RESEARCH IN LOOSE BOUNDARY HYDRAULICS AT AUCKLAND UNIVERSITY

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SUMMARY

This paper describes briefly recent research in loose boundary hydraulics done at the University of Auckland. Most of the work is experimental; phenomena have been observed and measurements made and explanations consistent with the laws of mechanics and the observations have been deduced. Some of these explanations are verbal while others use the more precise language of mathematics. The relationship between these studies and river hydraulics is emphasised. The value of and the need for field measurements in the same terms as the experimental and theoretical studies are also emphasised.

INTRODUCTION

Loose boundary hydraulics is concerned with open-channel flow in which the geometry of a channel is determined by the flow of the water it carries. Such flows are difficult to analyse because of the lack of fundamental knowledge about them and because of their inherent complexity. They have considerable practical importance in relation to rivers.

In respect of fundamental knowledge the situation is improving rapidly as research in numerous centres progresses. One of these centres is the University of Auckland and it is the purpose of this paper to outline briefly the ideas that are developing there. The research at Auckland is concerned with the causes of bed features and their effect on resistance to flow, the rate of bed load transport, the relationship between fluid turbulence and bed load movement, river meanders and models of channels with mobile beds. The research described is the work of post-graduate students and academic staff, generally working towards higher degrees.

BED FEATURES

The occurrence of ripples, dunes, standing waves and antidunes on the bed of a channel with loose boundaries is well known. The origin and properties of some of these bed features have been

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studied at Auckland by Raudkivi (1963 and 1965). He has drawn clearly defined conclusions regarding the formation of bed features and their properties, which affect any theoretical analysis which might be made. Raudkivi's conclusions are at variance with assumptions used by Kennedy (1963) and Reynolds (1965) in their valuable theoretical analyses. Raudkivi's conclusions in relation to the work of Kennedy and Reynolds are discussed first.

The analyses of Kennedy and Reynolds are concerned with the flow over a flat bed and the conditions under which an arbitrary periodic disturbance to the bed profile will amplify. Amongst other things they find the wave length of the disturbance that will amplify at the fastest rate and this is related to the wave length of the bed features which develop.

Significant among the assumptions they use are the following:

(1) There is a phase displacement between the fluid velocity and the local bed load transport rate

(2) The resistance to flow is zero, or can be regarded as compensated by a uniform slope.

The first assumption is arbitrary and there is no experimental evidence to confirm it or to deny it. The analysis fails to give a result without it, however.

With regard to the second assumption, Raudkivi's detailed measurements of the local shear stress on the surface of a ripple show that it varies from place to place. Therefore the resistance to flow cannot be regarded as being compensated by a uniform valley slope.

Observations also show that the bed features do not grow over the whole bed, as is implied by the concept of amplification of a disturbance of a particular wavelength. On the contrary, an initial local disturbance is the origin from which a series of ripples is propagated down stream.

Raudkivi explained the growth of a train of ripples by postulating a chance piling up of sand grains on the bed. As the water flows over this pile, it separates from the sand surface and an eddy forms on the lee side of the pile (Fig. 1).

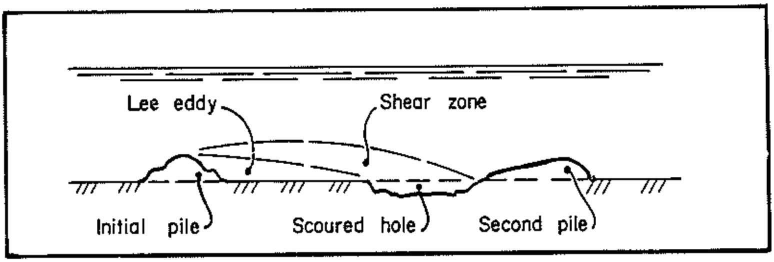


Fig. 1 — EDDY FORMING ON LEE SIDE OF PILE.

