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ANALYTICAL RELATIONS IN THE TIME FREQUENCY OF THE TOTAL RIVER RUNOFF AND ITS GENETIC COMPONENTS

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ABSTRACT

One of the partial problems in the hydrological balance of the time runoff regime, which is solved in the representative watershed of the Czech Cretaceous massif, is described here.

Starting from the already investigated principle of correspondence of *m*-daily river runoff values and *m*-daily ground-water excesses, which contribute to the total runoff, the law of relation between both values is derived. Having the shape of a power function it is expressed in the bilogarithmic functional net as a straight line with three sections. Each of these sections is valid for one of the specific, temporally limited water resources of water-courses. It is possible to carry out the solution for one hydrological year and other, even longer periods (decade, geohydrological period, long-term periods). This way the new analytical formulation method of relations in the time frequency of genetic runoff components and of the total runoff becomes a means which enables, besides other things, the division of ground-water excesses of various origin, the determination of their values and safeguarding in the considered period. It enables also other information concerning ground water, circumstances important for more precise planning and the perspective hydro-economical balances, their exploitation with regard to other interests of the national economy and their artificial enriching.

INTRODUCTION

In the Czechoslovak national programme of the International Hydrological Decade is included also the research of the hydrological regime of space distribution, the time frequency of ground-water excesses and surface-water runoffs in important regions, and their balance.

Within this task, besides other things, were included comprehensive measurements in the field, the research of the hydrological regime of water storage in space and time within the selected watershed of the Mohelka River, with an area of 176.2 km^2, in the promontory of the Luzice Mountains in the north-western hydroproductive part of the Czech Cretaceous massif's main geosyncline.

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The final study concerning the hydrological research in this important region (Slepicka, 1969), documented by charts, numerical and graphical appendices and tables, contains information about geomorphological, geological and hydrographical conditions; it includes also in several comprehensive chapters the detailed progress and results of conditions concerning problems, i.e. 1) the research of the hydrological regime of gradual creation, distribution and changes of ground-water excess and surface-water runoff within the watershed of the main Mohelka River and further eight partial watersheds; 2) the research of the time frequency in the hydrological regime of ground-water excesses and surface-water runoffs and the long-term trend in their creation; 3) the balance of regional regime; 4) hydro-economic conclusions about the hydrological and water-management importance, and exploitation possibility of the region.

This chapter contains a short report about the solution of the second partial problem, concerning the studied time frequency regime of ground-water excesses in relation to the total river runoff in the representative watershed of the Mohelka River.

HYDROLOGICAL IMPORTANCE OF THE GEOLOGICAL GROUND STRUCTURE IN THE PROCESS OF RUNOFF CREATION

The creation of the gradual river runoff in space and time is a complicated phenomenon. A series of various factors participate. Their influence reveals itself in the hydrological process by various space and time assertions, with a varied starting time, intensity and length of duration. Their action is felt in the total hydrological effect, in the fluctuating value of the total runoff. The influence is, therefore, also in the shape of the respective runoff line.

In this case we shall not occupy ourselves with a detailed specification and classification of the basic natural conditions which enable the occurrence and storage of ground water from infiltration. Let us consider, however, in connection with this problem, at least the circumstances which are governing the second phase of this complicated process, i.e. the flow in the water-bearing massif and the outflow of ground-water excesses which openly or furtively are feeding the watercourses discharge.

We can judge that the simplest recession curves will occur in small simple watersheds and most probably during dry years, where river runoffs are just fed by ground-water excesses. But not even such a small watershed of the lowest order will fulfill this assumption if its geological structure is not uniform, the development is uneven or if its structure is secondarily tectonically damaged.

Should we have in mind permeable regions, with sedimentary complexes of great capacity, such as some of the Cretaceous massifs in this country, very rich in ground waters, their genetic runoff process may become complicated, already on behalf of facial changes in the sedimentary massif. Facial changes are the cause of stratificated division in the vertical profile on layers of various water capacity and permeability. Also various successive facial changes in the region, with a view to the physical-mechanical character of the sediments within the same horizon, will change hydraulic conditions. Both contribute to the basic primary character of the hydrological regime of water circulation and are also the cause of the primary distribution of ground-water streaming. However, besides this, another secondary hydraulic influence in the runoff process is felt, the influence of tectonic lines in the regional net of tectonic fractures, with their heaving, drainage and conveying effects. This occurs especially in the research of the space regime in Cretaceous regions, but in other geological formations as well. Whether open conveying dislocations or underground barriers out of their effusive basalt fillings, etc., are coming into consideration, they complicate by their hydraulic effect the primary regime, like the visually and hydrometically easily perceptible surface inflow or withdrawal from a watercourse. Some fractures can connect underground even far-away regions. They bring or divert sometimes from the watercourse considerable quantities of water, and more rapidly than the primary porous streaming permits. Such secondary influences in the hydrological water circulation process can, therefore, complicate the primary porous streaming in various places, during various periods and to various degrees of the final effect. Besides the laminar regime (Slepicka, 1964) in the original sedimentary solid rock with linear law

