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COMPUTATION MODEL FOR RADIATIVE FLUXES*

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ABSTRACT

A model which enables the monthly mean intensities of the component fluxes of the radiation balance to be computed is described. Standard climatological (screen-height temperature and humidity and the cloud amount) and locational (both spatial and temporal) data are the basic inputs to the model. An accuracy analysis, in which computed and instrumentally observed values are compared, indicates that the accuracies of the two methods are comparable.

The benefits of knowing the intensities of the individual and net radiative fluxes in hydrological research are reviewed. The ability of the model to produce values of the direct and diffuse shortwave fluxes means that the values valid for horizontal surfaces may be corrected to take local site characteristics into account. Thus the model has an application on both the regional and basin scales.

NOTATION

a	slope of surface (degrees).
b	slope of adjacent surface (degrees).
C_1	regression coefficient.
C_2	regression coefficient.
C_3	regression coefficient.
E	mass of water evaporated or transpired.
e	difference between population and sample means.
f	net horizontal outflow of moisture.
G	change in heat energy storage.
g	change in soil moisture storage.
H	sensible heat energy.
LE	energy used in evapotranspiration.
$L\uparrow_h$	outgoing longwave radiation from horizontal surface.
$L\uparrow_s$	outgoing longwave radiation from sloping surface.
$L\downarrow_h$	longwave radiation from atmosphere received by a horizontal surface.

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$L\downarrow_s$	longwave radiation from atmosphere received by a sloping surface.
L'	longwave radiation reflected by adjacent surface and received by sloping surface.
L^*	longwave radiation emitted by adjacent surface and received by sloping surface.
N	number of observations.
O	small, normally insignificant, components of the surface energy balance (e.g. photosynthesis).
Q_h	incoming direct shortwave radiation on a horizontal surface.
Q_s	incoming direct shortwave radiation on a sloping surface.
q_h	incoming diffuse shortwave radiation on a horizontal surface.
q_s	incoming diffuse shortwave radiation on a sloping surface.
R	net radiation.
R_h	net radiation for a horizontal surface.
R_s	net radiation for a sloping surface.
r	precipitation.
$S\uparrow_h$	total shortwave radiation reflected by a horizontal surface.
$S\uparrow_s$	total shortwave radiation reflected by a sloping surface.
$S\downarrow_h$	total shortwave radiation on a horizontal surface.
$S\downarrow_s$	total shortwave radiation on a sloping surface.
S'	total shortwave radiation reflected by adjacent surface and received by sloping surface.
t	number of hours before $(-)$ or after $(+)$ true solar noon.
t_h	time of sunrise $(-)$ or sunset $(+)$ for a horizontal surface.
t_s	time of sunrise $(-)$ or sunset $(+)$ for a sloping surface.
w	angular velocity of earth's rotation (15 degrees per hour).
X_1	horizontal location co-ordinate.
X_2	horizontal location co-ordinate.
Y	vertical location co-ordinate.
Z_h	angle of incidence of direct shortwave radiation on horizontal surface.
Z_s	angle of incidence of direct shortwave radiation on sloping surface.
z	confidence coefficient (critical value).
α	albedo.
γ	change in longitude between the slope and its equivalent parallel horizontal surface.
δ	declination of sun.
ε	emissivity of surface.
θ	latitude of horizontal surface.
θ'	latitude of an equivalent horizontal surface parallel to the sloping surface.
σ	standard deviation.

φ bearing of maximum slope (degrees).
ω inclination of maximum slope (degrees).

INTRODUCTION

The significance of the net amount of radiant heat energy (net radiation) available in any hydrological system may be seen by the appearance of both the net radiation and the energy required for any evapotranspiration within the same system in the familiar energy balance equation:

$$R=H+LE+G+O \tag{1}$$

Evapotranspiration is normally a significant energy and mass sink within a hydrological system. Hydrologists may look upon the process as a mass sink and evaluate its magnitude by considering it as a residual term in the water balance equation:

$$E=r-f-g \tag{2}$$

An alternative, and nonetheless rewarding, approach is to consider evapotranspiration as an energy sink and determine its magnitude by way of the energy balance equation. The link between these two approaches is the latent heat of vaporization. The product of this and the mass of water involved in the evapotranspiration should, but seldom does, equal the use of energy attributed to the evapotranspiration process by means of the energy balance approach. The potential reasons for the inequality are too numerous to be presented here, except for one which represents the basis for the present study.

