LONGITUDINAL DISPERSION OF FLUID PARTICLES IN MOUNTAIN STREAMS: 1. THEORY AND FIELD EVIDENCE.

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ABSTRACT

The longitudinal dispersion of fluid particles in mountain streams was investigated in a series of 49 experiments. Mean properties of the dispersion process were studied over a maximum downstream distance of 2250 m, a mean velocity range of 0.32–1.57 m/s, and a mean flow width range of 2.7–21.8 m. It is conclusively shown that for distances of up to 800 mean channel widths in these steep, turbulent streams, the spread or standard deviation of an initially concentrated mass increases linearity with distance, and not as its square root as necessary for the application of conventional mixing models. Consequences of this linearity are an ever-increasing dispersion coefficient along the channel and a persistent asymmetrical concentration distribution.

INTRODUCTION

This paper deals with the longitudinal dispersion of passive contaminants (salt tracers) through natural channels – mountain streams in particular. Interest in the general topic of dispersion continues to increase with the demands for determining both acceptable levels of efficient inputs into natural waterways and canals, and the concentration pattern of accidental inputs of toxic materials. Further interest is associated with the use of tracer methods from stream gauging and time-of-travel studies in turbulent flows.

A primary focus of these studies has been the attempt to characterize the longitudinal mixing of a river by a dispersion coefficient K_x, and to relate this coefficient to bulk flow and channel geometry parameters. These coefficients are then used to predict the peak concentration decay, the rate of spreading of a pollutant, and its overall concentration pattern.

In a three-dimensional open channel flow, dispersion results from the combined action of a velocity distribution and turbulent velocity fluctuations. A mathematical formulation of the movement and spreading of a dissolved substance in a turbulent shear flow is inherently complex, and recourse to various simplified models has been necessary. In cases where the pollutant (tracer) does not alter the density of the fluid medium, Taylor's (1954) analysis for turbulent diffusion in pipes has been applied. However, application of this analysis to natural streams has met with mixed success. Non uniform streams have produced longitudinal dispersion coefficients which can vary significantly from those based upon theoretical considerations, and concentration distributions which consistently vary from the Gaussian. Various explanations for these differences have been advanced, the most frequent being the presence of a nonuniformity of channel cross section (Glover, 1964; Thackston, 1966; Church, 1967; Fischer, 1966, 1973a) and dead zones (Krenkel, 1960; Hays, 1966; Thackston and Schnelle, 1970). One premise of these explanations is that only the accuracy of Taylor's analysis is affected. In dead-zone studies, transport terms for material being exchanged between the main flow and the dead zones are added to Taylor's model.

THEORY: TAYLOR'S ANALYSIS

The concept of using coefficients to characterize the dispersion process inherent in a turbulent flow was introduced by Taylor (1954). Earlier, Taylor (1921) had proved that if a velocity of an individual fluid particle is a stationary random function of time with zero mean, then the mean square position of an ensemble of fluid particles initially concentrated at $x = 0$ and $t = 0$ is given by

$$\sigma x^2 = 2<u>^2 \int_0^t \int_0^\xi R(\xi)\,d \quad dt \tag{1}$$

where x is the direction of the mean motion (downstream), t is time, x^2 is the variance of the concentration distribution for a fixed x, $<u>$ is the mean cross-sectional velocity, and $R(\xi)$ is the Lagrangian autocorrelation function. For sufficiently large times, $R(\xi)$ can be expected to approach zero, and equation (1) reduces to

$$d/dt(\sigma_x^2) = 2<u>^2 T_L \tag{2}$$

where T_L is the Lagrangian integral time scale. This result shows that the spread or standard deviation of an initially concentrated mass increases with the square root of time, and is dependent upon the mean properties of the motion being stationary random functions of time.

Later, at the suggestion of G. K. Batchelor, Taylor (1954) applied this analysis to the dispersion of fluid particles in uniform flow in a pipe where distance and velocity were measured in a frame of reference moving with the mean cross-sectional velocity. The velocity of a particle in uniform pipe flow is a stationary random function because the statistical properties of the flow are the same in any longitudinal position and the particle contained within a uniform cross section samples all parts of the cross section. Taylor (1954) noted that under these circumstances, solutions to the ordinary diffusion equation were applicable. The basic diffusion equation is

$$\partial c/\partial t + <u>\partial c/\partial x = D\partial^2 C/\partial x^2 \tag{3}$$

where C is the concentration at t, and D is the dispersion coefficient. The solution of equation (3) for the initial conditions of a slug of concentration at $x = 0$ is

$$C = Bt^{-1/2}\exp \quad -(x-<u>t)^2/4K_x t \tag{4}$$

where B is a constant proportional to the amount of dispersing material, and K_x is the longitudinal dispersion coefficient. For a fixed time the concentration distribution along the channel, described by equation (4), is Gaussian, and provided the evolution of the concentration distribution is relatively slow in comparison to its convection, then the concentration as a function of time for a fixed x is also approximately Gaussian. Taylor also demonstrated that the dimensionless dispersion coefficient, K_x/ru_*, is a constant: i.e. $K_x/ru_* = 10.1$, where r is the radius of the pipe, and u_* is the shear velocity. Elder (1959) extended Taylor's analysis to two-dimensional flow with vertical velocity variations, and using a logarithmic velocity profile found that $K_x/du_* = 5.9$, where d is the cross-sectional flow depth. Further experiments in two-dimensional flows by Fischer (1966), Thackston and Krenkel (1967), and Sayre and Chang (1968) have produced values of K_x/du_* up to 13. However, Fischer (1973a) stated that these differences between experimental and theoretical results are small and could be explained by: (1) the sensitivity of K_x/du_* to the form of the velocity profile, (2) the presence of a viscous sublayer, (3) the value of the dimensionless time, and (4) the presence of walls or internal secondary circulations.

