TIME ANALYSIS OF RAINFALL ON AN URBAN CATCHMENT

V. J. BIDWELL*

ABSTRACT

This paper describes an analysis of short-time-increment rainfall at the University of Auckland. Use has been made of autocorrelation techniques proposed by Grace and Eagleson (1966). Practical methods of data recording and processing, suitable for digital computer analysis, are outlined. Some autocorrelation results are given for one year of 10-minute rainfall values for the Albert Park gauge at Auckland.

INTRODUCTION

Modern developments in hydrological techniques are making increased use of statistical analysis of rainfall events, and the use overseas of synthetic sequences of occurrences is becoming widespread in river basin planning. Synthetic sequences are used to obtain long-term records with the same fundamental statistical parameters as the historical data. Treatment of the drainage basin as a system which produces output stream flow in response to input precipitation involves a synthetic rainfall record of some analytical form. Long-term synthetic records are also required for analytical modelling of water-resource systems.

For urban catchments, the time of concentration, run-off, and other time parameters involved is relatively short and thus the rainfall increment must be short — in this analysis it is 10 minutes.

OVERSEAS RESULTS

Grace and Eagleson (1966) developed a method for synthesis of short-time-increment rainfall sequences. An arbitrary division of the rainfall time series into storms of varying periods was made by use of a critical time lag T_L. This lag is a measure of the rainfall persistence, and the probability of occurrence of any rainfall event is assumed to be independent of an event which occurred a time T_L or greater beforehand. The value of T_L was deduced by means

* Postgraduate student, University of Auckland.

of a statistical analysis of pairs of rainfall values with varying time lags between them. These pairs of rainfall values were tested for independence by means of the rank-correlation coefficient. The rainfall values are ranked in order of magnitude and the sum of the differences between the ranks of the pairs is then obtained. For complete independence this coefficient had an expected value of zero. The historical record could then be divided into storms by assuming that a rainfall event within a time T_L of any other event was part of the same storm. Thus, probability functions of storm duration and time between storms were obtained. Machine generation of random selection from these distributions gave the synthetic sequence of storm durations. The authors then correlated storm duration and total rainfall depth from the historical record to give the synthetic values using an arbitrary classification of storms into three types.

Distribution of rain within a storm was evaluated by means of an "urn" model in which N balls of one colour (representing the total rainfall depth) are distributed into a slots (equivalent to the length of storm) together with balls of another colour to give changing degrees of correlation as the distribution proceeds. This method was used on historical records for the summer (May to August) for areas in Vermont (U.S.A.) and Nova Scotia.

METHOD OF INVESTIGATION

A complete year (1965) of 10-minute rainfall values was extracted from autographic records for Albert Park, Auckland. This record was processed and analysed in order to obtain the critical time lag T_L described above. This analysis has been carried out for three-monthly sets of data and the results compared. Evaluation of the product-moment correlation coefficient was done for two six-month records, and the results also compared.

Some conclusions and explanations of the results are given here, with particular regard to differences in climatic conditions compared with the overseas results.

The 10-minute rainfall values were extracted from the recorder charts of the tilting-siphon rain gauge at Albert Park. These data were punched on IBM cards for computer use. The most efficient way of doing this was to punch the data as two-digit integers, 36 values to a card (equivalent to a six-hour record). Only non-zero values were punched, zero positions being left blank. Each card also had a record number. Computer programmes for sorting and correlation of data, used by Grace and Eagleson, were modified to suit the IBM 1130 computer at Auckland University.

