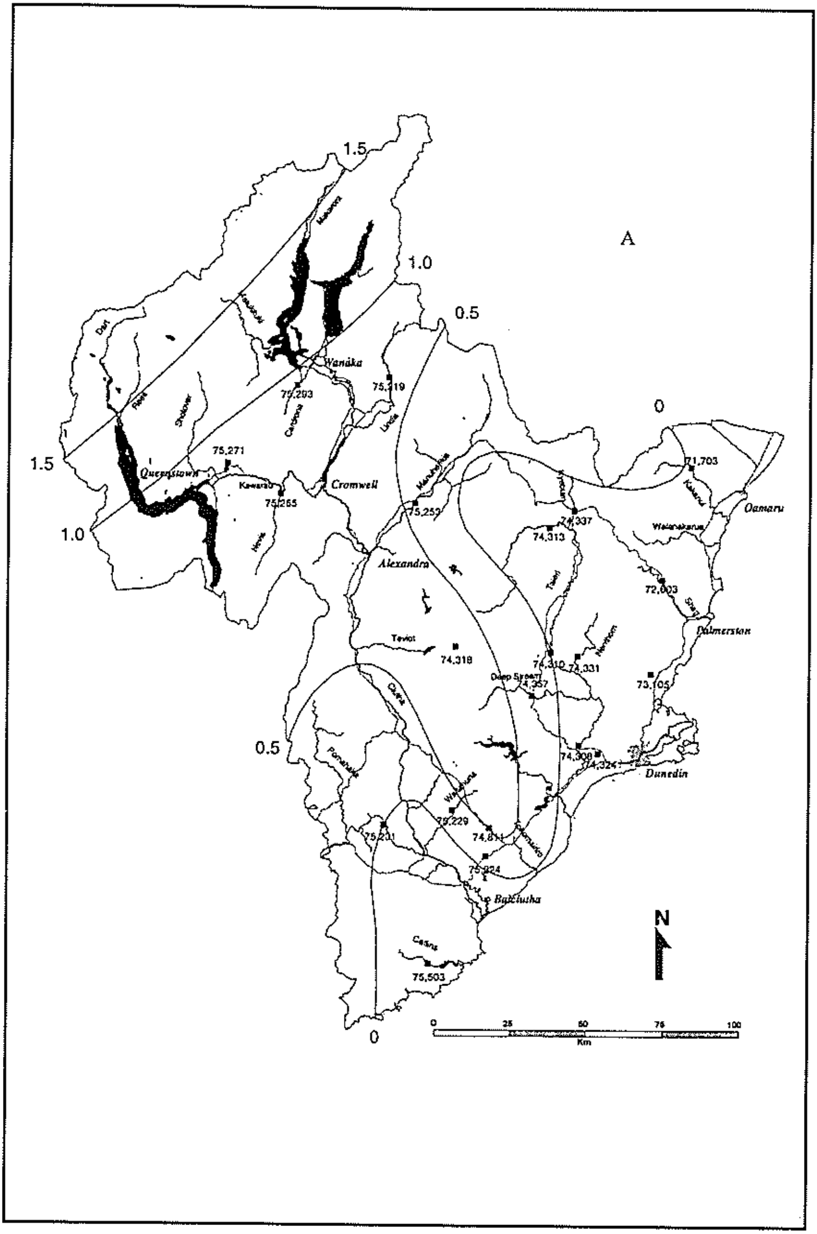

Figure 6A – Contours of log SQ7,m from (A) Pearson's (1995) contour map and (B) results from this study.

