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NEW ZEALAND REGIONAL FLOOD FREQUENCY ANALYSIS USING L-MOMENTS

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ABSTRACT

Linear moments (L-moments) of statistical distributions do not raise data to powers of 2, 3 and 4 as required for variance, skewness and kurtosis respectively, and thus give better parameter estimates for data containing outlying values. L-moment ratio analogues of coefficients of variation, skewness and kurtosis are more reliable for discerning homogeneous regions and identifying likely parent statistical distributions. L-moments are estimated for 275 annual maximum flood peak series from New Zealand to illustrate their use in regional flood frequency studies.

INTRODUCTION

There are many ways of fitting statistical distributions to sample data, e.g. graphical, methods of moments, maximum likelihood, least squares, maximum entropy, probability weighted moments, etc. All methods quantify parameters of statistical distributions using the sample data. The method of L-moments equates linear combinations of the sample data to corresponding theoretical expressions involving parameters of statistical distributions in order to specify these parameters. It is mathematically equivalent to the method of probability-weighted moments, which was used by McKerchar and Pearson (1989, 1990) for New Zealand floods. Conventional moments; i.e. second, third and fourth moments raise raw data to powers of 2, 3 and 4 respectively, to obtain estimates of standard deviation, skewness and kurtosis; L-moments avoid non-linear transformations of data. Use of non-linear transformations can lead to distortions, and hence poor parameter estimates, when there are outlying values in the data.

Recent hydrological literature on statistical theories for dealing with annual maximum flood series (e.g. Hosking *et al.* 1985a; Lettenmaier and Potter 1985; Wallis and Wood, 1985; Lettenmaier *et al.* 1987; Cunnane, 1989; Wallis, 1989; Potter and Lettenmaier 1990; Hosking, 1990; Hosking and Wallis, 1990) has shown that probability-weighted moments and L-moments often are superior to standard estimation techniques, particularly for regional studies.

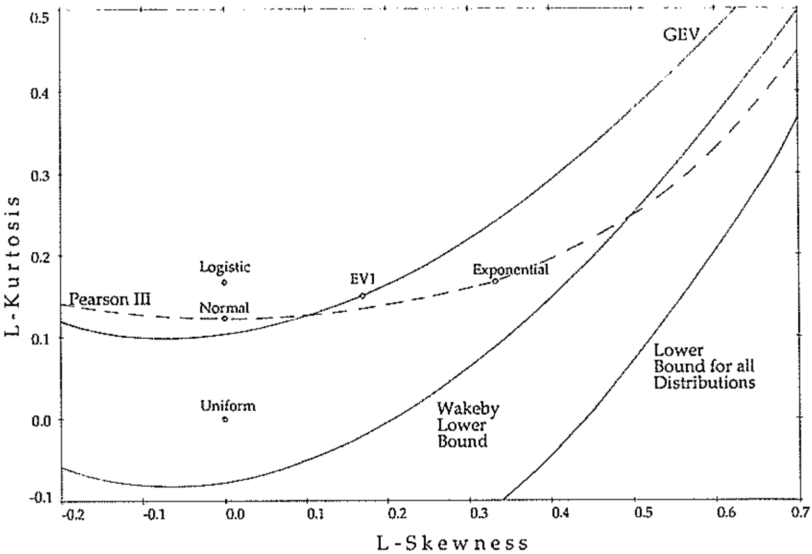

FIG. 1—L-moment ratios of some common statistical distributions. The EV2 distribution is the GEV curve to the right of the EV1 point, and the EV3 is the GEV curve to the left.

Population L-moment ratios (L-CV, L-skewness, L-kurtosis), analagous to coefficients of variation, skewness and kurtosis, are mathematically bounded. However conventional moment ratios are unbounded, which is a disadvantage since sample estimators are always bounded (Kirby, 1974), and so cannot attain the full range of population values. However, sample L-moment ratios can take on all feasible values for population L-moment ratios (Hosking, 1990). This advantage can be utilised, using L-moment ratio diagrams comparing sample values from a number of sites with population values of several statistical distributions. Hosking and Wallis (1991) have developed statistical tests for decisions on regional groupings and likely distributions for flood frequency analysis based on L-moment ratio diagrams.

In this paper L-moments are briefly defined (they have been extensively defined elsewhere e.g. Hosking, 1990; Chowdhury *et al.,* 1991), and then applied to regional flood data from New Zealand.

