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Dependence of flood peak magnitude on catchment area

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Abstract

The relationship between flood peak and catchment area is examined and exceedance probabilities of an envelope line for these variables is derived for Westland, New Zealand.

In particular, dimensional and Rational Method analysis, together with statistical evidence, is used to show that flood peak magnitude is proportional to catchment area raised to the power of 0.75. The analyses also support the mathematical form of the empirical TM61 formula and its recommended use in New Zealand for checking estimates of flood peak magnitude in small, ungauged catchments.

Using flood records from Westland, an envelope line is derived from a logarithmic plot of maximum recorded flood peak magnitude against catchment area for Westland. The expected exceedance probability of the envelope and the exceedance probability of the expected envelope are estimated. Values obtained are consistent with assessments provided by station year and index flood methods.

Consequently, where envelope lines can be derived, they provide a useful preliminary estimate or a quick check on other estimates of flood peak magnitude and its exceedance probability obtained, for instance, from rainfall-runoff models or flood frequency analysis.

Keywords

Flood peaks: catchment area: flood runoff: envelope curve: flood estimation: statistical hydrology

Introduction

Numerous empirical formulae have been developed relating flood peak magnitude to climate and catchment characteristics such as rainfall, area and slope. Almost all of these formulae are power laws, and they include catchment area, often as the only independent variable. The Dickens formula of 1865 (Alexander, 1972) is probably the earliest published; the best known are the Rational Method (Pilgrim and Cordery, 1993) and, in New Zealand, Technical Memorandum No. 61 (TM61) (Campbell, 1953; National Water and Soil Conservation Organisation, 1975) which is based on Cook's Method (Hamilton and Jepson, 1940). The various formulae apply to specific regions and have largely been derived using limited data, and in applications they require the use of constants and coefficients that must be evaluated by judgment, albeit within guidelines. The Rational Method continues to be widely used internationally, particularly for urban flood design, and TM61 is often employed in the design of small structures such as culverts and minor bridges in ungauged New Zealand catchments up to 1000 km^2 in area. Nowadays, TM61 is best used only as a check method (McKerchar and Macky, 2001).

Another class of empirical flood formulae attempts to predict maximum flood peak magnitude for a given catchment. They take the form of an envelope curve or line on a logarithmic plot of flood peak magnitude against relevant catchment area for a suite of basins. No account is taken of flood frequency, expressed for instance as exceedance probability. The Myers-Jarvis formula (Chow, 1964) is a classic example. Curves based on New Zealand data are presented by Schnackenberg (1949) and Pearson (1992). Envelope curves or lines provide a rudimentary but nevertheless useful summary of flood experience, and they are best employed to supply a preliminary estimate or a rough check on other estimates provided, for instance, from rainfall-runoff models and flood frequency analysis. Strictly speaking, they should be developed and used only in homogeneous flood regions – where all catchments have similar climatological and physiographic characteristics, and where any effects of changing climate are insignificant during the time period of interest.

Two questions are addressed herein. First, is there any theoretical basis for the empirical formulae (particularly TM61, because of its continuing importance in New Zealand) and what is the value of the catchment area exponent (because area is an independent variable in all formulae)? Second, does recent work on the exceedance probabilities of envelope curves (Vogel *et al.*, 2007) have potential for use in New Zealand?

The aim is to investigate whether some traditional approaches supported or enhanced by more modern analysis remain useful for rapid estimation or for checking on other estimates of flood peak magnitude.

Theory

Two approaches, the Rational Method and dimensional analysis, are taken to examine the dependence of flood peak magnitude, Q_p, on catchment area, A, for basins where surface runoff dominates the generation of flood peaks. Formulae for predicting exceedance probabilities of envelope curves for homogeneous flood regions are then reviewed, with a focus on regions where ranked annual flood series fit an Extreme Value Type I (EV1) (or Gumbel) probability distribution (Beirlant *et al.*, 2005).