This is that direct measurements of the energy terms are of necessity made at points with a need for subsequent spatial extrapolation and integration, while the water balance approach has spatial integration as an integral part of the method, although evaluation of the precipitation input does require some manipulation of the basic data.

A study will be made in this paper of a method for providing spatially integrated values of the radiative terms in the energy balance equation, taking topographic influences into account. Subsequent studies will have to concern themselves with the influence of topography on the other components of the energy balance.

The present paper falls into three sections:

(a) The description, in general terms, of a model developed as a result of an earlier study (Hay, 1970). The model enables monthly mean values of the component and net radiative fluxes to be calculated for horizontal surfaces which have no obstructions to their natural horizons (i.e. largely hypothetical surfaces in the case of drainage basins).

(b) A description of the methods by which the above model may be extended so that the influence of the actual topographic characteristics of an area may be taken into account when calculating the magnitudes of the radiative fluxes in those areas (i.e. the spatial extrapolation phase).

(c) To indicate how, once the spatial extrapolation of the radiative fluxes has been performed, the magnitude of the fluxes over a given area may be determined (i.e. the spatial integration phase).

RADIATIVE FLUXES ON UNOBSTRUCTED HORIZONTAL SURFACES

In an earlier study (Hay, 1970) monthly mean values of the component and net radiative fluxes for 175 locations in Canada and Alaska were calculated. The input data required for this study may be placed in three categories, as follows:

(a) Standard Climatological Data
    - screen height temperature
    - screen height vapour pressure
    - cloud amount

(b) Non-standard Climatological Data (optional)
    - cloud reflectivity
    - cloud absorptivity
    - surface albedo

(c) Locational Data
    - latitude
    - longitude
    - date

For many of these parameters it is possible, with minimum modification to the model, to use indirect methods of quantifying the basic input fields. For example, other measures of humidity may be converted to vapour pressure, while actual bright sunshine hours as a percentage of the possible duration may define the extent of cloud cover. In addition, cloud type frequencies may be used to provide information on the absorptive and reflective properties of cloud, and a knowledge of the surface cover types may permit a satisfactory approximation of surface albedo.

The information on radiative fluxes provided by the model consists of monthly mean values for horizontal surfaces of the following fluxes:

Incoming Shortwave Radiation	
direct	Q_h
diffuse	q_h
total	$S\downarrow_h$
Outgoing Shortwave Radiation	$S\uparrow_h$
Outgoing Longwave Radiation	$L\uparrow_h$
Incoming Longwave Radiation	$L\downarrow_h$
Net Radiation	R_h

A comparison of calculated and instrumentally observed fluxes led to the construction of Table 1, which indicates that values of the fluxes calculated using the model will be within the limits of instrumental accuracy at least 60 percent of the time. Thus the computed values provide a reasonable basis for extending the computations to take into account the influence of topography.

TABLE 1 — Comparison of root mean square errors in calculated and observed fluxes.

	RMSE (cal $cm^{-2}day^{-1}$)	
Radiative flux	*Calculated*	*Observed*
total shortwave	±18.8	±17.5*
direct shortwave	±20.2	±30.1**
net	±35.5	±20 to ±30***

* after Titus and Truhlar (1969).

** after WMO Radiation Commission (1961).

*** after Boyd and Reynolds (1963).

TOPOGRAPHIC INFLUENCES ON RADIATIVE FLUXES

Kondratyev (1965) presented a formula for the radiation balance of a slope:

$$R_s = Q_s + q_s + S' - S\uparrow_s + \varepsilon L\downarrow_s - L\uparrow_s + \varepsilon L' + \varepsilon L^* \tag{3}$$

and gave relationships between the terms on the right-hand side of the above equation and the corresponding terms for a horizontal surface. It is obvious, therefore, that the possibility of adapting the previously described model to take topographic influences into account relies on its ability to provide accurate values of the component fluxes of the net radiation. The procedure is to consider the effect of topographic influences on each of the component fluxes, and by summing these 'corrected' fluxes for any given surface the net radiation for that surface may be determined by applying equation (3).