The effective lower boundary of the water flowing along the channel is a strongly sheared zone between the lee eddy and the main flow over it. Where this shear zone reattaches to the sand boundary, turbulent agitation makes the grains more mobile so that a hole can be scoured. Further down stream where the turbulence is less, the scoured grains cannot be transported by the mean flow and they stop moving to make a second pile. The same process operates to create a third pile and so on. As the velocity of the water increases, the features at first grow in size, a limiting size at any velocity being determined by the capacity of the drag at the crest to sweep the sand into the lee of the ripple. The erosion phase, when the bed features increase in length and decrease in height as the velocity increases is believed to be due to an increase in the drag at the crest. Anti-dunes, which occur at still higher velocities, have not been studied at Auckland.

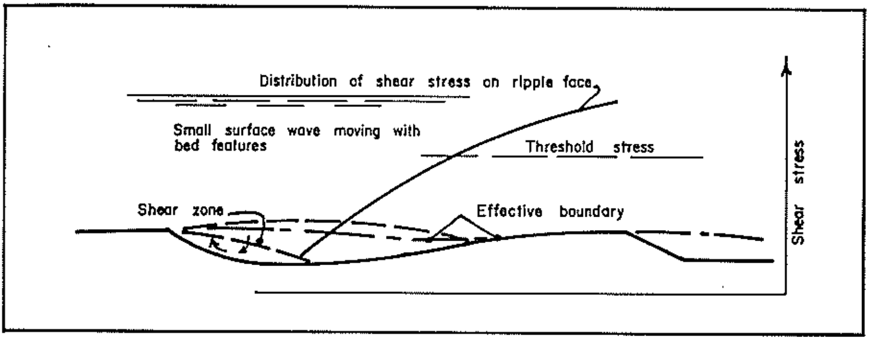


Fig. 2 — CHARACTERISTICS OF FLOW OVER A RIPPLE.

The essential features of flow over a ripple (Fig. 2) are that

(1) the lower boundary of the main flow is partly the ripple surface and partly the shear zone over the lee eddy.

(2) the water surface does not slope uniformly. There are waves on the surface, of small amplitude and bearing a phase relationship to the sand boundary which depends on the characteristics of the flow.

(3) the local drag stress rises from zero in the wake reattachment region to a maximum at the crest.

(4) sand is transported forward for a significant part of the upstream face where the local drag is less than the Shields threshold drag.

(5) the weak reverse flow under the lee eddy sweeps some sand back to the advancing face.

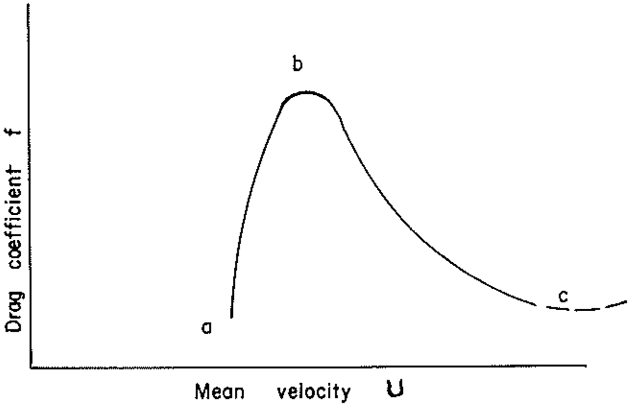
The most significant point here is the part played by turbulence generated in the shear zone in agitating the grains so that they can be moved forward by a sub-threshold drag. At any place on the face of the ripple, forward movement of the sand depends on the turbulence (which decreases towards the ripple crest) and the local surface drag (which increases towards the ripple crest).

The surface undulations must also be noted, for they have an obvious effect on the measurements made to determine the slope of a river.

RESISTANCE TO FLOW

Raudkivi (1963 and 1965) also studied the resistance to flow caused by a rippled bed. He showed how the drag coefficient f varies with mean velocity and in this aspect of the work, his observations confirm those of research workers abroad (see for example Vanoni and Brooks, 1957).