L-MOMENTS

Population L-moments

Population L-moments (λ_r, r = 1, 2, 3, etc) are defined as linear combinations of expected values of order statistics (Hosking, 1990). The first L-moment (λ_1) is the mean of a statistical distribution, and is identical to the first conventional moment. The second L-moment (λ_2) is a linear measure of spread or dispersion analagous to standard deviation. L-moment coefficient of variation is defined as L-CV = $\tau_2 = \lambda_2/\lambda_1$. Other L-moment ratios are $\tau_r = \lambda_r/\lambda_2$ for r = 3, 4, 5, etc. Hosking (1990) shows that τ_3 and τ_4 are measures of a distribution's skewness and kurtosis respectively. τ_3 is called L-skewness and τ_4 is called L-kurtosis. L-moment ratios are bounded so that $|\tau_r|<1$ for r = 3, 4, etc, and for r = 2 when the statistical distribution extends only over positive values (e.g. floods).

For any statistical distribution, L-moments can be defined in terms of the distribution's parameters, e.g. for the two-parameter Extreme Value Type I (EV1) distribution, where cumulative distribution function $F(x) = \exp[-\exp[-[x-u]/\alpha]]$ and u and α are its parameters: $\lambda_1 = u + 0.5772\alpha$ and $\lambda_2 = \alpha/n2$. Hosking (1990) lists corresponding expressions for nine other distributions. These expressions can be used with sample L-moments to estimate distribution parameters. Figure 1 shows different statistical distributions in the L-kurtosis–L-skewness plane. Two-parameter distributions plot as single points in this plane (e.g. EV1), three-parameter distributions as curves, and four or more parameter distributions as areas.

Sample L-moments

Unbiased, asymptotically normal sample estimators of population L-moments λ_r from an ordered n-sample $x_1 \leq x_2 \leq \ldots \leq x_n$ for r = 1, 2, 3, 4 are:

$$l_1 = \sum_{i=1}^{n} x_i / n$$

$$l_2 = \sum_{\text{all } i>j} (x_i - x_j)/(n-1)$$

$$l_3 = 2 \sum_{\text{all } i>j>k} (x_i - 2x_j + x_k)/n(n-1)(n-2)$$

$$l_4 = 6 \sum_{\text{all } i>j>k>l} (x_i - 3x_j + 3x_k - x_l)/n(n-1)(n-2)(n-3)$$

l_1 is the usual sample mean $\bar{x}$. L-CV is naturally estimated by l_2/l_1, L-skewness by l_3/l_2, and L-kurtosis by l_4/l_2.

The physical meaning of l_2, l_3 and l_4 is illustrated in Figure 2. For l_2, if any two values tend to be close together (Fig. 2a) then l_2 will be smaller than if they are far apart (Fig. 2b). Thus l_2 measures dispersion of a sample. For l_3, if the lower two values of any subsample of three tend to be closer (Fig. 2c) then l_3 will be positive, and if the upper two are closer to each other (Fig. 2d) l_3 will be negative. For symmetrical distributions, such as a normal distribution, l_3 will be close to zero (λ_3 exactly zero). For l_4, if middle values in any subsamples of four tend to be close (Fig. 2e) then l_4 will be positive, indicating heavy tails or positive kurtosis. (Fig. 2f) gives negative l_4.

L-moment ratio diagrams

An L-moment ratio diagram of L-kurtosis versus L-skewness compares sample estimates of the dimensionless ratios τ_4 with their population counterparts for a range of statistical distributions (Fig.1). L-moment diagrams are useful for