In three-dimensional open channel flow, velocity variations exist in both the vertical (y) and transverse (z) direction. Fischer (1966) showed that the transverse velocity variations are the primary producers of dispersion in natural streams. The longitudinal dispersion coefficient is then determined as

$$K_x = -\frac{1}{A} \int_0^w q'(z) \int_0^z \frac{1}{\epsilon_z d(z)} \int_0^z q'(z)\, dz dz dz \qquad (5)$$

in which

$$q'(z) = \int_0^z u'(y, z)\, dz \qquad (6)$$

where $u'(y, z)$ is the velocity of the cross section relative to the mean flow velocity, i.e. $u(y, z) = u(y, z) - \langle u \rangle$, w is the channel width, ϵ_z is the transverse mixing coefficient, and A is the cross sectional flow area. Chatwin (1971) noted that Taylor's one-dimensional model, equation (4), could be manipulated to give

$$[t \log_e (B/Ct^{1/2})]^{1/2} = (-x/2K_x^{1/2}) - (\langle u \rangle/2K_x^{1/2}) \qquad (7)$$

The left-hand side of equation (7) is plotted against time. If all points fall on a straight line, then Taylor's analysis is applicable. A dispersion coefficient can be determined from the intercept, and the velocity of the peak concentration can be determined from the slope. The value of the constant B was estimated to 1 or 2 percent by fitting several valves of B and accepting by eye the value which gave the smoothest curve near the peak. For large quantities of data, Day (1974) has shown that B can be estimated quickly by assuming that $B = C_P(T_P)^{1/2}$, where C_P is concentration and T_P the time peak, which is true only when the concentration distribution is symmetrical. Errors produced from this approximation are very small (Day, 1974). Chatwin used this analysis to show that the results of pipe flow experiments satisfied Taylor's theory, but that Fischer's (1966) open-channel experimental data did not. He suggested that this might be true because these experiments did not extend sufficiently far downstream for the statistical properties of the fluid particles to become independent of time.

The advantage of Chatwin's method for determining K_x is that it is based upon the shape of the concentration distribution. Taylor's model is fitted only to the leading edge of the concentration distribution, eliminating any distortions associated with trails. Plots of Chatwin's analysis also offer a quick visual comparison between the shape of the concentration distribution and Taylor's model. Only when all the points of the concentration distribution fall as a straight line does the theory apply.

Sayre (1968) numerically solved Aris's (1956) moment equations to show that although the approach to a Gaussian distribution was relatively slow, the variance rapidly satisfied the Taylor predictions that the growth of variance and distance was proportional to time. Sayre's results suggest that the most critical test of Taylor-type predictions is in the measurement of variance.

STUDY OBJECTIVES

Discrepancies between theoretical and field results are not surprising, as natural channels probably never meet the required assumptions of uniform cross section, steady flow and a constant mass balance. Most natural streams exhibit variations in plan and cross-sectional geometry which may give rise to non-homogeneous flow structures. Fischer (1973a) stated that meanders could cause a ninefold reduction in the dispersion coefficient. The studies of Fukuoka and Sayre (1973) into sinuous laboratory channels have shown that the presence of a regular sinuosity introduced a periodicity into the dispersion process. Also, zones of stationary and slowly moving water distributed along the flow boundary may lead to the entrapment and subsequent slow release of fluid particles. Dead-zone studies by, for example, Hays (1966) and Thackston and Schnelle (1970) have indicated that the shape of the concentration distribution is significantly different when these zones are known to exist.

Field experiments into longitudinal dispersion, being mainly concerned with larger streams, have suffered logistical limitations. Probably as a direct consequence of these limitations, few studies can offer a comprehensive definition of the dispersion pattern along a channel. The objective of this study was to supply, for a single channel type, such comprehensive definition for both a range of flow conditions and a range of stream scales. Results from this study have been previously discussed by Day (1975).

EXPERIMENTAL DESIGN

Selection of Test Reaches

As common to many doctoral studies, the field programme had to be developed within many constraints. Limitations in manpower and equipment confined the study to small mountain streams in the vicinity of the University of Canterbury Field Station at Cass. With its location along the eastern flanks of the Southern Alps, in approximately the centre of the South Island, this field site offered ready access to a wide number of small mountain streams. These streams are characterized by steep slopes, heavily armoured beds, low sinuosity, channel braiding associated with bar formation and general channel instability, and a high variable runoff regime.

During the field study 12 test reaches were selected, although not all 12 were under study at any one time. As is well known, these streams are laterally mobile, and four of the test reaches were abandoned when braiding developed within the study section. Other sites had to be abandoned in preference to those channels which were most readily accessible during the infrequent and short-duration rainfall events which sponsored the necessary moderate to high flow events. Of the original 12 sites, only 5 proved suitable. A summary of basic channel data, as well as the latitude and longitude, is presented for each test reach in Table 1. The test reaches on the Bruce, Craigieburn and Porter Streams are shown in Figs. 1 to 3, respectively.