$$v = kI \tag{1}$$

up to the postlinear law

$$v = (\eta/\sigma)^{m-1} k^m I^m \tag{2}$$

with the exponent within the limits

$$\tfrac{1}{2} < m < 1 \tag{3}$$

there applies in these secondary privileged ways (in lines of open dislocations, in the complicated net of tectonic fractures), also the turbulent flow regime, according to the quadratic law

$$v = c\sqrt{I} \tag{4}$$

According to various hydrometeorological and geohydraulic situations, there occur various combinations of hydraulic regimes in space and time. There are resulting many flow conditions and runoff changes. These are the circumstances which show how difficult it is to generalize natural conditions and how delicate it may be to relate values determined on one point, for stating magnitudes (coefficient of permeability, of infiltration, etc.) for the whole region. This makes a reliable realization of some balance processes based on them difficult; consequently, the results may be considered as mere estimates, having an informative value only.

Up to the present time it is practically impossible to follow these phenomena in detail in nature. Often data about the net of tectonic fractures, about their position, course, or even about their existence, are not sufficient. It is, therefore, necessary to begin at the final hydrological effect and to analyse it suitably, in order to elucidate their existence and effect.

The final hydrological effect of the complicated process is the variable successive river runoff in the space of the considered region, i.e. values of the variable successive runoff in time. Natural factors, their effect in the process of the hydrological runoff, have to appear at last in the shape of the respective characteristic hydrological lines, demonstrating the graphical expression of that collective final effect.

This is in the first case the line of successive profile discharges, which demonstrates the development of discharges along the route of the watercourse L, i.e. of successive watershed surfaces A:

$$Q=\mathrm{f}(L) \qquad \text{i.e. } Q=\mathrm{f}(A) \tag{5}$$

This is the starting point for the solution of problems of hydrological space regimes in the watershed (Slepicka, 1964).

In the second case this is the line of successive time discharges, called also "runoff hydrograph". It demonstrates the successive discharge Q in the time function t

$$Q=\mathrm{f}(t) \tag{6}$$

in the watercourse profile, governing hydrologically the considered watershed. The line is also the basis for the solution of problems concerning the runoff time regime in the watershed (Slepicka, 1957, 1969).

THE ACTUAL PROBLEM

Let us further consider the second case, the creation process of runoff in time. This is a complicated phenomenon. We shall not try to follow it in detail, eventually to reconstruct the application of participating factors, e.g. by means of a physical simulation or mathematical analysis, even if we shall have in mind one of the problems of the total runoff genetically. The introduction to these problems, which are dealt with in another task concerning genetic laws, is described in another study (Slepicka, 1970).

Let us consider in this connection rather the final moment of the process in the total runoff, in order to find out which of the resources finally creates the total river runoff. This is evidently first of all the total surface flood runoff resulting from atmospheric precipitation, further the inflow of ground water created during the precipitation period by bank infiltration in the time of the increased flood stage and by surface infiltration of precipitation in the vicinity, and lastly, there are overflows of ground water from ground-water basins (water-bearing strata), which during certain periods of the process are thus naturally drained.

Each of these watercourse resources has its time, place and magnitude of effect in the process, another intensity of manifestation, development and duration within the creation period of the total river runoff. This of course must at last appear in the position and shape of corresponding curves of the total runoff Q and time lines of ground-water excesses P which are derived from them and consequently also in the duration lines T_Q and T_P.

In case of a sufficiently detailed construction of lines T_Q derived from hydrographs for a certain period, for the hydrological year, it is obtained as a system of partial, mutually connected duration lines for individual partial runoff periods. The duration line is the simpler and relatively lower or inversely more complicated and higher, the simpler and more complicated are the hydrometeorological and runoff conditions in the given period and the geological structure of the region.