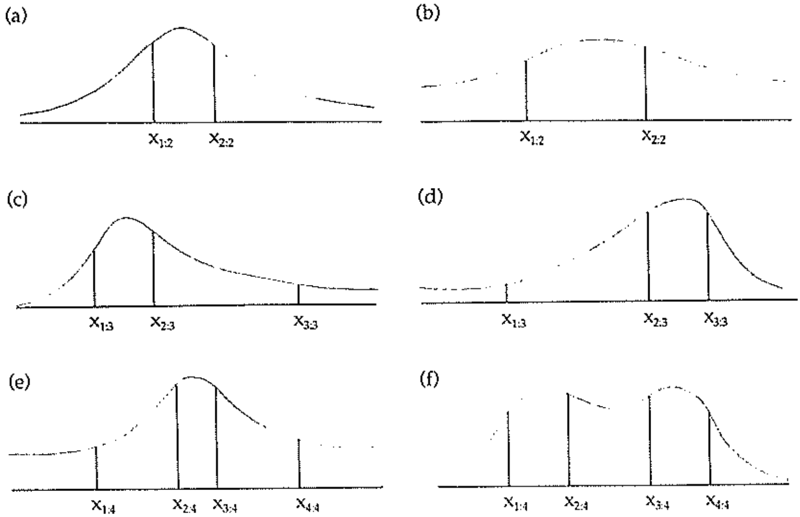

FIG. 2—Configurations of typical samples of sizes 2, 3 and 4. Probability density function [dF/dx] shapes are shown over the x axes.

discerning groupings of sites with similar flood frequency behaviour, and identifying the statistical distribution likely to adequately describe this behaviour. L-CV versus L-skewness plots are also useful for discerning groupings. Hosking and Wallis (1991) statistical tests are based on L-moment ratio diagrams.

NEW ZEALAND EXAMPLES — L-MOMENTS

Beable and McKerchar (1982) split New Zealand into geographical regions to estimate mean annual flood ($\bar{Q}$, "index flood") and dimensionless flood quantiles of the three-parameter Generalised Extreme Value (GEV) distribution. With more flood data available, McKerchar and Pearson (1989, 1990) reviewed New Zealand regional flood frequency and advocated use of the two-parameter EV1 distribution for annual maximum flood peaks, in conjunction with contour maps of two flood statistics: mean annual flood and dimensionless T = 100-year return period flood ($Q_{100}/\bar{Q}$). Hosking *et al.'s* (1985b) at-site statistical test based on probability-weighted moments was used for deciding if the GEV's shape parameter k was zero (EV1). McKerchar and Pearson's procedure circumvented the issue of defining regions, which its predecessor (Beable and McKerchar, 1982) had instituted.

L-moments were calculated for McKerchar and Pearson's set of 275 annual maximum flood series, each series having 10 or more annual maxima with a mean record length of 21 years. Figure 3(a) shows a plot of L-kurtosis versus L-skewness for this data. The scatter of this plot appears greater than the scatter exhibited by a Monte Carlo experiment (similar to those conducted by Wallis, 1989) that generated 275 EV1 flood series each with a record length of 21 years (Fig. 3b). Using Hosking and Wallis' (1991) goodness-of-fit test the GEV