Tracer Measurements and Field Procedures.

Common salt, NaCl, was used as the tracer. The salt dilution was hand-injected as a slug whose volumes ranged from 3 to 48 litres with concentrations varying from 23.6 to 121 g/*l*. During lower flow events when access to the stream was possible, the tracer was injected into the zone of maximum flow. When access was restricted, the injections were made as far out from the bank as safely possible.

The time–concentration curves were traced by means of a recording conductivity meter. The conductivity probes were weighted and placed in the channel with the same criteria as those of the injection design. The probes were always placed within 2.5 cm of the bed.

Each dispersion test consisted of a series of time–concentration curves which followed the tracer from initiation to the maximum possible downstream distance, always beyond the mixing length. The frequency of sampling sites decreased away from the injection site; a typical sampling design and test results are shown in Fig. 4. All injections for any test sequence were kept constant in volume and concentration.

TABLE 1 – Basic channel data.

Test reach	*Latitude**	*Longitude**	*Maximum channel length (m)*	*Slope*	*Mean bed particle Size† (cm)*
Bealey	42°57′35″	171°34′20″	660	0.0094	15.3
Bruce	43°01′00″	171°39′18″	775	0.0203	6.9
Craigieburn	43°08′20″	171°45′32″	780	0.0234	10.6
Porter	43°14′30″	171° 3′55″	825	0.0176	10.0
Thomas	43°12′50″	171°43′30″	2250	0.0273	5.6

* Refers to middle of test reach.
† Surface sample by line grid oriented along the channel.

Analytical Procedures

As the irregular channel cross sections precluded the use of current meters, the basic flow parameters are extracted from tracer measurements, and the relative salt dilution method is used for this purpose. Adequate critiques and outlines of this method of dilution gauging are available in the literature (e.g. Church and Kellerhals, 1970). It should be noted, however, that the mean velocities used here are mean particle velocities through the reaches and differ from the mean cross-sectional velocity typically used in dispersion work.

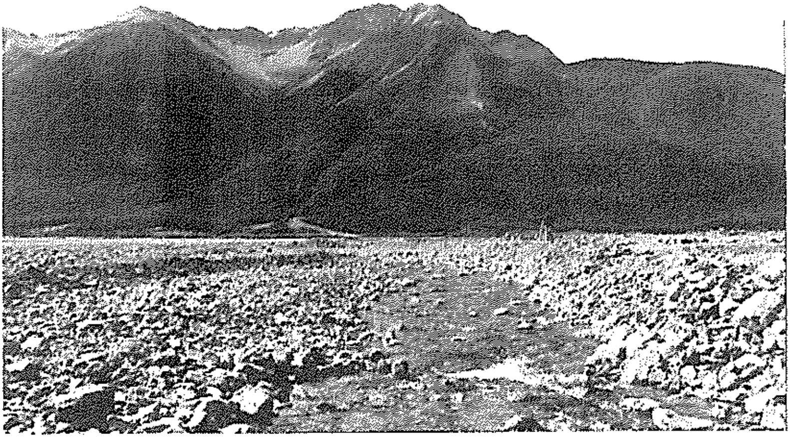

FIG.1 – Bruce test reach.

FIG.2 – Craigieburn test reach.

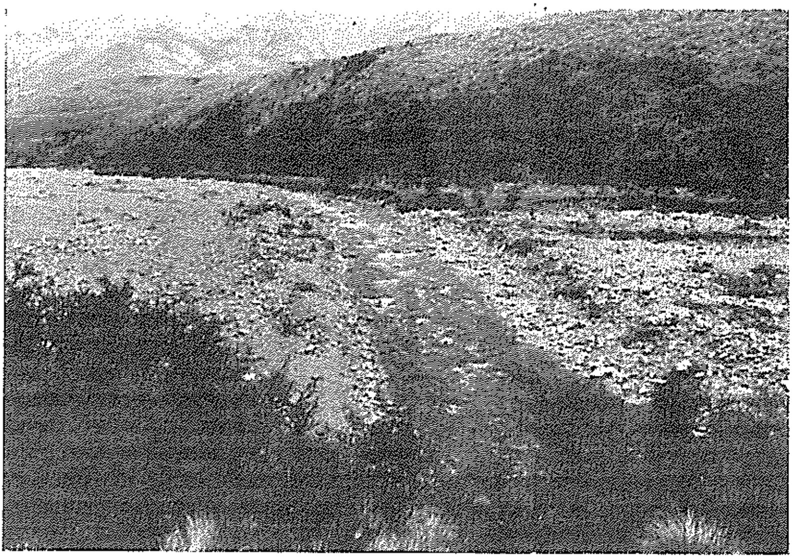

FIG.3 – Porter test reach.

