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GRAPHICAL CALCULATION OF AQUIFER TRANSMISSIVITIES IN NORTHERN CANTERBURY, NEW ZEALAND

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ABSTRACT

A graphical technique is used to determine transmissivities and steady-state flow rates for groundwater flow in northern Canterbury. The results of the analysis appear to be in reasonable agreement with what is known about the geology and geological history of the region. Maximum well yields, obtained from drillers' records, also tend to support the final results. Hence, it is suggested that this technique might be useful as a tool in the analysis and management of groundwater aquifers in other parts of New Zealand.

INTRODUCTION

The quantitative analysis of any regional groundwater aquifer system requires an *a priori* knowledge of certain aquifer 'constants' which appear in the partial differential equations and boundary conditions that model the flow. These aquifer 'constants' are actually functions which characterize the influence of the aquifer upon a relatively broad range of flows and only reduce to true constants for an aquifer which can be idealized as homogeneous. The transmissivity is probably the most basic of these aquifer functions, since a knowledge of its distribution is a prerequisite to the analysis of both steady and unsteady flow in any inhomogeneous aquifer. The purpose of this paper is to show how a graphical technique that was initially proposed by Nelson (1961) has been used to compute transmissivities and flow rates in the northern parts of the Canterbury Plains from a piezometric contour map obtained earlier by Wilson (1973). It is believed that this technique might be useful in other areas of New Zealand as a tool in the analysis and management of groundwater aquifers.

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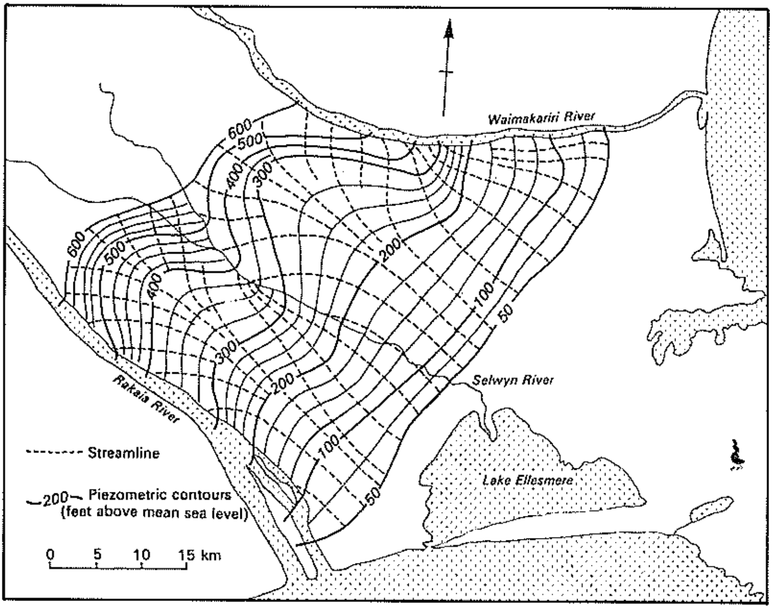

FIG. 1 — Streamlines and piezometric contours for a portion of the Canterbury Plains. (1 ft=0.305 m.)

THEORETICAL CONSIDERATIONS

A map of the region under consideration is shown in Fig. 1. The x and y co-ordinates will be measured in a horizontal plane that coincides with the bottom aquifer boundary, which is assumed impermeable, and the z co-ordinate will be measured as positive in the upward direction. It is assumed on the basis of well-drilling data that the piezometric head, h, is independent of z, which is equivalent to assuming that vertical velocity components are negligible. This latter consideration fixes the eastern boundary of the region shown in Fig. 1, since the aquifer or aquifers contained between this boundary and the sea are known to violate the assumption of constant piezometric head in the vertical direction. The permeability, k, will most generally be a function of x, y and z, so that Darcy's law gives for the horizontal discharge velocities, u_x and u_y,