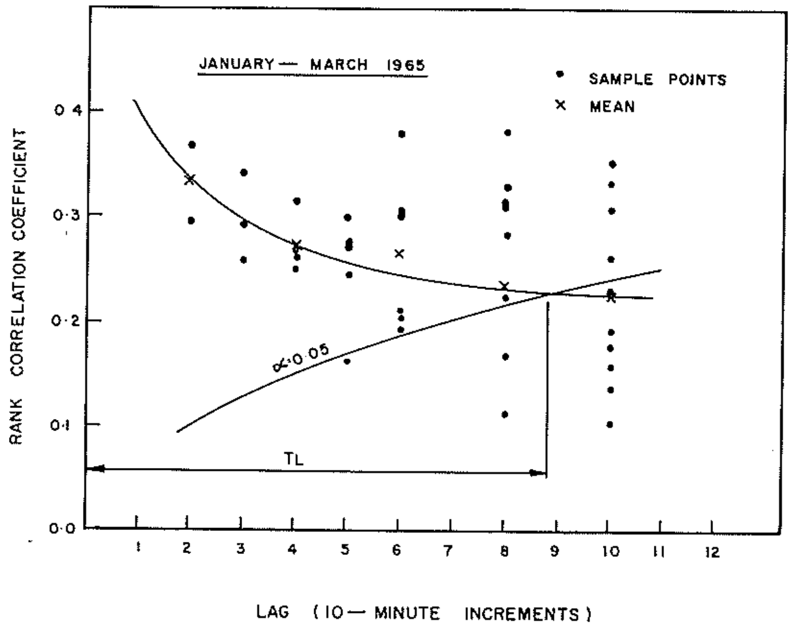

Fig. 1 — Application of the rank correlation coefficient test.

EXPERIMENTAL RESULTS

Fig. 1 illustrates the application of the rank-correlation coefficient as a test for rainfall persistence. A curve has been fitted to the means of sample points for the record of January–March 1965. Note that the number of sample points equals the lag value. This is a result of the sampling requirements of the rank coefficient and prevents the test at larger lag values being affected by any correlation at small lags. The critical lag T_L is deduced as being approximately at the intersection of the correlation curve and the line designated $\alpha = 0.05$. It is assumed that below this line there is a probability of 0.95 ($= 1 - \alpha$), and that a non-zero value of the rank coefficient is due to sampling and not to any dependence between the pairs of values at that lag. Other significance levels could be chosen, e.g. $\alpha = 0.01$. The value of the rank coefficient for a given α is a function of the sample size, and increases as the sample size decreases.

In Fig. 2 the sample means are given for each of the three-month groups of data. It can be noticed that all four sets exhibit a similar wavy curve. The expected curve shape (based on overseas results) is that shown in Fig. 1.

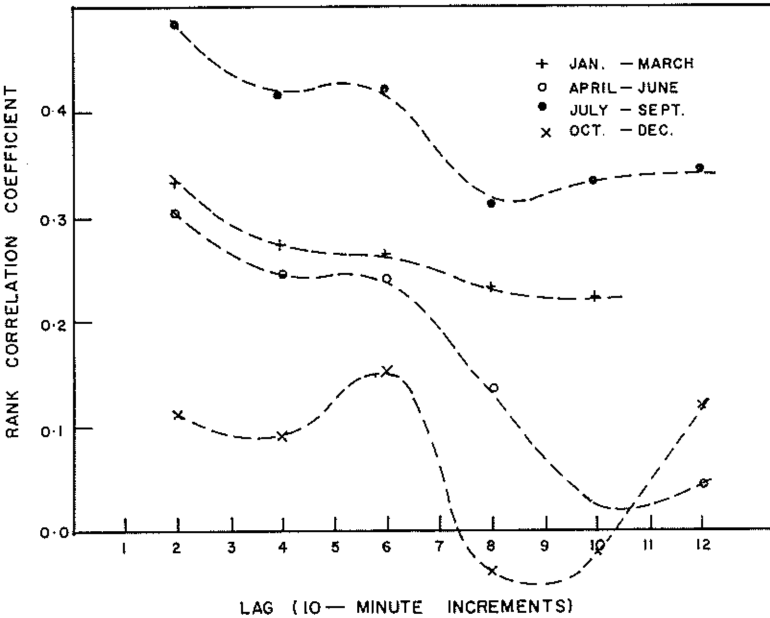

Fig. 2 — Variation of the rank correlation coefficient with lag.