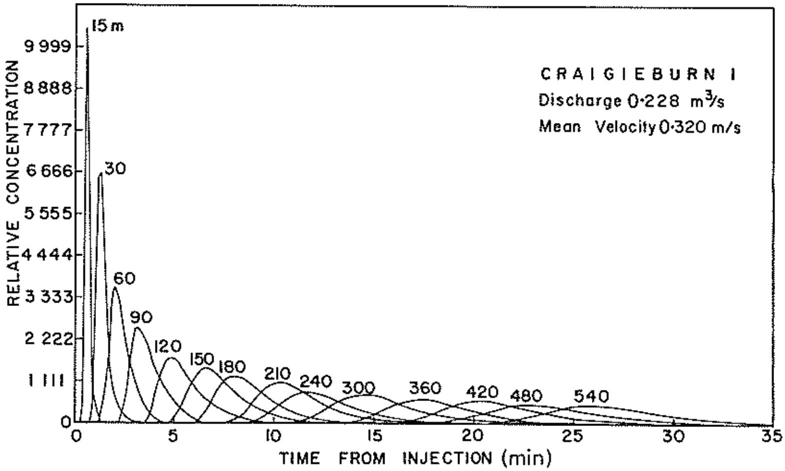

FIG.4 – Longitudinal dispersion characteristics for Craigieburn test 1.

EXPERIMENTAL RESULTS

Hydraulic Data

The hydraulic data for the 49 tests, representing 702 individual tracer waves, are summarized in Table 2. Mean velocities range from 0.32 to 1.57 m/s and mean flow widths range from 2.7 to 21.8 m. The mean flow width, w, was determined from 8–13 equally spaced width measurements distributed along the channel.

TABLE 2 – Hydraulic data and mixing lengths.

Test*	Discharge (m^3/s)	Mean velocity (m/s)	Mean flow width (m)	Length† (m)	Number of waves	Mixing length (m)	Convective Length (m) $t' = 0.4$	Convective Length (m) $t' = 1.0$
Bealey								
1s	15.00	1.20	21.8	540	9	360	5381	13 452
2s	8.93	1.06	19.7	660	10	500	5029	12 573
3s	10.54	1.02	21.1	600	10	500	4571	11 428
4s	8.48	0.94	20.1	600	10	600	4332	10 829
5s	7.89	0.98	19.2	600	10	600	4618	11 546
6s	12.31	1.27	20.6	600	10	360	5770	14 424
7c	6.03	0.85	18.4	600	11	500	1832	4 580
Bruce								
1c	0.92	0.67	6.2	600^{1}	17	120	297	743
2c	0.95	0.61	6.5	664^{1}	18	100	267	668
3c	0.87	0.57	6.4	650^{2}	18	125	242	605
4c	0.82	0.59	6.3	675^{2}	17	100	251	628
5c	0.60	0.47	6.0	500^{1}	17	100	208	520
6c	0.56	0.51	5.6	500^{1}	21	75	215	538
7s	6.28	1.11	7.2	775^{1}	18	125	684	1 711
8s	7.29	1.57	9.1	600	11	125	926	2 314
9s	4.24	1.40	8.2	600	12	75	1007	2 517
10s	6.08	1.37	8.7	600	12	150	1013	2 533
Craigieburn								
1c	0.23	0.32	4.9	540^{2}	14	50	155	360
2c	0.65	0.43	6.1	540^{2}	16	50	137	343
3c	0.38	0.43	5.9	780^{2}	19	75	139	348
4c	0.99	0.58	6.4	780^{2}	19	90	178	445
5c	0.43	0.38	5.5	480^{2}	15	240	147	368
6c	2.63	1.00	7.6	780^{2}	13	150	291	728
7c	1.57	0.80	6.7	780^{2}	16	240	229	573
8s	4.93	1.32	8.5	476	11	200	854	2 136
9s	3.20	1.13	7.9	693	15	90	735	1 838
10s	5.34	1.11	10.2	693	15	*150*	*841*	*2 102*
11c	1.71	0.85	6.7	693^{2}	15	*150*	*248*	*620*

Test*	Discharge (m^3/s)	Mean velocity (m/s)	Mean flow width (m)	Length† (m)	Number of waves	Mixing Length (m)	Convective Length (m) $t' = 0.4$	Convective Length (m) $t' = 1.0$
Porter								
1c	0.35	0.65	4.3	700[1]	14	50	300	750
2c	0.36	0.60	4.4	825[2]	18	100	275	688
3c	1.81	0.90	7.0	825[1]	16	175	347	868
4c	1.69	0.88	6.9	825[2]	16	125	322	805
5c	1.64	0.84	6.8	825[1]	17	150	358	895
6c	1.40	0.80	6.2	525[1]	16	100	281	703
7c	2.79	0.98	8.0	600[1]	17	100	384	960
8c	2.33	1.01	7.2	825[1]	16	100	346	865
9c	3.34	1.25	7.6	825[1]	12	100	409	1 023
10s	9.13	1.46	11.4	550	10	100	1162	2 906
11s	6.65	1.19	10.8	550	11	100	984	2 459
12s	5.24	1.19	9.7	550	12	100	990	2 475
13s	5.69	1.17	10.0	525	12	100	886	2 214
Thomas								
1c	0.15	0.55	2.9	350[2]	14	75	131	328
2c	0.12	0.51	2.7	500[2]	18	40	126	315
3c	0.37	0.54	3.8	500[2]	19	50	161	403
4c	0.21	0.51	3.3	350[2]	15	50	133	333
5c	1.58	0.99	4.9	525[2]	13	75	138	345
6c	1.14	0.88	4.7	500[2]	14	75	147	368
7c	1.59	0.65	5.0	525[2]	12	75	136	340
8c	0.94	0.86	2.8	2250[2]	11	75	140	350

* In this column the s represents side injection where $l = \frac{3}{4} w$, and the c represents centre injection where $l = \frac{1}{2} w$.