$$u_x(x,y,z) = -k(x,y,z)\,\frac{\partial h(x,y)}{\partial x} \tag{1}$$

$$u_y(x,y,z) = -k(x,y,z)\,\frac{\partial h(x,y)}{\partial y} \tag{2}$$

Integration of equations (1) and (2) with respect to z over the saturated thickness, $B(x,y)$, of the aquifer gives

$$q_x(x,y) \equiv \int_0^B u_x \mathrm{d}z = -T(x,y)\frac{\partial h(x,y)}{\partial x} \quad (3)$$

$$q_y(x,y) \equiv \int_0^B u_y \, \mathrm{d}z = -\ T(x,y)\frac{\partial h(x,y)}{\partial y} \quad (4)$$

in which q_x and q_y are discharges per unit arc length in the xy plane and the transmissivity, T, is defined as

$$T(x,y) \equiv \int_0^{B(x,y)} k(x,y,z)\,\mathrm{d}z \quad (5)$$

The equation of continuity will be taken in the form

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0 \quad (6)$$

so that equations (3) and (4) can be substituted into equation (6) to give the classical Dupuit equation of steady-state groundwater flow:

$$\frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial h}{\partial y}\right) = 0 \quad (7)$$

Several very important assumptions have been implied by writing equations (1) to (7). First, the mathematical model described by these equations assumes that u_x, u_y, k, T and h are continuous and differentiable functions. This assumption is not satisfied in the physical model. For example, lenses of rock, silt, clay or gravel can cause very rapid – if not discontinuous – changes in these variables. In practice, this means that any variable calculated from these equations can only be compared with the corresponding experimental value after the experimental value has been averaged over a region whose characteristic dimension is large compared to the size of any impermeable, or extremely permeable, lenses. Thus, these equations cannot be used to predict the amount of water that might be obtained from a well before it is drilled and tested. Neither, in general, should the value of T for use in equation (7) be calculated from a single pumping test in any given area. On the other hand, it can be expected that these equations will reliably predict flow rates, transmissivities and piezometric contours over relatively large regions such as the Canterbury Plains. Second, the variable time has been omitted from these equations, which implies steady-state conditions. These steady-state conditions apparently exist in the Canterbury Plains if the dependent variables are averaged over a period of years. In other words, the piezometric contours shown in Fig. 1 are known to have remained relatively constant over the past 25 years. On the other hand, any transmissivity distribution calculated from equation (7) should still be suitable for use in the unsteady form of equation (7) if the changes in saturated aquifer thickness, B, are not large compared to B itself.

Equations (1) and (2) imply that lines of constant h are orthogonal to the streamlines, which are lines that are tangent to the velocity vector at all points. Hence, the streamlines are sketched as dashed lines in Fig. 1 from the requirement that they be orthogonal to the experimentally measured piezometric contours. Two points should be made about these streamlines. First, streamlines in the upper left portion of Fig. 1 that appear to end abruptly actually do continue eastward to the sea. These streamlines were discontinued in the drawing because extending them to the 15-m contour would have made it difficult to distinguish between individual streamlines in the eastern portion of the region. Second, it was necessary to sketch in many more streamlines than the number that were finally used in the calculations. As the number of streamlines was increased, the correct streamline pattern became more and more apparent. Then the intermediate streamlines were erased in order to reduce the amount of computation required for the remainder of the analysis.

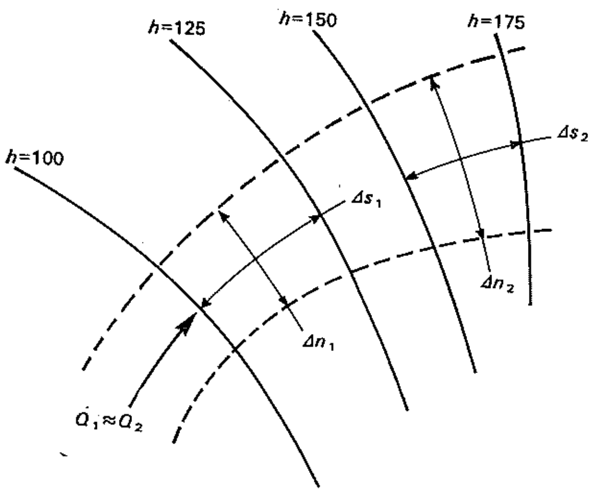

FIG. 2 — Portion of a typical streamtube in Fig 1.