Rational Method analysis

Consider steady uniform rain falling with intensity, I, on a catchment and assume that before rain commenced the catchment was in an average state of wetness. Flood discharge will increase with time until it reaches Q_p. The time to peak can be considered as the time taken for runoff from the most distant part of the basin to reach its outlet, that is, time of concentration, t_c. At the time of peak flood let the proportion of rain running off to total rainfall be C_f, the runoff co-efficient. Then, ignoring baseflow,

$$Q_p = C_f IA \tag{1}$$

which is the Rational Method formula. Since t_c is proportional to catchment length, assume that

$$t_c = a\,A^{0.5} \tag{2}$$

where a is a constant. Now, from analyses of extremes of rainfall data it is well known that extreme rainfall intensity with a given probability of exceedance is inversely proportional to the square root of duration (see for example Stedinger *et al.*, 1993)

$$I = I_o(t_{co}/t_c)^{0.5} \tag{3}$$

in which I_o is the extreme rainfall rate for a standard duration t_{co}. From Eqs. 1, 2 and 3 it follows that

$$Q_p = b\,A^{0.75} \tag{4}$$

where

$$b = C_f\, I_o\,(t_{co}/a)^{0.5} \tag{5}$$

In essence, b is a rainfall excess parameter with a given probability of exceedance. Similar results are presented by Johnstone and Cross (1949) and Alexander (1972), although the latter author obtained a value of 0.7 for the exponent in Eq. 4, owing to the use of regression analysis in the derivation.

No account is taken in the above of area reduction factors that enable point rainfalls to be adjusted to give mean intensities over given areas. For example, in the United Kingdom, Faulkner (1999) recommends that a 12-hour point rainfall intensity should be multiplied by 0.97 to yield a mean intensity over 10 km^2 and by 0.86 to yield a mean intensity over 1000 km^2. Experience in New Zealand (Tomlinson, 1978) is that depth-area characteristics of severe storms are extremely variable, especially for more severe storms, and that area reduction factors should be used with caution. Accordingly, the effect of catchment area on areal mean rainfall intensity estimates, at least for catchment areas up to 1000 km^2, is probably of second-order concern.

No account is taken either of variability in rainfall, runoff, and catchment properties. For an analysis which includes such variability see, for instance, Woods and Sivapalan (1999).

Dimensional analysis

The characteristic parameters of a dimensional analysis of flood runoff must describe the fluid (taken here as water, with any sediment content ignored), rainfall, catchment features and flow. Fluid properties are determined by kinematic viscosity, v, and water density, ρ. Rainfall is determined by depth of rainfall excess, D, for a given duration and exceedance probability, F, or return period T (where $F = 1/T$) and both D and Q_p have the same value of T. Antecedent catchment wetness is ignored. Catchment features can be described by A, slope, S, and a dimensionless index, C_p, of the physiographic properties of the catchment, including, particularly, infiltration and shape. Finally, the flow is defined by acceleration of gravity, g. From the theory of dimensional analysis (Yalin, 1971) any mechanical quantity such as Q_p, may be expressed as

$$Q_p = f_1 (v, \rho, g, D, A, S, C_p) \quad (6)$$

in which f_1 is an unknown function. Note that t_c is not included in Eq. 6, as it is a function of A, S and C_p. With ρ, g, A as basic quantities, application of the Buckingham *Pi* theorem to Eq. 1 yields five π terms:

$$Q_p / g^{0.5} A^{1.25} = f_2 (v/g^{0.5} A^{0.75}, D/A^{0.5}, S, C_p) \quad (7)$$

Observe that mass, as represented by ρ, does not enter the problem.

Following unit hydrograph theory, we assume that $Q_p \sim D$, so that Eq. 7 may be expressed as

$$Q_p = g^{0.5} A^{0.75} D f_3 (v/g^{0.5} A^{0.75}, S, C_p) \quad (8)$$

The first π term is dimensionless viscosity, which varies little and may be ignored. Moreover, Beable and McKerchar (1982) found that mean annual flood is statistically independent of S in New Zealand, so that term can also be omitted. With these assumptions Eq.8 may be modified to give the final result:

$$Q_p = [g^{0.5} D f_4(C_p)] A^{0.75} \quad (9)$$

Envelope curves

Vogel *et al.* (2007) analyse exceedance probabilities of an envelope curve for a homogeneous region consisting of m statistically independent flow measuring sites, each with n concurrent years of record of annual maximum flood peak. It is assumed that the regional data are identically distributed and each flood peak series is stationary. A Myers-Jarvis type envelope curve of the form

$$Q_e = c A^{0.5} \quad (10)$$

is adopted, in which Q_e is the maximum annual flood peak discharge recorded at a site and c is a constant.

The probability, p, of exceeding the envelope curve at a site is the joint probability of two events: (1) the occurrence in year n+1 of a flood larger than any previously recorded (E_1), sometimes termed "flood of record" and (2) given the occurrence of E_1, that this flood exceeds the envelope at the site (E_2). We then have

$$p(E_1E_2) = p(E_1)p(E_2|E_1) \qquad (11)$$

Note that if the flood in year $n+1$ falls on the envelope line (on logarithmic scales), then $p(E_2|E_1) = 1$.