† In this column the *1* represents those tests which extend downstream of Fischer's convective length, and the *2* represents those tests which extended downstream of both Fischer's and Chatwin's convective lengths.

Mixing Lengths

For dilution gauging, the mixing length, x_m, is the channel length required for the tracer to sample the flow cross section completely and evenly. Past this length no further dilution occurs, and the amount of dilution can be determined accurately from a one-dimensional description of the tracer wave. Mixing lengths were estimated by locating the distance required for the dilution of the tracer mass to become constant. These estimates are listed in Table 2.

Fischer (1966) showed by dimensional analysis that the time required for complete mixing is

$$T = l^2/\epsilon_z \tag{8}$$

when l is a characteristic length. For comparative purposes a dimensionless time can be defined in terms of bulk channel parameters

$$t' = t0{,}23\, rv_*/l^2 \tag{9}$$

Fischer states that $t' = 0.4$ is a reasonably practical criterion for the end of the convective period. Chatwin (1972) suggests that $t' = 1.0$ is required for the concentration distribution to approach Gaussian. Both convective length criteria are calculated for each test and are listed in the last two columns of Table 2. Estimates are expressed in terms of channel length, and are based upon

$$x_m = 1.8\, l^2 \langle u \rangle / rv_* \tag{10}$$

The characteristic length, l, is defined as $\frac{1}{2}w$ for centre injections and $\frac{3}{4}w$ for side injections.

The mixing length estimates based on field data are characteristically shorter than those calculated by Fischer's criteria. Twenty of the tests extend downstream of both Fischer's and Chatwin's mixing-length estimates. All tests with the exception of the Bealey data extend well beyond the actual mixing length as determined from the field data.

LONGITUDINAL SPREADING OF FLUID PARTICLES

Variance Trends

The longitudinal variance trends are shown for the Bruce and Thomas test reaches in Figs. 5 and 6 respectively. These data are presented in a dimensionless form. Determining dimensionless mixing lengths as x_m/w, it can be shown that all these reaches exhibit complete mixing by about $35x_m/w$. The flow width is used to determine the dimensionless distance, as the transverse velocity variations are considered to be the main contributor to differential advection in the longitudinal direction, and also, the largest eddies contributing to the turbulent component are related to channel width. It is quite evident from the Bruce and Thomas data (Figs. 5 and 6) that an expontential relationship exists between variance and distance for all measured distances, and certainly well beyond the mixing length. Similar trends have been shown for the Bealey, Craigieburn, and Porter test reaches (Day, 1974). There is no indication that the rate of spreading

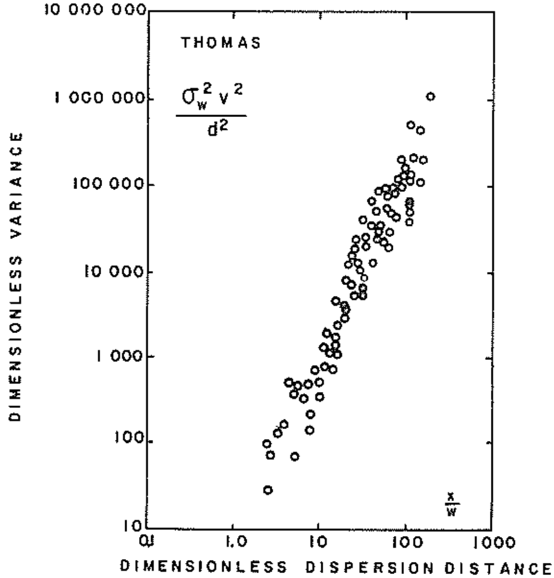

FIG.5 – Variance trends for Bruce test reach data.

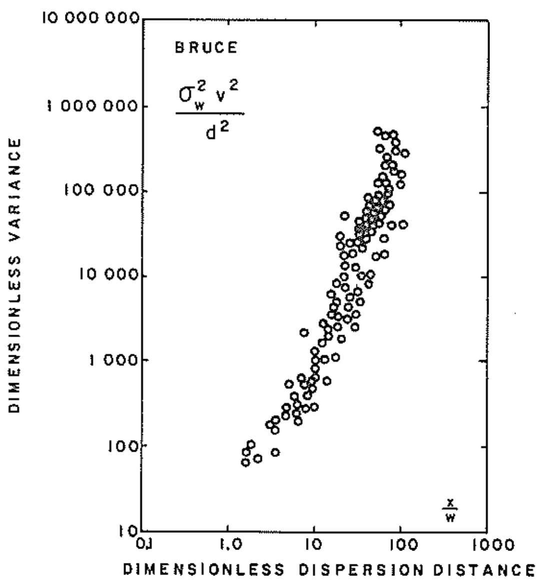

FIG.6 – Variance trends for Thomas test reach data.

decreases as the tracer particles pass downstream of the mixing length. A regression analysis on the logarithmically transformed dimensionless data sets is shown in Table 3. All equations explain at least 89 percent of the variance and are significant at the 5-percent level. The standard errors of estimate are approximately 0.3 log units. The 95% confidence band is shown for each slope and intercept.