The representative watershed of the Mohelka River is just suitable for the solution of the studied problem. There are sets of daily average runoff values at hand for the final profile of the watercourse for many years. The watershed itself is not characterized by simple natural conditions; this enables us to obtain not exceptional, but a more general expression of the studied relations, which can be more suitable even for other practical applications.

The Mohelka River watershed is geologically built in the frame of a broader region by sediments of the Cretaceous formation. These are variously permeable layers of sandstones of the middle Turonian on relatively impermeable sandy marls of the lower Turonian. The original sedimentary solid rock in the broader region is, however, dislocated by tectonic fractures, which participate to a high degree in the process, as is known from the solution of the watershed space regime (Slepicka, 1969). These are on the one hand conveyable dislocations, on the other hand swelled shifted impermeable layers, effusive basaltic veins, etc. The ground is furrowed by deep valleys. The one-sided hydrographical net is composed of four right-bank tributaries, running in the generally south-west direction of the ground-water flow towards the Mohelka River, the main watercourse of the watershed with its channel on the bottom of canyon-like valley; it flows with its longer middle section across the direction of ground-water flow in the water-bearing sandstone layers, into which the valley is deeply sunk underneath the level of the mountain plateau. The length of the watercourse $L=36.7$ km; the area of the watercourse $A=176.2$ km^2.

From the set of daily discharge values, the lines of successive time discharges (hydrographs), $Q=\mathrm{f}(t)$ have been plotted for the hydrological years. By excluding the ground-water excesses, their hydrographs $P=\mathrm{f}(t)$ have been constructed. For both runoff categories it has then been possible to construct the respective duration lines

$$T_Q \equiv Q_{(m)} = \mathrm{f}(m) \tag{7}$$

i.e.

$$T_P \equiv P_{(m)} = \mathrm{f}(m) \tag{8}$$

where $Q_{(m)}$, $P_{(m)}$ represent m-daily runoffs.

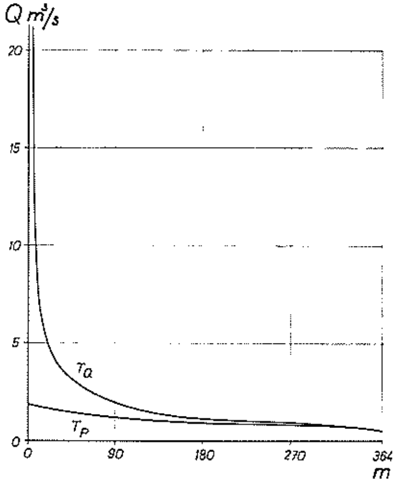


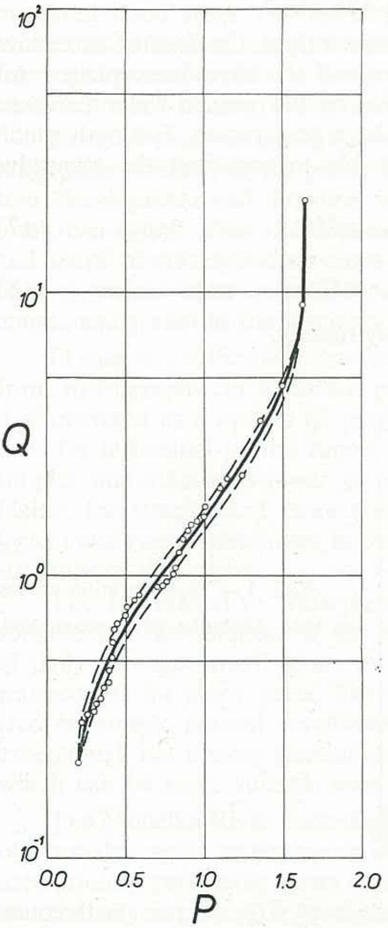
FIG. 1 — Flow-duration curves for 1946, Mohelka River watershed.

It is known that runoff values Q_a, Q_b in two watercourse profiles, with the same interval of duration (m), are corresponding values. From Fig. 1, where the line T_p is annexed to the line T_Q, it is possible to deduce that the quantity of ground-water excesses and the quantity of the total runoff, to which belongs the same interval of duration (m), are corresponding values. Based on this principle and on equations (7) and (8), it can be found that between both these values there exists a functional relation

$$P_{(m)}=\mathrm{f}(Q_{(m)}) \tag{9}$$

Natural laws, especially those relating to runoffs, usually have the shape of potential functions. If this assumption is to be valid for the considered runoff relation (9), it should be expressed in the bilogarithmical functional net as a straight line.