Raudkivi extended the overseas results by making observations of the drag coefficient in the region ab on Fig. 3a where the curve rises very steeply.



a. threshold
c. transition flat bed
a-b. bed features growing
b-c. bed features decaying

Fig. 3a — TYPICAL DRAG-COEFFICIENT CURVE.

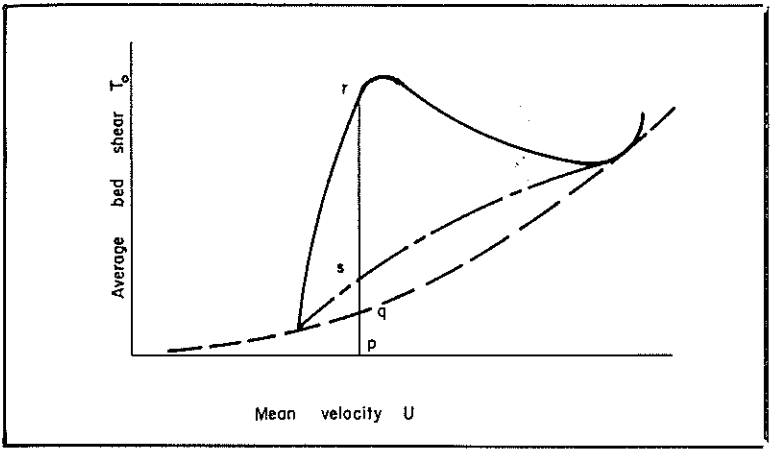


Fig. 3b — CORRESPONDING VARIATION IN AVERAGE DRAG.

The drag coefficient is defined by the equation $To = \frac{f}{8} pu^2$

where To = total bed resistance to flow averaged over area of bed
p = density of water
u = mean velocity

Curves like those shown in Figs. 3 are typical of fine grained materials. The resistance offered by a gravel bed would probably show a less pronounced peak at b in Fig. 3a and hence would not show a range where the shear stress falls with increasing velocity.

The total drag in Fig. 3b can be subdivided into two parts — pq and qr. The ordinate pq is the drag, or resistance to flow, arising from a flat bed of the grains of the channel bed. The dashed curve is the parabola

$$To = \frac{fo}{8} pu^2$$

where fo depends only on the relative roughness of the flat bed and does not vary with u.

The other part of the total drag qr can be identified with the form drag the water exerts on a ripple. By considering the equilibrium of the water under the shear zone (Fig. 2) it can be seen that the total shear in the shear zone balances the normal pressures the sand boundary exerts on the water. (The shear between the sand boundary and the rotating water in the lee eddy is assumed to be negligible.) The resultant of these normal pressures is the equilibrant of the form drag exerted on the ripple by the flowing water, and the shear between the lee eddy and the main flow acts to retard the main flow. Thus, one sees that the surface drag increases regularly with velocity, while the form drag at first increases as the bed features grow and decreases to zero as they decay to the transition flat bed.

Such relationships as these should exist in any channel with a loose boundary and it would be a valuable addition to the literature of this subject it they could be detected for full-sized channels and the results published.

It must be emphasised that such curves are for a particular channel only. Raudkivi has studied a large collection of published data with a view to establishing a generally valid relationship. He found the parameters

$$\frac{u}{\sqrt{(u_*^2 - u_{*c}^2)}} \quad \text{and} \quad \frac{pu_*^2}{vs^*d}$$

to give the least scatter, but his result, which will be published shortly, is at present tentative.

In these parameters,

u = mean velocity
u_* = sheer velocity
u_{*c} = threshold shear velocity
vs^* = submerged specific weight
d = grain size

BED LOAD TRANSPORT

Bed load transport is by means of translation of bed features. With reference to Fig. 4, sand is scoured off the upstream face ab of a ripple and is swept over the crest at b to be deposited in the lee of the ripple bc. In an ideal system, the ripples advance at a uniform rate without changing their shape. Then, measurement of the shape of the ripple and its speed of advance would give the average rate at which the bed load is moving. However, bed features of the kind discussed so far are usually superimposed on much larger sand waves moving along the channel and at any point along the channel, the mean bed over which the ripples move, rises and falls. Furthermore, the ideal train of regular ripples is rarely seen outside laboratory flumes.