The scatter evident in Figs. 5 and 6 (and others given by Day, 1974) represent both sampling design inadequancies and measurement errors. For small distances the concentration distributions are influenced by the conditions of injection and the upstream flow properties. Even though the injections were made as near to instantaneous as possible, exact replication was impossible. Variations in the geometry of the injection site caused by high flows would also increase the scatter, at least until the mixing length was reached. It is also possible that in some tests the probes might have been placed in dead zones located downstream of large bed elements. Furthermore, the waves were measured with, and their properties abstracted from a single probe record. Inadequate definition of the tails also can lead to considerable errors in the variance calculations.

The slopes of the regression equations reflect the structure of the dispersion process. For the five test reaches the slopes range from 1.84 for the Craigieburn to 2.3 for the Bruce, with an overall mean of 2.06. These slope values indicate that the spread or standard deviation of an ensemble of fluid particles dispersing in natural river channels increases approximately linearly with distance. The Bealey data indicate that this linearity develops within the mixing length, while the other data sets indicate that this structure is maintained for all measured distances. There is no evidence to suggest that the spreading eventually becomes proportional to the square root of distance as required for Taylor's analysis.

Peak Concentration Decay

Decay rates of the peak concentration are also determined by regression analysis of the logarithmically transformed data. The mean decay rates for each test reach are: Bealey, −1.27; Bruce, −1.24; Craigieburn, −1.07; Porter, −1.19; Thomas, −1.02. Individual decay rates range from −0.9 to −1.4, with an overall mean of −1.17 and a standard deviation of −0.21. For most tests there are insufficient data to define the decay rate within the mixing length. The Bealey data, however, measured mainly within the mixing length, indicate a decay rate greater than all but that of the Bruce data.

These data indicate that peak concentrations decay at a rate of approximately x^1, similar to the rate of longitudinal spreading.

Longitudinal Dispersion Coefficient

Two samples of Chatwin's analysis for the determination of K_x are shown in Figs. 7 and 8. The Thomas test shown in Fig. 7 represents the longest test sequence. In this plot, Taylor's Gaussian distribution is shown as a straight line and as fitted through the leading (downstream) edge of the concentration distribution.

Several features are evident. The leading edge is well represented by the regression equation, and hence by Taylor's model. In all cases the equations explain at least 90% of the variance. It is quite evident that the tracer particles resident in the leading edge rapidly adopt a Gaussian form which is maintained for all measured times and distances. In some cases, near injection, the regression model fits almost the complete wave.

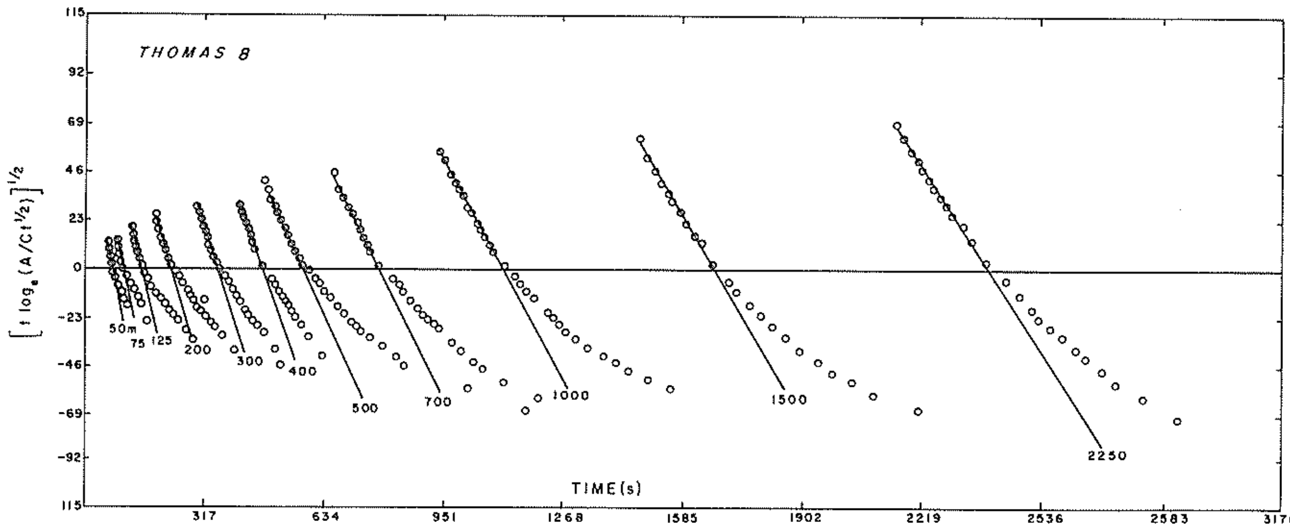

FIG.7 – Chatwin's analysis for Thomas test 8.