The task is further solved for two actual situations, i.e. for



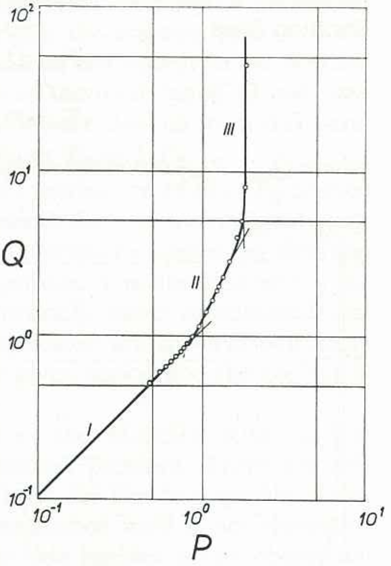


FIG. 2 (left) — Semi-log plot of m-daily values $Q_{(m)}$ and $P_{(m)}$ for the year 1944 Mohelka River watershed.

FIG. 3 (right) — Log-log plot of m-daily values $Q_{(m)}$ and $P_{(m)}$ for the year 1944 Mohelka River watershed.

two extreme hydrological years in regard to the runoff, e.g. for the "dry" year 1944 and "wet" year 1946, in the middle of the representative decade 1941–1950.

On Fig. 2 are plotted in the semilogarithmical net m-daily values $Q_{(m)}$ and $P_{(m)}$ for the year 1944. For the set of points, boundary lines and the centre line are constructed. The centre line represents an equilibrated line of a genetic relation between the values $Q_{(m)}$ and $P_{(m)}$ for hydrological situations, which occurred during the considered period (hydrological year). Fig. 3 in the bilogarithmical net confirms the assumption of the power function shape

$$P_{(m)} = aQ_{(m)}{}^{b} \tag{10}$$

It is expressed by a broken line to which the plotted points are closely lined up.

The division into three sections points to three validity ranges of partial laws. They are mutually connected with short transitory curves. Each of the validity ranges has its own functional dependance expression (10). After the evaluation of parameters a and b they have, in the given case, the shape

$$\left.\begin{array}{l} \text{I} \;.\;.\;.\;.\;.\; P = 0.75Q \\ \text{II} \;.\;.\;.\;.\;.\; P = 0.80Q^{0.5} \\ \text{III} \;.\;.\;.\;.\;.\; P = 1.45Q^{0.0147} \end{array}\right\} \tag{11}$$

Therefore, each abscissa expresses a dependence for a certain functional period with a limited value region, which belongs to a certain category of water resources, to a certain genetic component of the total runoff. It results, from the graphical picture of the functional relation, that there are genetic regions of the total runoff, the extent of which are measured by the value of ground-water excess duration, i.e. their individual types in the duration process of the respective total runoff quantities. The section I for the range of the lowest runoff values, practically expresses the linear dependence between the total m-daily runoff $Q_{(m)}$ and the m-daily excesses of the ground water $P_{(m)}$, which are participating on it. These are evidently overflows from the deep ground-water storage. The following intermediate section shows a lower increase of the total runoff out of the ground-water contribution. It belongs to the genetic phase, when the outflow is fed by ground-water inflows created during the higher runoff stages by infiltration into the bank zone and by infiltration of precipitation in the neighbouring area. The last genetic phase of the total runoff represents section III.

It shows, during the highest river runoff values, a very low contribution out of the inflow of ground water. In the given case this amounts only to 2.5% of the value, reached in both previous cases.

The intensity of the ground-water contribution in the total runoff within the individual ranges of the functional power relation is given by the value of the direction of the tangent line, which expresses the value of the exponent b in equation (10), for individual genetic phases. The direction of the tangent line $\mathrm{tg}\alpha = b$, can probably gain the value within the limit

$$0 < b < 1 \tag{12}$$

The value $\mathrm{tg}\alpha = b = 0$ may occur in the case when, in this range of the total runoff creation, no contribution from the ground-water storage will appear, as it is for certain cases also presumed by Kudelin. The straight line expressing the dependence in the bilogarithmic net would be parallel with the axis Q. In the given case this is a very small value, $\mathrm{tg}\alpha = b \rightarrow 0$. This would mean that in the considered milieu of Cretaceous sandstone layers even during a dry year, the watercourse is fed by overflows from the ground-water storage; however, during high stages and runoffs in the period, the same decrease considerably, with the increasing of total discharge, on a minor value. During the flood runoff, especially in the culminating phase of the floodwave, the inflow can have even a very small value.