The treatment of all but the first terms on the right-hand side of equation (3) requires several simplifying assumptions, and thus these terms will be discussed collectively. On the other hand, the derivation of the conversion for instantaneous fluxes of the incoming direct shortwave radiation is exact. Therefore this term will be treated separately in the following section.

Direct Shortwave Radiation on a Sloping Surface

The instantaneous ratio of the direct shortwave radiation falling on an inclined surface to that falling on a horizontal surface is given by:

$$\frac{Q_s}{Q_h} = \frac{\cos Z_s}{\cos Z_h} \tag{4}$$

In terms of the equivalent slope concept (Lee, 1962), this same ratio can be expressed, following Rouse and Wilson (1969), as

$$\frac{Q_s}{Q_h} = \frac{\cos \theta' \cos \delta \cos (wt+\gamma) + \sin \theta' \sin \delta}{\cos \theta \ \cos \delta \cos (wt) + \sin \theta \sin \delta} \tag{5}$$

Care must be taken to see that the sloping surface never has a daylength longer than that for a horizontal surface at the same latitude. The limiting conditions are (Frank and Lee, 1966):

$$t_h = \pm[\cos^{-1}(-\tan \theta \tan \delta)]/w \tag{6}$$

$$t_s = \pm[\cos^{-1}(-\tan \theta' \tan\delta) - \gamma]/w \tag{7}$$

However, in most drainage basins, daylength for all surfaces will be even further reduced because of shading by adjacent surfaces (i.e. the local horizon will not be the horizontal plane). Lee (1964) has investigated alternative approaches to the solution of this problem. One solution is through a complete topographic analysis with the evaluation for each slope segment of the losses caused by topographic shading. This procedure may follow a method used by Garnett (1935). Such an analysis is laborious, since individual solutions for each location within the catchment must be obtained.

Lee (1964) has suggested an alternative method which he judged to be equally satisfactory and which is more easily implemented. He defines a plane that will intercept an amount of direct shortwave radiation equivalent to that intercepted by the entire drainage basin. This is done by describing the location of equidistant points along the perimeter of the basin using a three-co-ordinate system and calculating by least-squares multiple regression the best fitting plane, which has the form:

$$Y=C_1+C_2X_1+C_3X_2 \tag{8}$$

This gives the percent slopes C_2 and C_3 in the selected directions, X_1 and X_2, respectively. Thus the azimuth of maximum slope has a bearing (φ) from the X_1 direction that is given by:

$$\tan\varphi=\sin C_3/\sin C_2 \tag{9}$$

The maximum inclination (ω) is given by:

$$\sin\omega=\sin C_2/\cos\varphi \tag{10}$$

Lee calculated a radiation index (the ratio of total daily potential insolation to the maximum potential insolation at the site) for this surface and found that hourly values of the index did not diverge by more than 3 to 4 percent from the radiation indices calculated using the more thorough topographic analysis. Moreover, daily values were almost identical. Thus it appears that topographic influences (slope, aspect and shading) may be adequately accounted for by this expedient method when entire basin analyses are required. For intra-basin analyses the more laborious method based on hourly calculations of the ratio Q_s/Q_h with topographic shading taken into account will have to be used.

Diffuse Shortwave and Longwave Radiation Fluxes on a Sloping Surface

The derivation of the conversions to give the incoming diffuse and reflected shortwave fluxes and the longwave fluxes on inclined surfaces is based on the following assumptions:

(i) the radiant energy is isotropically distributed;

(ii) the thermal and optical properties of all slopes are identical.

Kondratyev (1965) has presented formulae for calculating the changes in the radiative fluxes brought about by the non-horizontal inclination of the slope in question. These may be written as:

$$q_s=q_h\cos^2\tfrac{1}{2}a \tag{11}$$

$$S'=(Q_h+q_h)\,\alpha\sin^2\tfrac{1}{2}a \tag{12}$$

$$S{\uparrow}_s=(Q_s+q_s+S')\,(1-\alpha) \tag{13}$$

$$L{\downarrow}_s=L{\downarrow}_h\cos^2\tfrac{1}{2}a \tag{14}$$

$$L'=(1-\varepsilon)\,L{\downarrow}_h\sin^2\tfrac{1}{2}a \tag{15}$$

$$L^*=L{\uparrow}_h\sin^2\tfrac{1}{2}a \tag{16}$$

The assumptions listed above are best satisfied if the equations are applied to values of the horizontal fluxes integrated over a week or a month. In addition the relationships also assume that surfaces adjacent to the slope in question are horizontal. As a consequence, these horizontal surfaces would not intercept any radiation originating in the sky hemisphere (incoming diffuse shortwave and incoming longwave radiation). Also, the energy either radiated or reflected by the adjacent surfaces is intercepted by the slope at decreasing angles of incidence (thereby increasing the intensity) with increasing inclination of the adjacent surfaces.

This description of this limitation of these relationships is in keeping with the method of experimentation usually employed to verify them – namely, to move the sensing surface of the radiometer through various angles of inclination to the horizontal in an area where the horizon is unobstructed, in accordance with the normal requirements of exposure for radiation sensors (for example, see Kondratyev and Manolova, 1960, and Benseman and Cook, 1969).

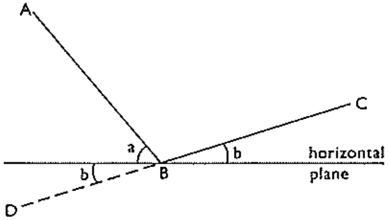

FIG. 1 — Diagrammatic representation of the inclined surfaces.

Within a typical drainage basin, the assumption that adjacent surfaces are horizontal is normally not justified. Fig. 1 shows a more typical, though still somewhat idealized, situation with two adjacent slopes AB and BC inclined to the horizontal at angles of *a* and *b* respectively. The radiation balance of the surface AB is being considered.

Since DC forms the local horizontal for the slope AB, under the assumed conditions of isotropic radiation the ratio of the flux of diffuse radiation falling on that surface to that falling on a horizontal surface would be given by:

$$q_s = q_h \cos^2 \tfrac{1}{2}(a+b) \qquad (17)$$

Similarly for the other relations given in equations (12) to (16). With non-horizontal adjacent surfaces the equations become:

$$S' \; = (Q_h + q_h)\; \alpha \sin^2 \tfrac{1}{2}(a+b) \tag{18}$$

$$S\!\uparrow_s = (Q_s + q_s + S')\;(1-\alpha) \tag{19}$$

$$L\!\downarrow_s = L\!\downarrow_h \cos^2 \tfrac{1}{2}(a+b) \tag{20}$$

$$L' \; = (1-\varepsilon) L\!\downarrow_h \sin^2 \tfrac{1}{2}(a+b) \tag{21}$$

$$L^* = L\!\uparrow_h \sin^2 \tfrac{1}{2}(a+b) \tag{22}$$

SPATIAL INTEGRATION

When equations (17) to (22) are used to determine the spatial distribution of the component and net radiative fluxes within a drainage basin it is necessary to undertake the calculations for a number of locations within the area. This is in contrast to the alternative approach suggested by Lee (1964) which is not applicable here since he was not concerned with fluxes generated within the drainage basin itself. The method of locating the points within the catchment is somewhat arbitrary and depends to some extent on the topographic characteristics of the catchment. Thus, the sampling design could vary from the establishing of points representing distinct slope facets (in a catchment where such differentiation is possible) through decreasing degrees of stratification within the sampling design to a stage where a fully random or systematic method of locating the points would be used. It can be seen that the choice of sampling design is partly a function of how readily distinct and relatively homogeneous topographic components may be recognized.

The number of sampling points used will greatly influence the reliability of the estimate of the total amount of radiant heat energy available within the catchment area. Here one important advantage of the present method over that of direct instrumental observation of the fluxes is immediately obvious. This is the ability to increase the reliability of the estimate by increasing the number of sampling points. The increased costs associated with such expansion are due solely to increases in analysis time rather than to increased expenditure on both instrumentation and analysis. Nevertheless, since the limited accuracy of the conversion formulae also influences the accuracy of any calculated radiative fluxes, an indefinite increase in the number of sampling points will not produce an ever-decreasing error in the estimate of the total radiant heat energy available within the catchment.