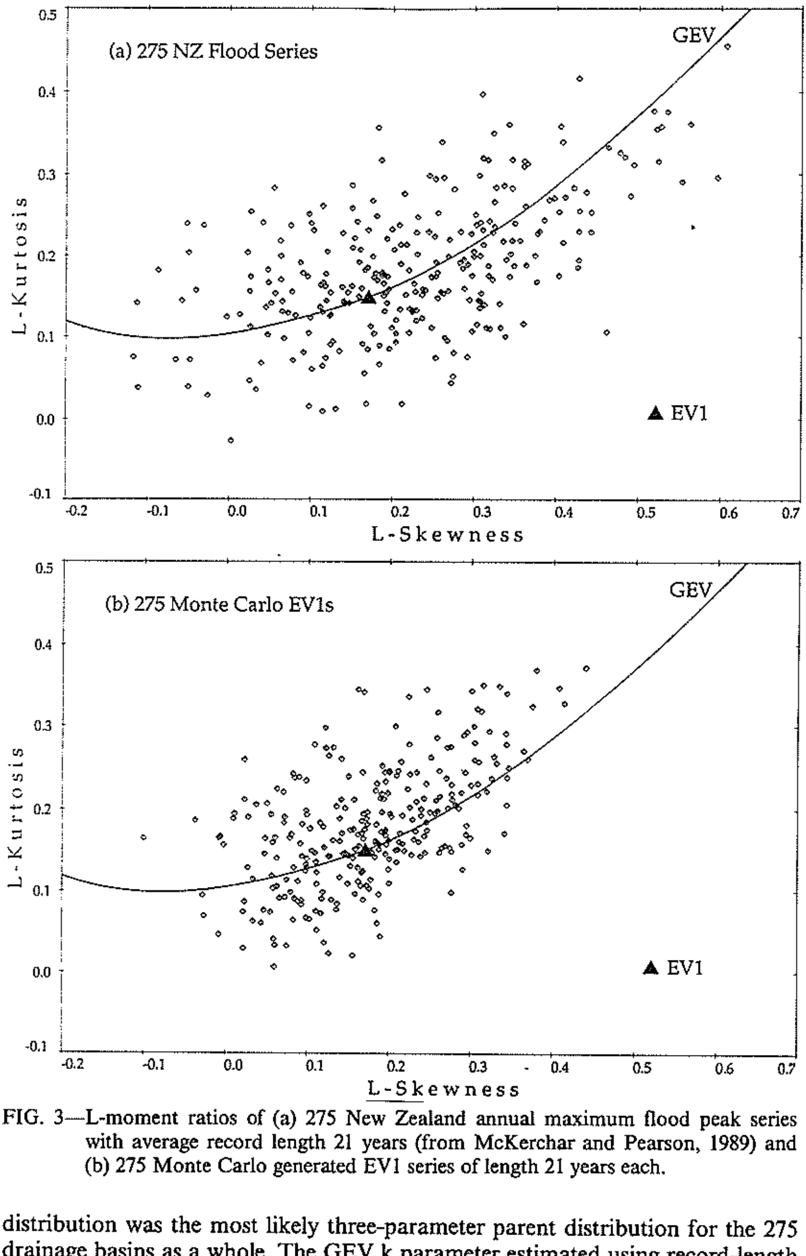

FIG. 3—L-moment ratios of (a) 275 New Zealand annual maximum flood peak series with average record length 21 years (from McKerchar and Pearson, 1989) and (b) 275 Monte Carlo generated EV1 series of length 21 years each.

distribution was the most likely three-parameter parent distribution for the 275 drainage basins as a whole. The GEV k parameter estimated using record-length weighted L-moments was –0.07 i.e. EV2 distribution, but not too different to EV1, particularly for quantiles of 100 years or less.

Two geographical regions of South Island (dry South Canterbury on the east coast and the wet West Coast) which have contrasting regional flood frequency extremes in Beable and McKerchar (1982) were further analysed. Table 1 gives sample L-moments and L-moment ratios for sites in each region.

TABLE 1—L-moments and L-moment ratios for annual maximum flood peak series from two South Island regions:
(a) South Canterbury (sites from Beable and McKerchar, 1982, South Canterbury region)
(b) West Coast (sites from Grey River south)

(a) SOUTH CANTERBURY

Site River	n (years)	l_1 (cumecs)	l_2 (cumecs)	l_3/l_2 (L-skew.)	l_4/l_2 (L-kurt.)
69505 Orari	27	274	132	.340	.081
69614 Opuha	52	203	79.3	.341	.195
69618 Opihi	52	153	72.8	.492	.332
69621 Rocky Gully	24	16.7	8.83	.640	.511
71103 Hakataramea	23	172	96.4	.590	.398
71116 Ahuriri	23	237	66.8	.325	.156
71122 Maryburn	17	3.96	1.47	.453	.499
71129 Forks	24	21.9	6.22	.302	.160
71135 Jollie	22	69.9	21.9	.409	.279

(b) WEST COAST

Site River	n (years)	l_1 (cumecs)	l_2 (cumecs)	l_3/l_2 (L-skew.)	l_4/l_2 (L-kurt.)
84701 Cleddau	16	1020	129	.066	.102
86802 Haast	17	3630	842	.317	.227
87301 Lake Moeraki	11	444	68.8	–.171	–.065
89601 Poerua	14	804	91.2	.091	.127
90604 Hokitika	17	1630	147	–.133	.081
90605 Butchers	15	25.3	3.9	.094	.025
91104 Taramakau	15	2260	413	–.004	.242
91401 Grey	20	3780	542	–.083	.220
91404 Grey	18	884	186	.250	.315
91405 Lake Brunner	18	727	58.6	–.111	.153
91407 Ahaura	19	1210	237	.146	.165

The L-kurtosis-L-skewness plot (Fig. 4) shows two distinct groups of points corresponding to each geographical region: the South Canterbury annual flood series have greater skewness and kurtosis than those of the West Coast. Record-length weighted average (τ_3, τ_4) points for each region indicate (Fig. 4) that South Canterbury is EV2, since its average point is close to the uppermost part of the GEV curve, and West Coast is more kurtotic than distributions shown in Figure 1.