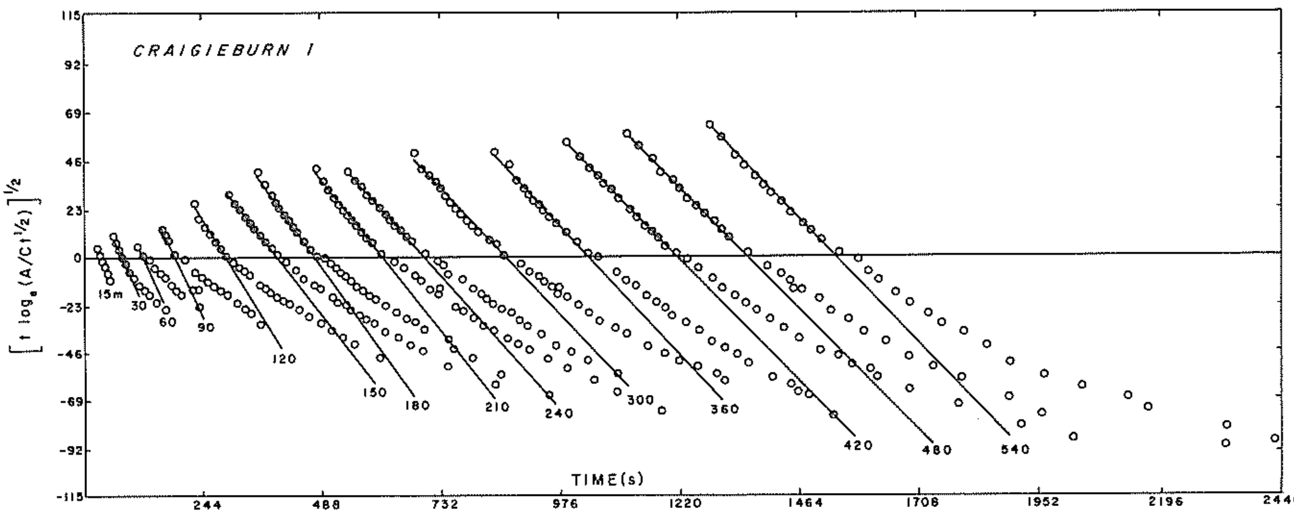

FIG.8 – Chatwin's analysis for Craigieburn test I.

It is, of course, incorrect to apply Taylor's analysis to these types of turbulent flow, whatever the mechanism or mechanisms involved. However, it is of interest to demonstrate the behaviour of K_x to illustrate probable sources of some of the discrepancies previously found between theoretical and field results.

A dimensionless dispersion coefficient is defined as K_x/vd and is shown in Fig. 9 for two test reaches. It is quite evident that the dispersion coefficient, instead of being constant along the reach, continues to increase. Similar results have been shown for the other test reaches (Day, 1974). Besides being a function of distance K_x/vd, for a set value of x/w, increases with a decreasing flow rate; i.e. higher values of K_x/vd occur with lower values of v.

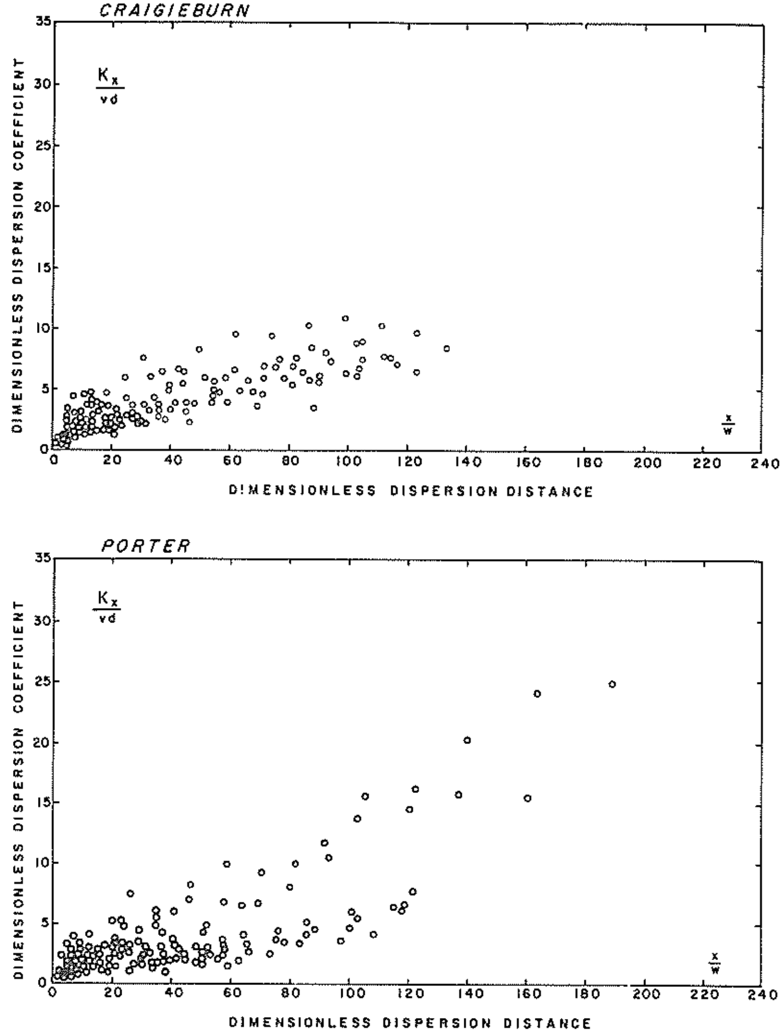

FIG.9 – Dimensionless dispersion coefficients.

TABLE 3 – Regression analysis of variance.