If certain assumptions are made (most importantly, that the fluxes are normally distributed and the sampling points are independent and randomly located) then the number of sampling points required can be given (after Budyko and Drozdov, 1966) by:

$$N = \left(\frac{z \cdot \sigma}{e} \right)^2 \tag{23}$$

A preliminary survey will give an estimate of the variability of the radiative fluxes within the catchment, while the remaining two parameters (i.e. z and e) will reflect the confidence one wishes to, or indeed may place in the results. The fact that the assumptions which underly this approach will frequently not be met means that the calculated sample size will normally have to be regarded as a minimum required to gain the desired degree of accuracy.

FURTHER STUDY

The formulae presented here may be derived only after a number of assumptions have been made. Potentially the most significant are those of isotropic radiation and identical thermal and optical properties for all surfaces. Limited studies to date (Kondratyev, 1965) indicate that for long time periods – in the order of weeks or months – these assumptions may not be ill-founded. The present author believes that a far greater challenge is to test the formulae in areas where a more complex topography exists (e.g. mountain and valley terrain).

To this end, it is the author's intention to establish a network of radiation sensors in the Chilton Valley, South Island, New Zealand, where the microclimatological station of the Geography Department, University of Canterbury, is located (Soons and Rayner, 1968). The numerical modelling procedures described in this paper will be implemented for locations within the basin where radiation sensors will be installed. In this way both a testing of the modelling procedures presented here and an evaluation of the radiation balance of the entire basin will be achieved. This will be a major advance towards the ultimate goal of developing complete energy and water balances for the basin with the ability to extend the results into adjacent areas.

APPLICATIONS

Jackson (1967), in a study of the water balance of the Native Forest Catchment at Taita, New Zealand, has shown the value of understanding the complex interactions between topography and the individual radiative fluxes on the one hand and the total radiant energy available for evapotranspiration and the water balance of an entire catchment on the other. He found that the combined effects of slope, aspect and albedo may make the difference between water surplus or deficit in any of the months October to March. This is despite the simplification of his calculations to the extent that spatial variations within the catchment of the longwave radiative fluxes were ignored. A study by Aizenshtat and Zuyev (1952) has indicated that even greater differences would be expected if the differing longwave radiation regimes of the various slopes had been considered.

A study by Rouse and Wilson (1969) has also shown the value of investigating the spatial variations in evapotranspiration rates and soil moisture changes in the light of topographic controls on the component fluxes of the radiation balance. They found that during the growing season in a mid-latitude locality (near Montreal, Canada) the LE/R ratios were high, moderate and low on north, horizontal and south exposures, respectively. This observation enabled the observed between-site variations in soil moisture and air and soil temperatures to be explained in physical terms.

However, once again it must be noted that the authors did not use the full range of formulae presented here, but chose to utilize an empirical relationship between net radiation and total shortwave radiation (not shown to exist for non-horizontal surfaces) once the latter flux had been evaluated by methods outlined in the present paper.

The two examples presented above, together with the general comments made at the beginning of this paper, are ample evidence of the significance of net radiation in water balance studies. The significance of net radiation at a point is no greater than that integrated over an entire catchment, and indeed may be much less if a water balance for the whole basin is being constructed.

Thus steps, similar to those presented here, must be made to extrapolate the results of the normally all too few radiation measurements to as many locations within the study area as are necessary to provide a realistic value of the radiant heat energy available for dissipation as sensible or latent heat.

SUMMARY AND CONCLUSIONS

This paper has attempted to show that it is possible, without the direct use of instrumentation, to determine the component and net radiative fluxes for sloping surfaces in areas of complex topography. The spatial integration of such values for an entire drainage basin has the potential to yield more accurate values of the amount of radiant heat energy available for sensible and latent heat transfer processes than could be obtained by the use of uncorrected but instrumentally observed data gathered at a small number of points within the basin.

Previous equations which were to be applied to correct for the slope of the surface are considered to be inadequate in the context of the topography normally found within drainage basins. Thus new formulae, which take into account the slope of the adjacent surface, have been presented. By considering both the interactions, at a point, of the radiant heat energy fluxes and other components of the energy balance and by spatially integrating point values of the radiative fluxes it is claimed that one may obtain greater insight into the complex energy/mass interactions taking place within a drainage basin.

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