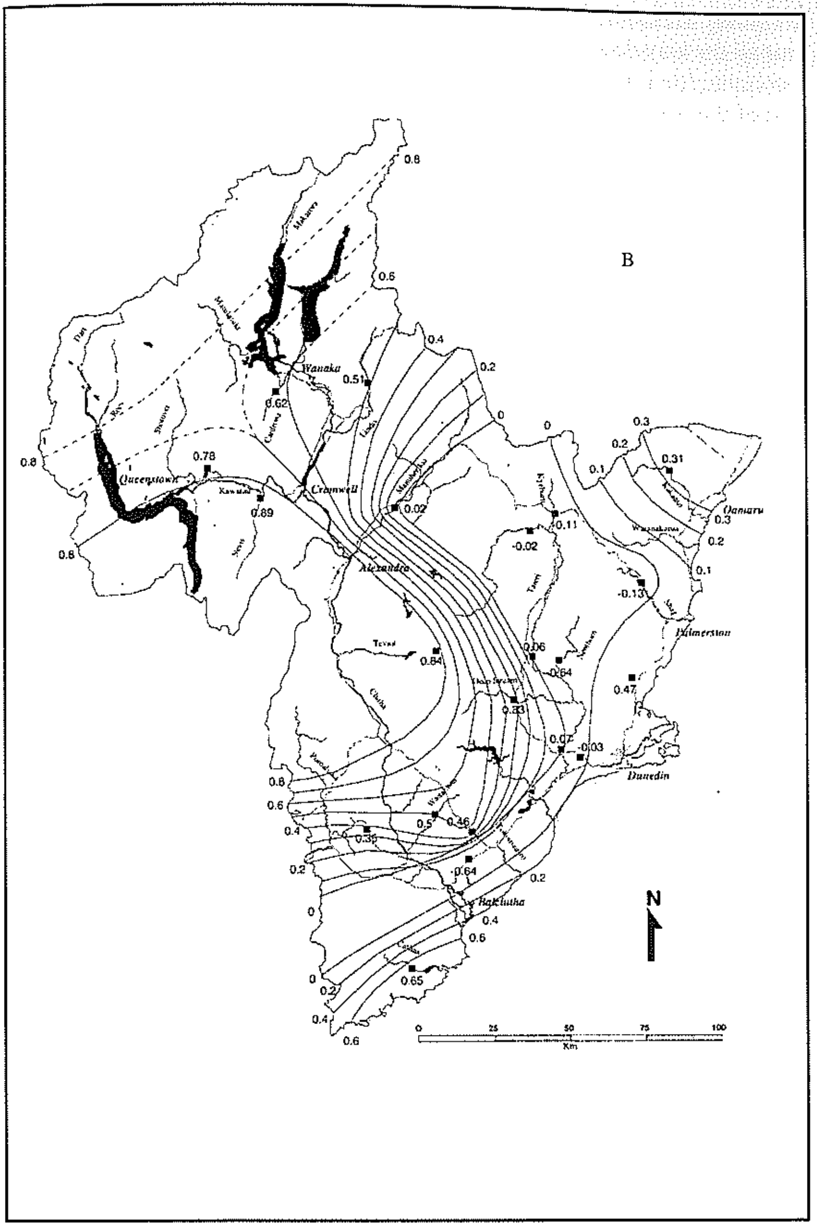
B
0.8
0.6
0.4
0.2
0
0
0.3
0.2
0.1
0.31
Oamaru
0.3
0.2
0.1
Makarora
Matukituki
Dart
Rees
Shotover
Wanaka
0.51
0.62
Cardrona
Lindis
1
0.8
0.78
Queenstown
Kawarau
0.89
Cromwell
Manuherikia
0.02
Kyeburn
0.11
-0.02
0.8
Nevis
Alexandra
Taieri
-0.13
Shag
Palmerston
Teviot
0.84
0.06
0.64
0.47
Clutha
0.33
0.07
-0.03
Dunedin
0.8
0.6
0.4
0.5
0.46
0.35
0.2
-0.64
0.2
Balclutha
0.4
0.6
0
0.65
0
0.2
0.4
0.6
N
0
25
50
75
100
Km

Figure 6B

are in the Maniototo and Strath Taieri, and also along a narrow band to the east all the way down south toward Balclutha, not too distant from the coast. However, the revised, more detailed map showed that most of the western part of the region, as far south as the upper Pomahaka, has high $SQ_{7,m}$ values, and that values also increase somewhat in the North Otago/Kakanui area and the Catlins area. This detail is not accounted for in the original map. The revised map shows an unusually steep gradient near the mid-reaches of the Tokomairiro River. This could indicate some error in the estimated low flow in the river at the West Branch and might warrant reanalysis of the flows there.