A portion of a typical streamtube is shown in Fig. 2. Central difference approximations of equations (3) and (4) can be used to write the total discharge at the centre of rectangular element 1 in the form

$$Q_1 = -T_1 \Delta n_1 \frac{\Delta h_1}{\Delta s_1} \qquad (8)$$

Similarly, the discharge at the centre of rectangular element 2 is

$$Q_2 = -T_2 \Delta n_2 \frac{\Delta h_2}{\Delta s_2} \qquad (9)$$

If it is assumed that Q_1 and Q_2 are large compared to any flow leaking into or out of the streamtube segment between elements 1 and 2, then Q_1 and Q_2 can be equated to give

$$T_2 = \frac{\Delta s_2}{\Delta n_2} \frac{\Delta n_1}{\Delta s_1} \frac{\Delta h_1}{\Delta h_2} T_1 \qquad (10)$$

It is a simple matter to modify equation (10) to include recharge by rainfall by setting the difference of Q_2 and Q_1 equal to the infiltration flow rate between elements 1 and 2. However, average rainfall infiltration rates are not known for northern Canterbury. Furthermore, unpublished results of an oxygen-18 isotope study carried out from 1971 to 1973 by Dr C. B. Taylor of the Nuclear Sciences Division of DSIR indicate that aquifer recharge in the Christchurch area comes almost entirely from high-country precipitation rather than rainfall upon the plains. Hence, rainfall infiltration was neglected in this work.

Equation (10) can be used to compute T at points along any streamtube in terms of one reference value along that particular streamtube. The results of these calculations for the piezometric contour map shown in Fig. 1 are plotted as a contour map of relative transmissivity in Fig. 3. In Fig. 3, a relative transmissivity of 0.4 on a particular streamline indicates that the transmissivity at that point is four-tenths of the transmissivity at the point where that same streamline intersects the eastern boundary. The relative transmissivities along each streamtube were plotted and smooth curves were drawn through the calculated values in order to obtain the contours shown in Fig. 3. Previous experience with this technique has convinced the writers that smoothing the final results in this way helps reduce errors which creep in as a result of graphical and numerical inaccuracies.

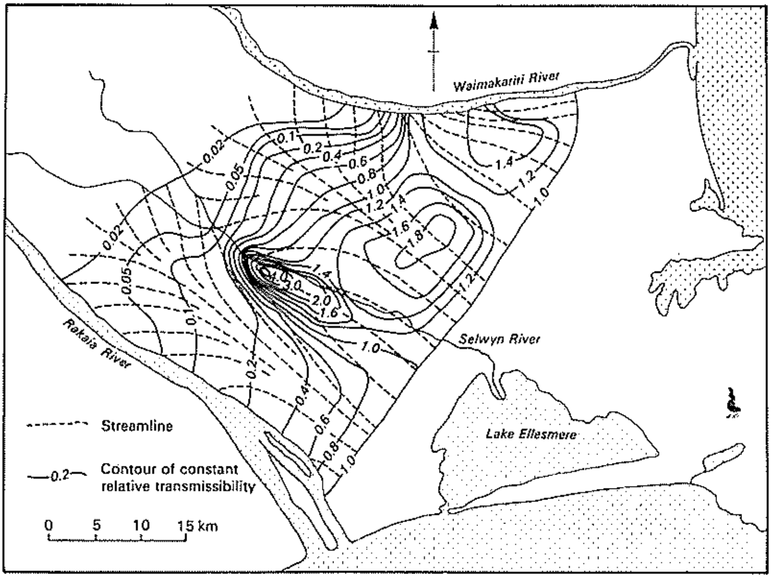

FIG. 3 — Relative transmissivities computed from the piezometric contour map shown in Fig. 1.