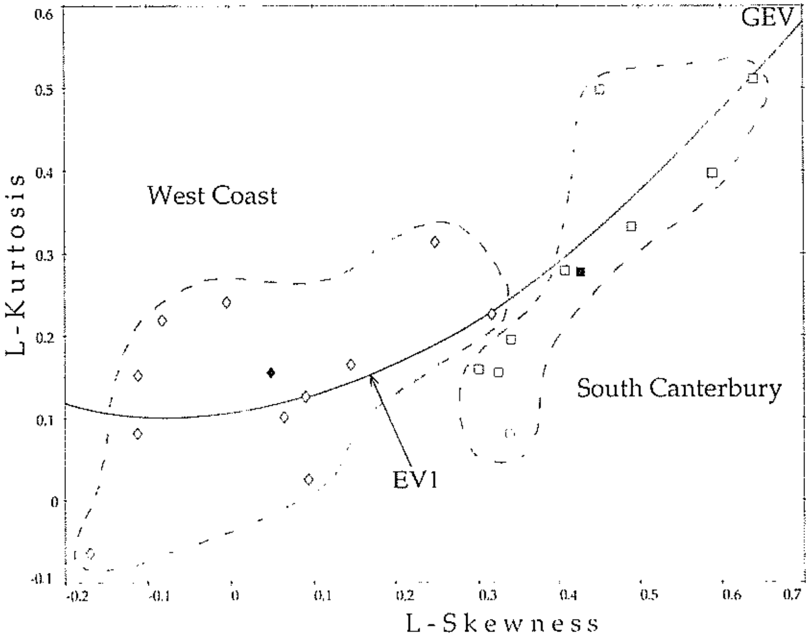

FIG. 4—L-moment ratios of annual flood series from two South Island geographical regions: West Coast and South Canterbury. Record-length weighted regional averages for each region are shown as solid symbols.

South Canterbury regional flood frequency

Record-length weighted, averaged, L-moment ratios were used to fit a dimensionless (i.e. representing $Q/\bar{Q}$) regional GEV distribution for South Canterbury annual maximum flood peak data. The GEV distribution function is:

$$F(Q) = \exp\{-[1 - k(Q - u)/\alpha]^{1/k}\}$$

where u and α are location and scale parameters respectively, and k is the shape parameter which specifies one of three asymptotic extreme-value distribution types: EV1 ($k = 0$), EV2 ($k < 0$) or EV3 ($k > 0$). Relationships between λ_1, λ_2, λ_3 and u, α, k given in Hosking (1990) are used with $l_2/l_1 = 0.417$ and $l_3/l_2 = 0.427$ to estimate dimensionless regional GEV parameters for South Canterbury: $u = 0.576$, $\alpha = 0.374$, $k = -0.365$. Dimensionless regional GEV quantiles are given in Table 2 and Figure 5 for various annual exceedance probabilities (AEPs). These can be multiplied $\bar{Q}$ (= l_1) to give regional flood estimates for each site e.g. 1% AEP flood (i.e. 100-year flood) estimate for Orari (site 69505) is Q_{100} = [5.04] × [274 cumecs] = 1380 cumecs. Three Hosking and Wallis (1991) tests based on L-moment ratios (including L-CV) indicated that South Canterbury was a reasonably homogenous region and that the EV2 distribution was the best statistical distribution for this region.

TABLE 2—Dimensionless regional GEV quantiles for South Canterbury. T is return period, T = 1/AEP.

AEP:	0.9	0.8	0.5	0.2	0.1	0.05	0.02	0.01	0.001
T(years):	1.11	1.25	2	5	10	20	50	100	1000
GEV Regional Quantiles:	0.31	0.42	0.72	1.31	1.88	2.58	3.80	5.04	12.3