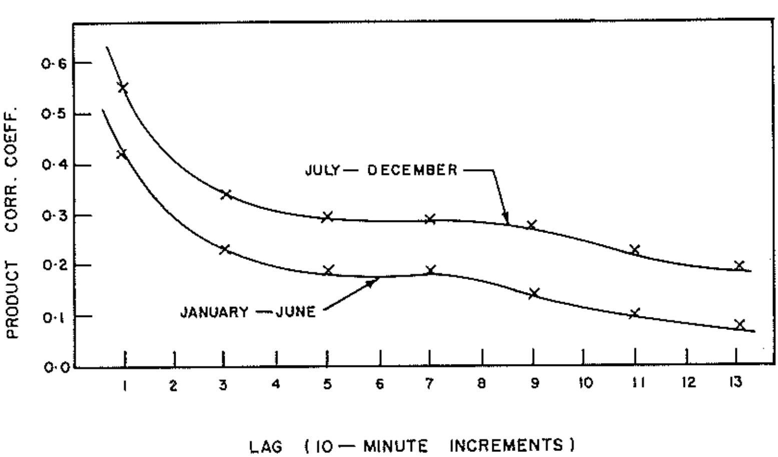

Fig. 3 — Variation of the product moment correlation coefficient with lag.

The variation of the product-moment correlation coefficient with lag is shown for the two six-month records in Fig. 3. Longer records could be analysed with this coefficient than with the rank coefficient because less computer storage is required. Note that the shapes of the curves are similar, although the values of the correlation coefficient are higher for the July–December record. It is difficult to comment on this difference because the population distribution within the two sets of data is unknown. For the same reason, any evaluation of the critical lag T_L is of doubtful reliability, as tests for these statistics rely on an assumption of a normal population distribution. It is interesting to note that the departure of the product-moment coefficient from a steadily decreasing curve coincides approximately with the humps in the rank-coefficient curve.

An attempt was made to obtain a correlation curve for daily rainfall, but the length of record considered (one year) was not long enough to obtain significant results.

CONCLUSIONS

One of the major differences between these results and those of Grace and Eagleson is the nature of the rainfall considered. The work presented here is an analysis of a complete year of rainfall record. Grace and Eagleson considered that since high-intensity rainstorms were very important in this work, they would consider only summer records (May to August inclusive). Because of the continental climatic conditions in the northern United States and Canada, the sudden intense rainfall usually associated with thunderstorms occurs mainly in these summer months. By contrast, complete yearly records would appear to be necessary in this country.

The length of record used here is not of sufficient length to give conclusive evidence of storm lengths or any seasonal variations. Quite large differences have occurred even in the sets of data analysed in the one-year record, and whether these are due to seasonal changes or otherwise is not readily apparent. It is intended that the analysis serve as an indication of the application of these techniques to rainfall records and at this stage no attempt is made to deduce information or propose definite conclusions, although it is hoped to obtain design figures in the future.

It is interesting to note the occurrence of the irregularities in the correlation curves and the similarity of their shapes for the four three-month sets of data. It is difficult to say whether this departure from a steadily decreasing curve is due to sampling or not. The consistency of shape in the four sets may be some indication of the internal structure of a rainstorm and could perhaps be caused by a pattern of intermittent showers or periods of higher intensity within a typical storm.

Further research in this field should include analysis of more records for a better determination of parameters such as the critical lag T_L. Similar statistical techniques could also be applied to hourly or daily rainfall in order to detect any periodicities and probabilities of events in the weather system.

Analysis of the internal structure of storms using techniques similar to those of Grace and Eagleson will be attempted with particular attention given to local climatic conditions. Included in this work will be development of techniques for synthesizing rainfall records suitable for local application to analysis of water resources and urban drainage systems.

The aim is to obtain an analytical model of the mechanism of run-off in a small urban catchment. By using the rainfall model as input data to the catchment system it is hoped to obtain output discharge data that will form a basis for design figures for urban drainage.

ACKNOWLEDGMENTS

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