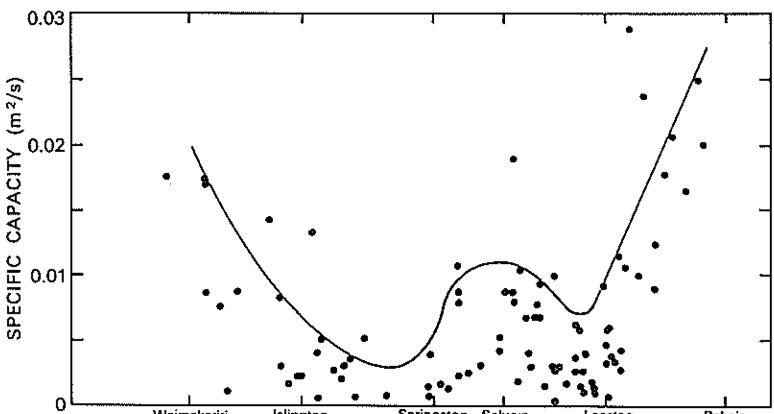

FIG. 4 — Equivalent specific capacities along the eastern boundary in Figs. 1 and 2.

The information which cannot be calculated from equation (10) is the variation of transmissivity across the streamlines. This missing information was derived by consideration of values of specific capacities in wells throughout the area considered. Well drillers determine these specific capacities by pumping a developed well at a constant flow rate for an unspecified period of time and then calculating the specific capacity as the ratio of flow rate to the final piezometric head drawdown. Walton (1970) shows that if all other variables are equal, then the specific capacity is very nearly directly proportional to the first power of T. Thus, the relative transmissivities in Fig. 3 were used to reduce specific capacities at points within the aquifer to equivalent specific capacities along the eastern boundary. These results are shown in Fig. 4. The large amount of experimental scatter in Fig. 4 is not surprising when it is pointed out that relatively close wells had specific capacities which, in places, differed by factors of four or five and occasionally as much as an order of magnitude. This scatter is undoubtedly a result of the facts that these wells were pumped for different, unspecified time periods, have different diameters, different screen lengths, penetrate to different depths in the aquifer and may be located in lenses of relatively permeable or impermeable material. However, Fig. 4 does show a reasonably well-defined envelope of maximum specific capacities, and since the specific capacity is normally a measure of the minimum value of T (Walton, 1970), it was decided to use the limiting curve shown in Fig. 4 as a way of calculating relative transmissivities along the eastern boundary in Fig. 3. Thus, transmissivities along the eastern boundary were assumed to be directly proportional to the envelope curve ordinates in Fig. 4, which – together with Fig. 3 – fixes the entire transmissivity distribution to within one unknown multiplicative constant.

On 5 February 1970, the Ministry of Works used stream gauging techniques to estimate that a flow of 9.36 m^3/s was lost from the Waimakariri River between the Halkett and West Melton Groynes, with a total flow at the Gorge Bridge of 61.17 m^3/s*. In this reach, the river is bounded by relatively impermeable outwash gravels on the north bank and by more permeable postglacial alluvium on the south bank. Hence, it is reasonable to assume that the entire seepage loss recharged the aquifer on the south side of the river. Thus, the one remaining multiplicative constant was chosen so that 9.36 m^3/s

* It would be better to use a loss resulting from a flow that is equalled or exceeded 50 percent of the time. The flow of 61.17 m^3/s, which is only about two-thirds of a 50-percent flow, was used in this study because it is the largest flow for which simultaneous gaugings are given by Dalmer (1970).