Summary and Conclusions

The log-normal and Weibull distributions have generally been recommended and used most often for low-flow analysis, but there are no clear guidelines on which methods should be used for which data. Analysis of 21 stations in Otago showed that although the log-normal estimate of $Q_{7,10}$ was usually higher than the Weibull estimate, only a small proportion of stations had differences greater than or equal to 20%. With the exception of one station, results for the EV1 distribution were always lower than for the other two theoretical distributions. Care should be taken when using the EV1 distribution for individual stations because negative values can be generated, implying zero flows. There was no clear pattern of the plotting position values being higher or lower than the theoretical estimates.

The Kolmogorov Smirnov test showed that the three distributions tested fit many data sets adequately, but could not be used to determine the best distribution. The Chi-Square test could not be used for most stations because the record lengths were too short, and might only be applicable to sites with long records. Therefore, these traditional tests do not appear to be useful for evaluating the best distribution. On the other hand, the L-moments method is very useful for evaluating goodness-of fit, as well as regional frequency analysis. It has many advantages over conventional methods. This method showed that the GPA and GLO distributions were best for most individual stations. Therefore, these distributions are recommended for setting minimum flows at these stations. However, they are not necessarily appropriate for sites in other areas of New Zealand or internationally. Testing and evaluation, similar to that performed in this study, would be required to determine the best distributions in other areas.

The unusually high discordancy for Nenthorn Stream (2.71) indicated that this station is an outlier and shouldn't be included in the group of homogeneous stations within Otago. The rest of the group can be considered homogeneous. The evaluation of discordancy and exclusion of some stations can improve the evaluation of homogenous regions and regional frequency distributions. The L-moments method showed that data from the group fit most distributions adequately. However, the best distribution for the group was the GEV distribution. The LN3 and 2-parameter EV1 distributions also fit the data very well. Any of these three distributions could be used with confidence for the group of sites in Otago. The GEV and LN3 distributions are recommended for individual ungauged locations, however, because the EV1 distribution underestimates flows at some sites. These distributions might also be appropriate for other regions, but this would require some testing.

Regression equations based on catchment characteristics can only be used for very rough estimates of low flows, but more detailed data from nearby stations is probably needed for ungauged sites, especially where minimum flows are proposed. The catchment characteristics that should be used in these equations to provide the best estimates appear to vary between sites and should be evaluated further. Contour maps can also be useful for evaluating general spatial trends across regions and providing rough estimates of low flows at ungauged sites. Detailed evaluation of historical discharge data from many gauged sites across the region can help to significantly improve these maps.

In the future, the L_{CV} values for stations could be mapped and contoured similarly to the $Q_{7,m}$ values. Values of L_{CV} from this map, along with $Q_{7,m}$ values from the revised contour map, could then be estimated for ungauged locations and used with the GEV distribution (with fixed L_{SK}) to estimate low flows for these locations. Alternatively, it may be possible to develop a relationship between the station L_{CV} values and precipitation as the independent variable. Again, L_{CV} values for ungauged locations could then be estimated and used with the GEV distribution to estimate low flows. These are areas of research recommended for the future.

In conclusion, this study has shown that different frequency analysis methods can produce considerably different low-flow results for individual stations. These differences can have significant implications for resource management and users when setting minimum flows. Therefore, it is important to evaluate the methods and differences in some detail and select the most appropriate method for individual stations and for regional analysis. The techniques demonstrated in this study can be applied to other areas in New Zealand and internationally to evaluate and select low-flow frequency analysis methods. They can also be used to provide a sound scientific basis for the delineation of minimum flows for the effective management of competing water uses and protection of instream values.

Acknowledgements

I would like to thank Charles Pearson of NIWA for providing the Hosking L-moments Fortran programmes, many useful suggestions, and review of a draft of the manuscript, and Roddy Henderson of NIWA for L_{SK} vs L_{KUR} curves for some of the standard distributions.

References

Bardsley, W.E. 1989: Graphical estimation of extreme value prediction functions. *Journal of Hydrology 110*: 315-321.

Bardsley, W.E. 1994: Against objective statistical analysis of hydrological extremes. *Journal of Hydrology 162*: 429-431.

Clausen, B.; Pearson, C.P. 1995: Regional frequency analysis of annual maximum streamflow drought. *Journal of Hydrology 173*: 111-130.