It is easily seen that

$$p(E_1) = 1/(n+1) \qquad (12)$$

and that $p(E_2|E_1)$ must depend on the cumulative distribution of the largest flood peak values for the site.

Two quite separate summary measures of $p(E_1E_2)$ can be defined: first, the expected exceedance probability of an envelope and, second, the exceedance probability of the expected envelope. If the objective is to make a probabilistic statement about a given envelope prescribed by historical observations, then the expected exceedance probability of an envelope (*EEPE*) should be employed. If on the other hand the statement concerns the expected, or average, envelope, then the exceedance probability of the expected envelope (*EPEE*) should be used. If the annual flood series at m regional sites can be fitted by a Gumbel or Extreme Value Type I (EV1) distribution (Beirlant *et al.*, 2005) then Vogel *et al.* (2007) show that

$$EEPE \approx 1/[(n+1)(m+1)] \qquad (13)$$

and

$$EPEE = 0.56/(mn+0.12) \qquad (14)$$

and that the ratio *EEPE/EPEE* tends to 1.78 as m becomes large (that is, greater than about 40). Equation 14 is, of course, the plotting position of Gringorten (1963) of the largest value in a data set of mn values. It is also the plotting position that could be used in an application of the station year method of flood frequency analysis (Clarke-Hafstad, 1942; Griffiths, 1989) where series data from all regional sites are lumped together as for a single site.

Finally, Castellarin *et al.* (2005) give formulae derived by Monte Carlo simulation for the number of independent sites in a homogeneous region consisting of m_r sites, each with n years of record, and where some sites may be intercorrelated:

$$m = \frac{m_r}{1 + \overline{\rho_c^w}(m_r - 1)} \qquad (15)$$

where

$$w = \frac{1.4(nm_r)^{0.176}}{(1 - \rho_c)^{0.376}} \qquad (16)$$

and ρ_c is the correlation coefficient in a linear correlation of data between any two sites, and a bar denotes an average.

Application

The theoretical results developed above are now discussed or applied in a New Zealand context, firstly to estimate flood peak magnitude in a given catchment and secondly to assess exceedance probabilities of an envelope curve for a homogeneous region.

Flood peak estimation

Equation 4 implies that the area exponent in an empirical flood prediction formula should be less than unity and close to 0.75. This is verified by many regional flood study results. In New Zealand McKerchar and Pearson (1989) found, using records from 343 catchments ranging in area from 0.01 to 10,000 km^2, that

$$Q_m \sim A^{0.8} \qquad (17)$$

in which Q_m is mean annual flood. This suggests that quoting flood percentiles as discharge per unit area is a practice that is likely to be misleading. Further, Eqs. 4 and 5 show that if C_f is constant and baseflow is negligible, flood data can provide an estimate of an average storm rainfall percentile over a catchment. This is a useful result for catchments where raingauges are sparse.

From Eq. 4, the maps in McKerchar and Pearson (1989) which contour values of $Q_m/A^{0.8}$ are equivalent to maps of b in Eq. 5, which in essence is a rainfall excess parameter with a given probability of exceedance. Conceptually, $C_f I_o$, could be estimated for each element of a catchment from knowledge of rainfall and rainfall losses, and then a pooled value derived for a catchment. In practice, however, rainfall losses are estimated as the difference between rainfall and quickflow, and so $C_f I_o$ is determined only for catchments above flow-gauging sites.

Equation 4, and especially Eq. 9, provide strong support for the TM61 formula

$$Q_p = 0.0139CRSA^{0.75} \qquad (18)$$

in which C, R, and S are physiographic, rainfall and shape factors respectively. In Eq. 9 one can choose $f_4(C_p) \approx CS$, in which case Eqs. 5, 9 and 18 are equivalent. In 1953, when TM61 was first introduced in New Zealand, there were insufficient local data available to calibrate and verify it. This lack of calibration and verification has never been resolved, so TM61 has been regarded with circumspection, if not ignored, by some hydrologists and engineers. However, this analysis shows that the form, variables and area exponent used in the TM61 formula (Eq.18) are consistent with the assumptions upon which it is based.