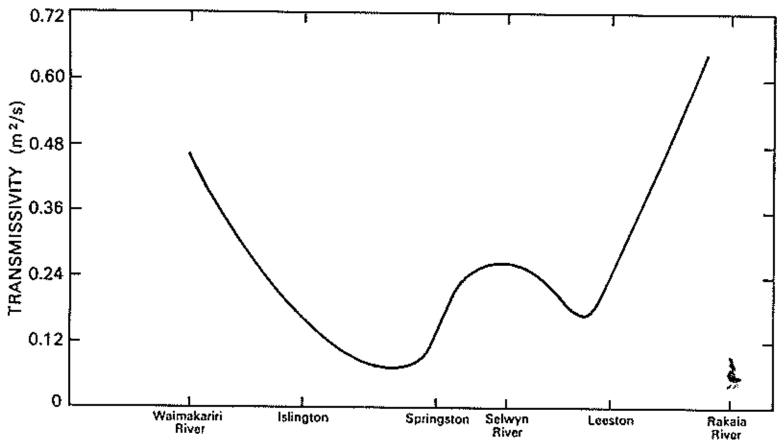

FIG. 5 — Transmissivities along the eastern boundary of Fig. 3.

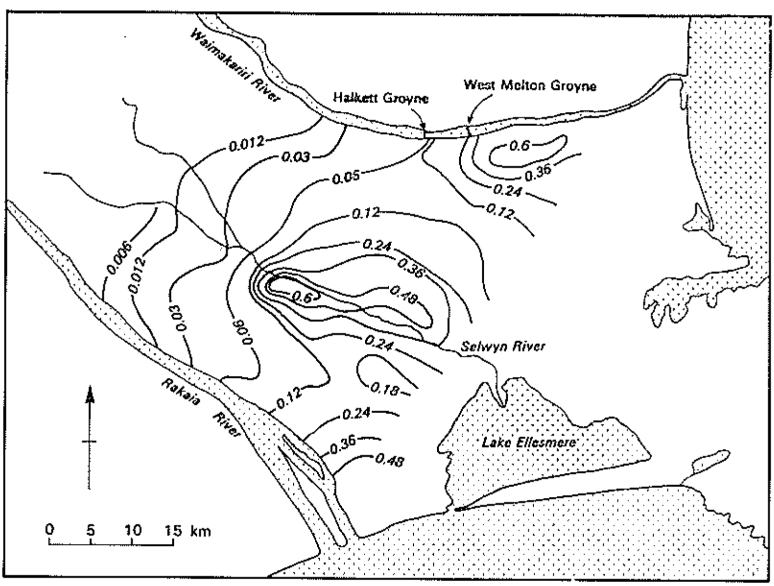

FIG. 6 — Transmissivities computed from Figs. 3 and 5 in m²/s.

passes through the streamtubes bounded by the two streamlines that originate at the Halkett and West Melton Groynes. The result gave a transmissivity distribution along the eastern boundary that is shown in Fig. 5, and a transmissivity distribution throughout the aquifer that is shown in Fig. 6.

The transmissivity distribution across the streamtubes might have been determined more satisfactorily by a carefully conducted series of pumping tests along any line that intersects all of the streamlines in Fig. 1. Thus, the graphical technique described here does not eliminate the need for pumping tests. It does, however, reduce considerably the number of pumping tests required, since transmissivities determined along any one line which intersects all of the streamlines are sufficient to allow calculation of transmissivities throughout the entire aquifer.