Grant, P.J. 1971: Low flow characteristics on three rock types of the East Coast, and the translation of some representative basin data. *Journal of Hydrology (NZ) 10*: 22-35.

Haan, C.T. 1977: *Statistical methods in hydrology.* The Iowa State University Press, Ames, Iowa.

Harrison, W. 1988: The influence of the 1982-83 drought on the river flows in Hawke's Bay. *Journal of Hydrology (NZ) 27*: 1-25.

Hosking, J.R.M. 1990: L-moments: analysis and estimation of distributions using linear combinations of order statistics. *Journal of Royal Statistical Society B 52*: 105-124.

Hosking, J.R.M.; Wallis, J.R. 1993: Some statistics useful in regional frequency analysis. *Water Resources Research 29*: 271-281.

Hutchinson, P.D. 1990: *Regression estimation of low flow in New Zealand.* Publication No. 22 of the Hydrology Centre, Department of Scientific and Industrial Research, Christchurch.

Joseph, E.S. 1970: Probability distribution of annual droughts. Proceedings of the American Society of Civil Engineers Journal of Irrigation and Drainage Division. Vol. 90, No. IR4, pp. 461-474.

Linsley, R.K.; Kohler, M.A.; Paulhus, J.L.H. 1982: *Hydrology for engineers,* McGraw-Hill, Inc. N.Y.

Matalas, N.C. 1963: *Probability distribution of low flows. Statistical studies in hydrology.* U.S. Geological Survey Professional Paper 434-A.

McKerchar, A.I.; Dymond, J.R. 1981: Low flows in Taranaki rivers. *Transactions of NZ Institution of Engineers 8*: 86-96.

McMahon, T.A. 1980: *Analyses of low flow data: A review.* Keynote paper presented to Workshop on Low Flows, Christchurch, 24-25 July, 1980.

MfE (Ministry for the Environment). 1998: *Flow guidelines for instream values – Volume B.* Ministry for the Environment, Wellington. 216 pp.

Mosley, M.P.; Pearson, C.P. 1997: *Introduction: hydrological extremes and climate in New Zealand. In* Mosley, M.P.; C.P. Pearson (*eds.*) *Floods and droughts: the New Zealand experience.* New Zealand Hydrological Society Inc., Wellington, New Zealand. pp. 1-14.

Nathan, R.J.; McMahon, T.A. 1990: Practical aspects of low-flow frequency analysis. *Water Resources Research 26*: 2135-2141.

Otago Regional Council 1998: *Proposed Regional Plan: Water for Otago.* Otago Regional Council, Dunedin.

Pearson, C.P. 1991a: New Zealand regional flood frequency analysis using L moments. *Journal of Hydrology (NZ) 30*: 53-64.

Pearson, C.P. 1991b: Regional flood frequency analysis for small New Zealand basins 2. Flood frequency groups. *Journal of Hydrology (NZ) 30*: 77-92.

Pearson, C.P. 1995: Regional frequency analysis of low flows in New Zealand rivers. *Journal of Hydrology (NZ) 33*: 94-122.

Pearson, C.P.; Davies, T. 1997. *Stochastic methods. In* Mosley, M.P.; C.P. Pearson (*eds.*) *Floods and droughts: the New Zealand experience.* New Zealand Hydrological Society Inc., Wellington, New Zealand. pp. 65-87.

Rodgers, M.W.; Thompson, S.M. 1992: *TIDEDA reference manual* (2nd ed.). Hydrology Centre Publication No. 24, Christchurch, 222 pp.

Snelder, T.; McKerchar, A.; Carter, G.; Henderson, R. 1997: *Assessment of the effects of surface water use policies in the Draft Regional Plan: Water for Otago.* NIWA Unpublished Report CHC97/79, prepared for Otago Regional Council.

Stedinger, J.R.; Vogel, R.M.; Foufoula-Georgiou, E. 1993: Frequency analysis of extreme events. *In* Maidment, D.R. (*ed.*) *Handbook of hydrology.* McGraw-Hill, NY. pp. 18.1-18.66.

Vogel, R.M.; Fennessey, N.M. 1993: L moment diagrams should replace product moment diagrams. *Water Resources Research 29*: 1745-1752.

Manuscript received: 15 March 1999; accepted for publication 10 January 2000.