On the contrary, the second boundary value $\mathrm{tg}\alpha = b = 1$ finds application during periods of low total runoffs; the river discharge is kept by draining the proper ground-water storage, in the given case from the thick water-bearing complex of the region's Cretaceous layers.

Between both marginal phases the intermediate range of river runoff creation is formed, as a result of a complicated process with prevalent infiltrated ground waters in the bank zone and near areas of the neighbouring region; the value of the exponent $b = \mathrm{tg}\alpha$ moves within the given limits (12). Overlapping effects of individual genetic components in space and time are evidently the cause of boundary ranges of individual components not being always completely expressive; transitory sections are created, which link by arches the prescribed straight lines in the log-log system.

Let us, as an opposite pole to conditions in the dry year 1944, analogically consider the development of runoff conditions in the wet year 1946, in the middle of the representative decade 1941–1950. In the bilogarithmical functional net in Fig. 4 the dependence (10) appears again as a broken line with three sections. Each represents a power function, valid for one of the resources on the total runoff. Their shapes are:

$$\left.\begin{array}{ll} \text{I} \;.\;.\;.\;.\;. & P=0.925Q \\ \text{II} \;.\;.\;.\;.\;. & P=0.89\ Q^{0.5} \\ \text{III} \;.\;.\;.\;.\;. & P=1.90\ Q^{0.0075} \end{array}\right\} \qquad (13)$$

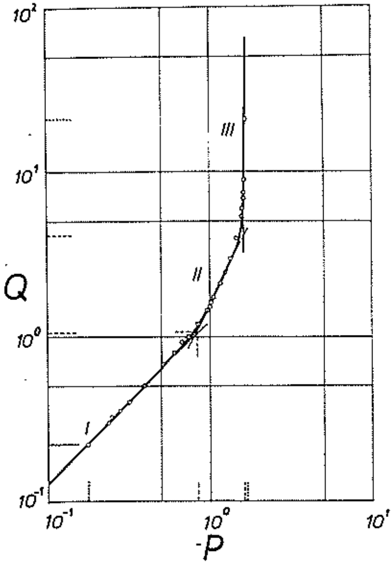


FIG. 4 — Log-log plot of m-daily values $Q_{(m)}$ and $P_{(m)}$ for the year 1946, Mohelka River watershed.

The sections are mutually connected by means of transitory ranges, which are a little broader than for the year 1944. This points to a more complicated runoff process in the hydrometeorologically richer situation of the wet year 1946. Nevertheless, even in this case is the relation between the m-daily total river runoffs and the m-daily ground-water contributions appropriate. In section I the exponent $b=1$ demonstrates the direct dependence of both runoff categories; in range II for the dry year 1944 and for the wet year 1946, and — as we have ascertained even for normal years — the exponent $b=\frac{1}{2}$. This may be considered as a certain characteristic of the resources or of the watershed. In the range III is the exponent at a higher total runoff in the wet year lower. A large prevalence of precipitation water is shown, while the contribution of the ground-water inflow is very low.

In considerably complicated situations and large watersheds, between ranges I and III there can occur, besides range II, an intermediate range IIa which represents the category of infiltrated ground-water inflows from distant areas, eventually also from neighbouring watersheds, from which they come to the local outflow localities and fronts after important open fractures, interconnecting underground areas.

CONCLUSION

By the above-mentioned process it is possible to divide the total river runoff on individual genetic components, together with the distribution of individual genetic categories of ground-water excesses, according to the safeguarding of their quantities; their quantitative contribution is limited in the total runoff together with the duration length of occurrence in the considered period (hydrological year, etc.).

Analytical results based on the new process according to the discovered principle testify on the one hand a satisfactory degree of obtainable precision, and on the other hand a reliability of the new methods for practical application of water-management planning in geohydrological forecasts, perspective balances and assured occurrence of ground-water excesses for water-supply purposes. The process can namely be successfully applied for longer periods (hydrological year, decade, geohydrological period, long-term periods).

A great advantage is that the basis for the solutions is formed by a set of data concerning daily runoffs within the considered period, which are already available at the State Hydrological Service, generally even for longer periods.

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