Equations (3), (4) and (6) imply the existence of a stream function, ψ, which can be defined in the following manner:

$$q_x = -T\frac{\partial h}{\partial x} = \frac{\partial \psi}{\partial y} \qquad (11)$$

$$q_y = -T\frac{\partial h}{\partial y} = -\frac{\partial \psi}{\partial x} \qquad (12)$$

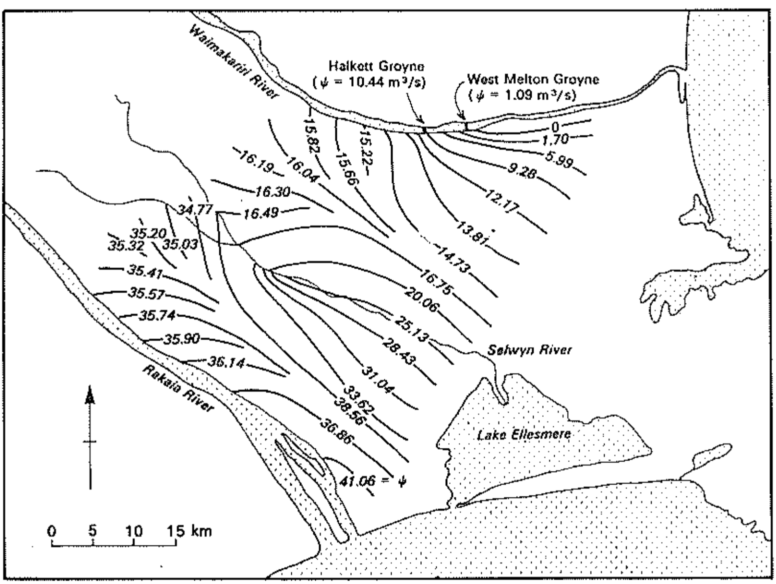

FIG. 7 — Streamlines and values of the stream function in m³/s.

It can be shown that lines of constant ψ coincide with the streamlines and that the difference between these constant values of ψ on any two streamlines gives the three-dimensional, volumetric flow rate between the two streamlines. Dividing equations (11) and (12) by T, differentiating equation (11) with respect to y and equation (12) with respect to x, and subtracting to eliminate h shows that ψ satisfies an equation that is quite similar to equation (7):

$$\frac{\partial}{\partial x}\left(\frac{1}{T}\frac{\partial \psi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{T}\frac{\partial \psi}{\partial y}\right)=0 \qquad (13)$$

Values of the stream function for the streamlines in Fig. 1 and transmissivities in Fig. 6 were determined by calculating the flow rate through each streamtube. (This is equivalent to solving equations (11) and (12), in finite-difference form, for ψ.) The final results are plotted in Fig. 7, which shows at a glance the distribution of average flow rates throughout the aquifer.

DISCUSSION

Quantitative Reliability of Results

The weakest step in the entire analysis is probably the use of the 9.36 m^3/s flow that was assumed lost to influent seepage from the Waimakariri River between the Halkett and West Melton Groynes. This loss is one of several measured by near-simultaneous gauging of the Waimakariri River. It has been used in calculations because it represents a loss from 61.17 m^3/s total flow at the Gorge Bridge, and this flow, which is about 67 percent of the mean river flow, is the highest river flow for which gauging figures are readily available. Since influent seepage rates increase with river stage, the mean seepage loss from the river is probably higher than the 9.36 m^3/s that was used in computations. Thus, computed flow rates and transmissivities in Figs. 6 and 7 probably err on the low side. As computed, the flow rates of Fig. 7 indicate a recharge in the Christchurch region that exceeds total discharges to wells and spring-fed rivers by approximately 3:1. If the figure of 9.36 m^3/s should later prove to be incorrect, a single multiplicative constant can be used to correct the results in Figs. 5, 6 and 7. For example, if it is found in the future that 12 m^3/s rather than 9.36 m^3/s is actually lost to the aquifer between the two groynes, then ordinates in Fig. 5, transmissivities in Fig. 6 and values of the stream functions in Fig. 7 should all be multiplied by the constant 12/9.36. Alternatively, if the transmissivity at some particular point is found –

perhaps by a series of carefully run pumping tests – to be 0.20 m^2/s instead of the 0.12 m^2/s given by Fig. 6, then ordinates in Fig. 5, transmissivities in Fig. 6 and values of the stream function in Fig. 7 should all be multiplied by the constant 0.20/0.12.