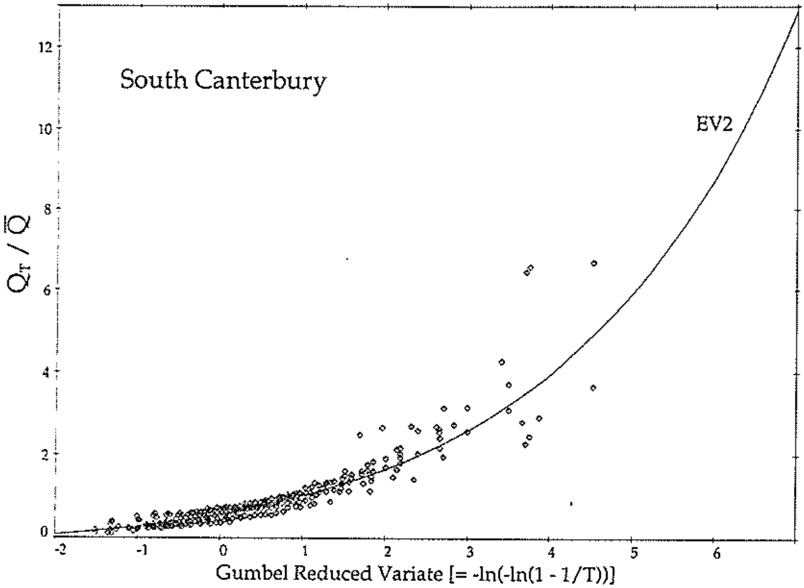

FIG. 5—South Canterbury regional flood frequency growth curve. 1% AEP or 100-year flood factor is 5.04, corresponding to a Gumbel reduced variate of 4.6.

West Coast regional flood frequency

From Figure 4, the GEV distribution does not appear kurtotic enough for the West Coast region. Hosking and Wallis (1991) tests indicated that the West Coast was a reasonably homogenous region and the generalised logistic (GL) distribution was the best three-parameter distribution for the West Coast. The GL distribution function F(Q) is,

$$F(Q) = 1 \;/\; [1 + \{\, 1 - k(Q - u)/\alpha\}^{1/k}]$$

where u and α are location and scale parameters respectively, and k is the shape parameter. Analogously to the relationship between the GEV and EV1 distributions, $k = 0$ for the GL distribution is the two-parameter logistic distribution. Record-length weighted, averaged, L-moment ratios (l_2/l_1 = 0.156, l_3/l_2 = 0.049) were used to fit dimensionless GL, GEV and Gumbel (EV1) distributions for West Coast annual maximum flood peak data.

The three-parameter GEV distribution fitted to the West Coast data implied an EV3 distribution (k = 0.197 > 0) and a finite upper bound of $Q/\bar{Q}$ = 2.22. The EV1 and EV2 distributions do not have finite upper bounds. The GL distribution specified by the fitted parameters (u = 0.987, α = 0.156, k = –0.049) has no upper bound (since $k \leq 0$). The West Coast regional 1% AEP dimensionless flood estimate from this distribution is 1.79, and the EV1 and EV3 regional 1% AEP estimates are 1.91 and 1.69 respectively. As noted above, the GL estimate is the best estimate for the West Coast from a three-parameter distribution. For all return periods there is little difference (approximately 7%) between the EV1 and GL estimates, and hence the advantage of going from the two-parameter EV1 distribution to the three-parameter GL distribution is negligible. The West Coast regional dimensionless EV1 flood frequency estimates are given in Table 3 and shown in Figure 6.

TABLE 3—Dimensionless regional EV1 quantiles for the West Coast.

AEP: T (years):	0.9 1.11	0.8 1.25	0.5 2	0.2 5	0.1 10	0.05 20	0.02 50	0.01 100	0.001 1000
EV1 Regional Quantiles:	0.68	0.76	0.95	1.21	1.38	1.54	1.75	1.91	2.42

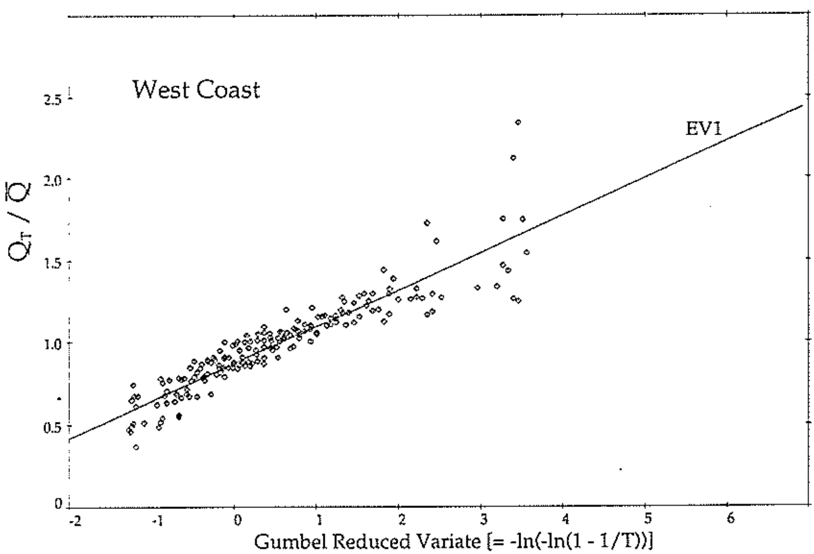

FIG. 6—West Coast regional flood frequency growth curve. 1% AEP or 100-year flood factor is 1.91, corresponding to a Gumbel reduced variate of 4.6.