Envelope curve

Based on evidence presented in Mosley (1981), Beable and McKerchar (1982), McKerchar and Pearson (1989) and Pearson (1991) regarding homogeneity, the Westland region of South Island, New Zealand, was selected to use as an example. Annual flood series at some 13 sites are reasonably stationary and data for each site are independent and identically distributed. In particular, the flood series can be well modelled by EV1 distributions. The period of concurrent record for this approximately homogeneous region runs from 1980 to 2004, that is, for 25 years.

A matrix of correlation coefficients was computed for the 13 sites (Table 1). All intercorrelations were checked to ensure linearity assumptions were met. The high values of ρ in some cases indicate strong statistical dependence between those sites. Equations 15 and 16 and data in Table 1 were used to estimate the number of statistically independent sites. The result is 6 (6.25 exactly). By arbitrarily choosing a threshold value of $\rho_c \leq |0.51|$ in Table 1 to indicate independence between sites, it is a straightforward exercise to select a suite of 6 independent sites. Those chosen were Karamea, Inangahua (Blacks Point), Taipo, Cropp, Hokitika and Haast.

Figure 1 is a logarithmic plot of maximum recorded flood peak magnitude (1980–2004) against catchment area for all 13 sites of Westland. An envelope line was fitted to data from the suite of 6 independent sites. It is defined by the equation

$$Q_e = 72.9\,A^{0.63} \qquad (19)$$

Note that the standard Myers-Jarvis exponent of 0.5 on A is inappropriate for this data set. With $m = 6$ and $n = 25$, Eq. 13 yields a value of $EEPE = 5.5 \times 10^{-3}$ ($T = 182$ years). Equation 14 gives $EPEE = 3.7 \times 10^{-3}$ ($T = 268$ years). The ratio $EEPE/EPEE$ is 1.47, lower than the limiting value of 1.78 mentioned above, because m is small.

To provide a check on the exceedance probability of the expected envelope ($EPEE$) estimate, a station year analysis was conducted using the suite of 6 independent

Table 1 – Cross correlation matrix of annual flood maxima (1980–2004) for Westland flood region, South Island, New Zealand. For details on site records see Walter (2000).

Site number	River and site	95102	93208	93207	93206	93203	91412	91407	91404	91401	91104	91103	90607	90612	86802
95102	Karamea at Gorge	1.00													
93208	Buller at Woolfs	0.63	1.00												
93207	Inangahua at Blacks Point	0.47	0.60	1.00											
93206	Inangahua at Landing	0.58	0.85	0.74	1.00										
93203	Buller at Te Kuha	0.64	0.86	0.56	0.83	1.00									
91412	Pattinson Creek at Weir	0.32	0.48	0.21	0.55	0.53	1.00								
91407	Ahaura at Gorge	0.66	0.62	0.54	0.54	0.56	0.02	1.00							
91404	Grey at Waipuna	0.68	0.62	0.66	0.63	0.60	0.12	0.90	1.00						
91401	Grey at Dobson	0.73	0.77	0.64	0.74	0.74	0.34	0.88	0.92	1.00					
91104	Taramakau at Greenstone Bridge	0.48	0.26	0.30	0.22	0.18	-0.02	0.73	0.64	0.64	1.00				
91103	Taipo at SH 72 Bridge	0.43	0.48	0.51	0.50	0.39	0.17	0.66	0.56	0.61	0.72	1.00			
90607	Cropp at Gorge	0.29	0.43	0.46	0.56	0.39	0.36	0.37	0.48	0.47	0.21	0.51	1.00		
90612	Hokitika at Gorge	0.35	0.27	0.01	0.15	0.25	0.21	0.41	0.30	0.33	0.35	0.22	0.18	1.00	
86802	Haast at Roaring Billy	0.00	0.14	-0.15	-0.03	0.10	0.13	0.32	0.19	0.18	0.30	0.50	0.29	0.49	1.00