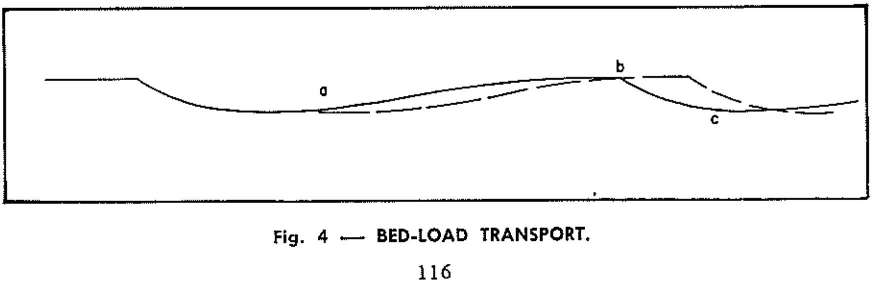
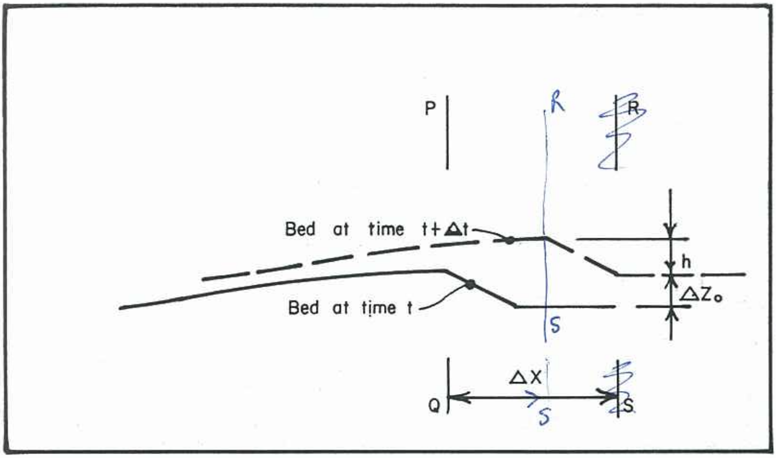


Fig. 4 — BED-LOAD TRANSPORT.

Measurements can be made leading to bed load transport rates if a large dune or sand "wall" can be detected and followed for a short time. For a distance up to about five times the dune or wall height measured downstream from the wall, the sand is not swept forward, being under the lee eddy. Thus, with reference to Fig. 5 a length of channel between sections PQ and RS could be regarded as a trap for the sand swept over the crest. Since the mechanism by which all features advance is by the sweeping forward of sand over the surface, trapping in this way will permit the transport rate to be measured. Thus, with symbols defined on Fig. 5, the total volume of sand trapped in time, Δt, is

$$L\Delta t = h\,\Delta x + \Delta Z_o\,\Delta x$$

$$L = (h + \Delta Z_o)\,\frac{\Delta x}{\Delta t}$$



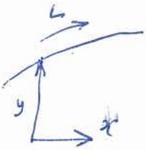


Fig. 5 — MEASUREMENT OF TRANSPORT RATE.

In this, L is the volume rate of bed load transport per unit width of channel. It must be emphasised here that the local rate of bed load transport must vary from place to place along the channel to ensure the translation of bed features. In fact, the rate of change of bed load transport rate with respect to distance $\left(\frac{\Delta L}{\Delta x}\right)$ proportional to $\delta y/\delta x$ equals the speed of migration of the feature if it travels at constant speed without changing its shape.

It will be seen that the equation above suggests a method of field measurement of bed load transport. Laboratory measurements are made by trapping and weighing or by sampling, and most published data have been collected in laboratories. It is desirable that field measurements should also be made. Such a project is being undertaken by the Waikato Valley Authority and their courtesy in allowing the University of Auckland to use the data is acknowledged.