IMPLICATIONS FOR NEW ZEALAND REGIONAL FLOOD FREQUENCY

L-moments analysis indicates that annual maximum flood peaks for South Canterbury are best described by the three-parameter EV2 distribution. This contrasts with the McKerchar and Pearson (1989, 1990) design procedure which prescribed the two-parameter EV1 distribution for annual maximum flood peaks (contour maps of two flood statistics, $\bar{Q}/A^{0.8}$ and $Q_{100}/\bar{Q}$, facilitated flood quantile estimates for any catchment in New Zealand). However, McKerchar and Pearson used biennial and triennial, rather than annual, sampling intervals for those sites in South Canterbury exhibiting EV2 behaviour. In most cases the biennial or triennial flood series were satisfactorily EV1, from which an estimate of Q_{100} was used for mapping. Regional EV2 Q_{100} estimates derived from Tables 1 and 2, and McKerchar and Pearson Q_{100} estimates for South Canterbury are compared in Table 4.

TABLE 4—Comparison of South Canterbury EV2 regional Q_{100} estimates from Tables 1 and 2 with McKerchar and Pearson (1989) EV1 at-site Q_{100} estimates derived from annual (1), biennial (2) or triennial (3) sampling intervals.

Site	EV2 regional Q_{100} (cumecs)	EV1 at-site Q_{100} (cumecs)	Sampling interval (years)
69505	1380	1040	1
69614	1030	721	2
69618	771	785	3
69621	84.3	110	3
71103	867	940	2
71116	1200	623	1
71122	20.0	12.5	1
71129	110	58.0	1
71135	352	227	2

The differences between the regional and at-site estimates (Table 4) illustrate the relative magnitudes of model error and sampling error in flood frequency analysis. Regional estimates are based on the model assumption that all sites in the region have identical, dimensionless, flood-frequency behaviour, i.e. homogeneous grouping. At-site estimates avoid this model error but have greater sampling variability, particularly when fewer flood peaks are used from longer sampling intervals. For South Canterbury the assumption of regional homogeneity could be erroneous since its cluster of L-moment ratios (Fig.4) is quite scattered, the same applies for the West Coast. However for both regions Hosking and Wallis (1991) tests indicated that this degree of scatter is expected from homogeneous regions. Alternative grouping procedures (Acreman and Wiltshire, 1989) could improve homogeneity beyond that of the arbitrary regional groupings analysed in this paper.

Until more experience is obtained through wider application of L-moments, the McKerchar and Pearson procedure provides the most reliable flood estimates for New Zealand, particularly for mean annual flood. However, for catchments with areas less than 10 square kilometres, the McKerchar and Pearson procedure is less precise. This was attributed chiefly to the predominance of larger catchments masking the individuality of smaller catchments, and to a lesser extent, to shorter lengths of record, larger errors in stage-discharge rating curves, the effects of urbanisation and afforestation, and larger variability of short duration rainfalls.

A new study is underway investigating alternative regional flood-frequency procedures to improve the precision of flood estimates for small New Zealand drainage basins. This involves L-moments and their application to small ungauged drainage basins. Regional approaches based on similarity of physical catchment characteristics (reviewed by Acreman and Wiltshire, 1989) are being considered.

Other studies underway include application of L-moments to New Zealand low flow frequency and an "index flood" procedure for Christchurch regional extreme-rainfall frequency.

CONCLUSIONS

L-moments plots are useful for discerning flood differences and confirming similarities in groupings of catchments. They allow specification of group dimensionless flood frequency distributions, which can be dimensionalised by estimates of mean annual flood (index flood approach). Application of L-moments theory to New Zealand flood data indicated that South Canterbury annual flood series are better described by the EV2 distribution rather than EV1.

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