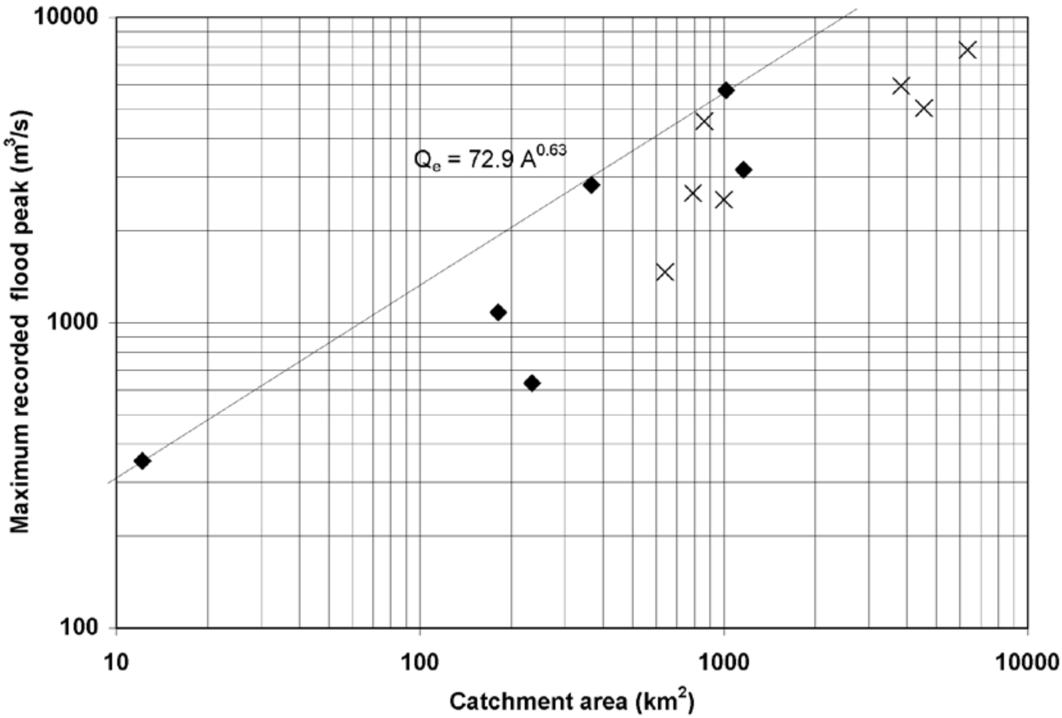

Figure 1 – Maximum recorded flood peak magnitude (1980–2004) versus catchment area showing envelope line (Eq. 19) for Westland flood region, South Island, New Zealand. (♦ denotes a statistically independent site)

sites. The largest dimensionless flood peak discharge, Q_p/Q_m, measured at these sites is 2.11, which occurs in the Cropp record. For this value the analysis yielded $F = 4.72 \times 10^{-3}$ ($T = 212$ years). As a further check, an index flood analysis (Beable and McKerchar, 1982) yielded $F = 5.41 \times 10^{-3}$ ($T = 185$ years). The exceedance probability of the expected envelope (*EPEE*) estimate and the two check methods all use Eq. 15 as a plotting position and in theory should be the same. Differences, which are acceptable, arise here because the check values are provided by the fit of EV1 distributions to a limited and quite variable data set.

The above results derive from a strict application of the methodology of Vogel *et al.* (2007), in that the assumptions of their analysis are very nearly satisfied. This Westland example demonstrates that envelope curves can be readily derived for homogeneous regions in New Zealand and, as mentioned previously, can provide a very useful check on more refined estimates of flood peak magnitude for a specific exceedance probability. Equation 19 could be used in Westland for design. The relevant exceedance probability (the expected exceedance probability of an envelope – *EEPE*) may be taken to be $F = 5.5 \times 10^{-3}$ ($T = 182$ years) which applies to all estimates of Q_e, given A, provided by Eq. 19.

Lastly, it is important to remember that stationarity of annual flood peak time series is assumed throughout this analysis: predictions thus apply only to a climatic regime similar to the one for which the various formulae were calibrated. For recent developments on how shifts in rainfall and flow regimes can be treated, see Gray *et al.* (2005) and Kwon *et al.* (2008).

Conclusions

Dimensional and Rational Method analysis, together with statistical evidence, demonstrates that flood peak magnitude is proportional to catchment area raised to the power of 0.75. The analyses support the form, choice of independent variables and continued use of the TM61 flood peak prediction formula for checking, within the limits prescribed for its application.

Exceedance probability for the envelope line to a logarithmic plot of maximum recorded flood peak magnitude at a site against catchment area can be estimated for Westland. The respective probabilities are the expected exceedance probability of the envelope and the exceedance probability of the expected envelope. The estimates are consistent with other assessments provided by station year and index flood methods.

Where envelope lines and their exceedance probabilities can be derived in homogenous New Zealand regions, they can provide a useful quick check on other estimates of flood peak magnitude and its exceedance probability derived using more sophisticated methods.

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