It seems likely that, in any particular channel, bed-load transport will be best correlated with mean velocity. Bed-load formulas generally give the transport rate in terms of bed shear. However, it is not clear which shear should be used and there is much scatter of observed data. Einstein (1950) subdivided the total shear into surface drag and form drag and assumed that the latter does not contribute to bed-load movement. Other authors have not been explicit, but they appear to use the total drag as the argument in their formulas. It is believed at Auckland that neither of these views is correct. In the first place, Raudkivi's measurements show that grains are moved down stream at sub-threshold shears if the turbulent agitation is violent enough. Since this agitation arises from the shearing that occurs in the shear zone over the lee eddy (Fig. 2) the form drag is seen to contribute to grain motion. However, not all the turbulence generated by the shearing of this water reaches the region where the wake reattaches itself to the bed. Some is convected and some diffuses into the main flow of water where it decays. Therefore, not all of the form drag can contribute to bed-load transport. The form drag qr on Fig. 3b can, from this point of view, be subdivided into two parts — qs, the effective form drag and sr, the dissipated form drag. It seems reasonable to expect that the rate of bed load transport would increase monotonically with both mean velocity and the sum of surface drag and effective form drag. As shown on Fig. 3b, both are compatible with a range of velocities through which resistance to flow decreases as velocity rises.

This is an aspect of the problem in which research is just beginning at Auckland. The ideas put forward here are at present qualitative and are not yet supported by measurement.

TURBULENCE

It will be seen from the foregoing that turbulence generated in the shear zone is of first importance. As such it has been studied by Walker (1961) and Sheen (1964). Walker worked on flow past a negative step in a two-dimensional closed channel. His measurements of mean-flow characteristics and turbulence properties are confirmed by similar studies elsewhere. Sheen studied turbulence generated in the wake of a solid form in the shape of a naturally created ripple. A third, more ambitious study is at present nearing completion. This is a theoretical and experimental investigation of the relationship between turbulence characteristics and the forces exerted on an individual grain. The theoretical study of the threshold condition is concerned with the capacity of an impulsive force to impart sufficient momentum to a grain to roll it over its neighbours. It is an extension of the theoretical work of White (1940), who investigated the effect of the mean flow on the stability of an individual grain. The experiment includes an attempt to measure the force exerted on a grain $\frac{1}{4}$in. in diameter. This involves considerable problems in the design of a transducer and in instrumentation.

This study will also shed some light on the effect that the size of the system has on the threshold of grain movement.

INSTABILITY

The ideas of hydrodynamic stability have been applied to loose boundary flows with some hope of success. The problem investigated concerns the response of a straight channel with loose boundaries and the appropriate bed features to an arbitrary disturbance. The following four partial differential equations have been written:

(1) Equation of motion down the channel.
(2) Equation of motion across the channel.
(3) Continuity equation for the water.
(4) Continuity equation for the bed material.

The variables in these equations are two velocity components, depth of water, water surface elevation, boundary shear and local rate of bed-load transport. In accordance with the ideas outlined above, the last two are assumed to be functions of the water velocity, so that the problem contains six unknowns and six equations. A perturbation is imposed on the system, and a condition for a solution to exist is established. This leads to the identification of the ranges of variables within which a channel will be stable or unstable. It is too early to say more than that the investigation looks promising and will be susceptible to the test of experiment.

The form which the unstable channel will take is of obvious practical interest and it depends on whether the perturbation includes a transverse velocity component or not. In the former case, the instability is seen as the origin of meandering. In the latter case, it is seen as the origin of the large sand waves referred to above. These are features with wave lengths comparable with meander wave lengths, but involving a vertical displacement of the bed which does not vary across the channel and on disturbance to the alignment in plan.

Associated with this theorectical study is a detailed investigation of the properties of a fully developed meander. Construction of the meander has been described elsewhere (Callander, 1966) and the measurements made include the topography of the bed, topography of the water surface and distribution of velocity.