Because of the large amount of scatter that is present, the envelope curve in Fig. 4 that was used to compute relative transmissivities must be assumed to have an intermediate degree of reliability. An extensive series of carefully planned pumping tests may be the best way to check Figs. 4 and 5. If Fig. 5 is later replaced or changed, then Figs. 6 and 7 will require correction based on the new data and information given in Figs. 1 and 3.

The piezometric contours upon which this study was based were derived by Wilson (1973) from water-level data in several thousand wells. In view of the volume of data used, the geometry of the water-table contours is probably fairly accurate, and the relative transmissivities of Fig. 3, which are developed from the contours, are regarded as being the most accurate of the results given here. One of the authors hopes in the near future to check and improve the results shown in Fig. 3 by solving equation (7) numerically for h as the relative distribution of T is changed systematically. This type of check would not provide conclusive evidence of the accuracy of Fig. 3 though, since Nelson (1961) points out that the solution of equation (7) for h is relatively insensitive to errors in the distribution of T. In fact, Nelson estimates that the resultant error in h is only about 13 percent of errors that are made in calculating T.

Relationship between Derived Transmissivity Pattern and Geology

Gregg (1964) and Suggate (1973) mapped geological boundaries across the section of plains covered in this report. All of the wells from which data for water-table contours and water availability have been derived are drilled into late glacial and postglacial surface deposits. The depth range of the wells is about 15 to 60 metres, and within this range all aquifers penetrated are likely to be in postglacial alluvium or young glacial outwash. Any discussion of the relationship between the transmissivity pattern of Fig. 6 and geological history therefore requires a summary of late glacial and postglacial river history.

Geological mapping in the Waimakariri sector of the plains has established a sequence of alluvial events that is almost certainly paralleled in other major rivers. During the last glaciation the river built an outwash fan of silt-laden greywacke gravels derived from sparsely vegetated foothills that were being rapidly eroded. Since the peak of glaciation, the vegetation cover on the foothills has increased as a result of progressive climatic improvement, and rivers have flowed on to the Canterbury Plains with progressively lower loads, have retained sufficient energy to entrench themselves in the inland plains, and have then built new fans downstream with material derived from upstream entrenchment. The boundary between entrenchment and fan building has moved progressively downstream with time.

On the basis of water availability, Wilson (1973) postulated that aquifer permeability is directly proportional firstly to the youth of the aquifer (because younger deposits are in general better sorted and less compacted), and secondly to the distance of the aquifer from the foothills (because sorting improves with distance of transport). Wilson also advanced evidence based on the geometry of the water table to suggest influent seepage to groundwater from the main rivers. The possibilities of recharge are present in middle and lower reaches of the rivers, where the flood plain of each river closely corresponds with a 'ridge' in the water table. The possibilities are heightened when the river's flood plain is bounded laterally by highly permeable beds which could act as pipelines for influent seepage. Thus, because each river's sector of the plains has been built by aggrading streams fanning out from apices that have moved downstream with time, and since successive fans are considered successively more permeable, the routes of greatest influent seepage should prove to be ancient channels fanning out from points on the river and increasing in permeability in a downstream direction. Transmissivity contours, and streamlines derived from them (Figs. 6 and 7), tend to support this interpretation. However, transmissivities also show up features that require some explanation. For example, Fig. 6 shows a coastward decline in transmissivity from the transmissivity highs near the Waimakariri and Selwyn Rivers.

Although an increase in permeability directly related to distance of transport from the foothills has been postulated, there is a complicating factor near the coast. This is the fact that during the postglacial period (about 5000 years ago), and during earlier interglacial periods, the coastline north of Banks Peninsula was at least several kilometres, perhaps as much as 10 km, inland from the present coast. Similarly, shorelines south of the peninsula extended inland about the same distance. During these periods of marine transgression, near-coastal river gradients were probably small, permitting accumulation of sands and finer sediments among the gravels. For example, well logs at Harewood Airport contain terms like "gravel and clay", "blue clay", "blue pug", "gravel and clay mixed" at regular intervals down to 60 m. Such an admixture of fine materials with gravels would tend to lower permeability.