LOOSE BOUNDARY MODELS

Work at Auckland in this field has been limited to a study in connection with a model for a flood-protection scheme. In the event, the model was built with rigid boundaries, the size of the grains and some cementing making it undesirable to have a mobile bed in the model. Theoretical studies were made, however, to identify the dimensionless groups which are relevant and to investigate the possibility of and the need for making them have the same values in model and prototype.

The relevant dimensionless groups appear to be the Froude number, the particle Reynolds number (which is proportional to the ratio of grain diameter to the thickness of the laminar sublayer), the entrainment function $\frac{pu_*^2}{vs^*d}$ and the resistance function $\frac{u}{\sqrt{u_*^2 - u_{*c}^2}}$. Yalin (1965) lists these (or their equivalents) and, in addition, the ratio of grain density to fluid density. It can be shown that all of these can be made equal in model and prototype only if the scale ratio is one — a conclusion of little practical value.

However, there is good reason to believe that the Froude number need not be equal in model and prototype. Provided both flows are subcritical and that the model Froude number does not get too close to unity, and provided no surface wave phenomena are involved, the model Froude number can be bigger than the prototype Froude number. There is much precedent for this, for descriptions of loose boundary model practice (see, for example Allen, 1952) usually include the procedure adopted for experimental adjustment of the velocity and discharge scale ratios after building the model for equal Froude numbers.

If the model and prototype Froude numbers are not equal, relationships leading to a choice of geometrical scales and of bed material can be established. The practical value of these conclusions is limited by the following facts:

(1) The scale ratios are small, so that the model must be relatively big. Hence the theory is of value only for a small prototype — e.g. a short section of a river or canal.

(2) The model bed material must have a specific gravity only slightly greater than one and its size has to fall within fairly close limits. Such a material is hard to find. Furthermore, it will rise into suspension readily; the motion of both water and grains is then affected by the density ratio, which should therefore be the same in the model as it is in the prototype. This contradiction has not yet been resolved.

CONCLUSION

This short report has attempted to show the subjects being investigated and their relationship to hydraulic engineering as applied to rivers. In particular, forms in which the resistance to flow and rate of bed load transport can be related to mean velocity have been suggested. The desirability of finding how these vary for individual channels has been emphasised. Such data are doubly valuable in that they are design tools in particular cases and they are elements which can contribute to a comprehensive understanding of loose boundary hydraulics.

REFERENCES

Allen, J. 1952: **Scale models in hydraulic engineering.** Longmans Green, London.

Callander, R. A. 1966: Construction of an experimental meander. **Proc. 2nd Australasian Conference on Hydraulics and Fluid Mechanics.** University of Auckland.

Einstein, H. A. 1950: **The bed load function for sediment transportation in open channel flows.** U.S. Dept. of Agriculture, Soil Conservation Service, Washington. Technical Bulletin No. 1026.

Kennedy, J. F. 1963: The mechanics of dunes and antidunes in erodible bed channels. **Journal of Fluid Mechanics** Vol. 16, Part 4, p.521.

Raudkivi, A. J. 1963: Study of sediment ripple formation. **Proc. ASCE** No. HY,6 Nov., Vol. 89, pp.15-33.

Raudkivi, A. J. 1965: Bed forms and resistance in fluvial channels. Ph. D. thesis, University of Auckland.

Reynolds, A. J. 1965: Waves on the erodible bed of an open channel. **Journal of Fluid Mechanics** Vol. 22, Part 1. May, pp.113-33.

Sheen, S. J. 1964: **Turbulence over a sand ripple.** M.E. thesis, University of Auckland.

Vanoni, V. A.; Brooks, N. H. 1957: **Laboratory studies of the roughness and suspended load of alluvial streams.** Sedimentation Laboratory. California Institute of Technology, Pasadena. Report No. E 68.

Walker, G. R. 1961: A study of the two dimensional flow of a turbulent fluid past a step. M.E. thesis, University of Auckland.

White, C. M. 1940: The equilibrium of grains on the bed of a stream. **Proc. Royal Society** Vol. 174, pp.322-38.

Yalin, M. S. 1965: Similarity in sediment transport by currents. Hydraulics Research Paper No. 6., H.M.S.O. London.