There is another stratigraphic change that might explain a coastward fall-off in transmissivity. This is the fact that well logs of the top 30 m to the west of Christchurch show gravels predominant, while well logs in central Christchurch show principally silts, sands, and peats in the top 30 m (Christchurch Formation of Suggate, 1958). These deposits are contemporaneous; the western ones are permeable, the eastern ones are generally impermeable and act as aquicludes.

Both high-transmissivity zones of Fig. 6 fit the concept of recharge from rivers, for both are situated in recent flood plains. In the case of the Selwyn River, the high-transmissivity zone more or less corresponds with the present position of the river. This is to be expected for an inter-fan consequent river like the Selwyn, which has been confined to a relatively narrow width of influence by fans of its larger neighbours. The northern high-transmissivity zone which is centred to the south of the present bed of the Waimakariri River is actually quite close to the 'mean' position of the river in its alluvial fan. This is because the river has been moving steadily northward over its lowest 35-km reach during the past 5000 to 10 000 years.

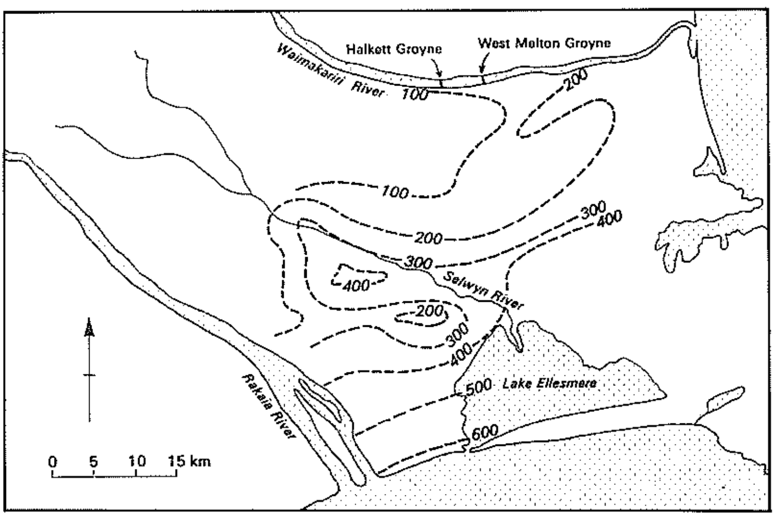

FIG. 8 — Contours showing well yield in units of 10^{-4} m^3/s.

Relationship between Well Yields and Derived Transmissivity Values

Fig. 8 shows contours of maximum well yields, which have been obtained from data contained in drillers' records kept by the Geological Survey. The writers believe that the geometry and location of these maximum-well-yield contours also tend generally to support the credibility of the transmissivity contours shown in Fig. 6.

Finally, the writers would like to end the discussion by noting that the North Canterbury Catchment Board, the water authority for the region, is currently embarking upon a programme to improve the precision of water levels, upon which the piezometric contours are based, and to determine by test pumping a more accurate distribution of transmissivity across the streamlines. Hence, more accurate field data should be available in the future to improve the results that have been derived here.

CONCLUSIONS

A graphical method, which was first proposed by Nelson (1961), has been used to obtain distributions of groundwater transmissivities and flow rates in the northern Canterbury Plains region. This method uses a piezometric contour map, which must be obtained from field measurements, to determine steady-state streamlines. The resulting flow net can then be used to calculate flow rates and transmissivity distributions if the variation of transmissivity is known along any line, or lines, that intersect all of the streamlines. The writers believe that these calculations have given results that agree reasonably well with what is known about the geology and well yields of northern Canterbury and that this method might be useful as a tool in the analysis and management of aquifers in other regions of New Zealand.

ACKNOWLEDGMENT

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