Test reach	*Degrees of freedom*	*Correl. coeff*	*Std error of estimate (log units)*	*Intercept*	*Regression constants*		
					*95% C.I.**	*Slope*	*95% C.I.**
Bealey	70	0.96	0.31	131.20	129.9–112.5	1.96	1.8–2.1
Bruce	161	0.95	0.33	5.64	4.5– 6.8	2.30	2.2–2.4
Craigieburn	168	0.96	0.25	22.18	21.0– 23.4	1.84	1.8–1.9
Porter	187	0.94	0.34	11.48	10.3– 12.7	2.05	1.9–2.2
Thomas	93	0.96	0.30	6.53	5.4– 7.9	2.13	2.1–2.3

C.I. is the confidence interval.

ASYMMETICAL CONCENTRATION DISTRIBUTIONS

The fitting of Chatwin's analysis has been introduced in the previous section. Figs. 7 and 8 both clearly show that once developed, the concentration distributions maintain a persistent asymmetry. In both diagrams, it can be seen that the asymmetry has developed within 200 m of the injection site. Day (1974) has shown that this asymmetrical form develops within the mixing length and possibly within the vicinity of the vertical mixing length.

The presence of asymmetrical distributions for measurements made in natural channels has long been recognized and many explanations advanced. Both Elder (1959) and Sullivan (1971) considered the laminar sublayer to play a role in producing skewness. Hays (1966) and Thackston and Schnelle (1970) show how trapping can affect the shape of the concentration distribution. Fischer (1966) dismissed both the laminar sublayer and the dead-zone concept in preference for his view that skewness develops during the convective period when the tracer particles adopt the form of the velocity profile.

The presistence of a skewed tracer distribution throughout the dispersion pattern indicates that the determining mechanism or mechanisms operate irrespective of time or distance. The role of the velocity profile in producing asymmetry exists only within the convective period, and consequently cannot be an adequate explanation. Also, the large relative roughness in the flow would create steep velocity profiles and would thereby diminish the effect of the velocity profile in producing asymmetry.

DISCUSSION WITH SUPPORTING DATA

The most logical, although unsupported, explanation for the distinctly non-Taylor-like dispersion in mountain streams is the presence of fluid traps distributed along the boundary of the main flow. The large roughness elements and the pool and rifle sequence of these streams were observed to retain then slowly release fluorescent dye back into the main flow. Once entrained in such traps, it must be impossible for these particles to ever catch up with the main tracer body retained in the main flow zone.

The linearity in longitudinal dispersion of fluvial particles is by no means restricted to the mountain streams studied here. Supporting data, although limited in extent, are available both from Godfrey and Frederick (1970) and from Yotsukura *et al.* (1970): the former's data included five natural channels and one canal, while the latter's data consist of one test on a section of a large meandering river.

The results of a regression analysis on the variance data from these sources are shown in Table 4. The data are in a dimensionless form similar to that of the test data of this study, except that the flow width, mean velocity, and flow depth are cross-sectional values. In all tests the slope is greater than 2.0, although only just so for the Clinch River and the Coachella Canal. The correlation coefficients are above 0.8, again with the exception of the Clinch River. The wide confidence bands are in part caused by the few degrees of freedom associated with the analysis. These supporting data, representing eight longitudinal dispersion tests in six natural rivers and canals, together with the 49 tests in the five streams of this study, offer convincing evidence that the spreading of fluid particles in natural channels increases approximately linearly with distance.

Although the limitations of applying Taylor's analysis have been recognized, only one other study has offered data which indicate the linear spreading in natural channels. Nordin *et al.* (1973) reported that the variance of the concentration data from Yotsukura *et al.* (1970) increased as the square of time. According to Fischer (1973b), the implication of this rate of spreading is that the turbulent fluctuations have a longer memory or correlation time than the period of observation. If this is true, then the integral time scale is not finite, and Taylor's model is inapplicable.

TABLE 4 – Regression analysis of supplementary variance data.

Data source	*Degrees of freedom*	*Correl. coeff*	*Std error of estimate*	*Regression slope*	*95% confidence band*
Godfrey & Frederick (1970)					
Cooper Ck (tests 1,6 & 11)	18	0.961	0.31	2.49	2.1 –2.89
Clinch R. (tests 2,7, & 10)	18	0.685	0.49	1.34	0.59–2.09
Copper Ck (test 3)	6	0.803	0.78	2.99	0.0 –6.07
Powell R. (test 4)	5	0.953	0.31	2.87	1.21–4.53
Clinch R. (test 5)	6	0.98	0.21	2.36	1.62–3.10
Coachella Canal (test 8)	5	0.99	0.05	1.45	1.27–1.63
Yotsukura *et al.* (1970)					
Missouri R.	4	0.97	0.16	2.17	0.60–3.74

Nordin *et al.* (1972, 1973) attempted to explain this rate of spreading by the presence of a Hurst phenomenon, possibly through the influence of such sporadic processes as intermittency or vortex action. These features have not, however, been demonstrated.

SUMMARY

The structure of the dispersion process in mountain streams is empirically defined, and it is shown that for distance up to 800 x/w, this structure precludes the application of Taylor's analysis. It is conclusively shown that the spread or standard deviation of an initially concentrated mass increases linearly with distance, and that the concentration distributions maintain a persistent asymmetry. Both features indicate that the mean properties of the motion never become stationary random functions of time. Unfortunately, until the determining mechanism or mechanisms are demonstrated, possible causes can only be speculated upon. The similarity in structure of the dispersion process among the rough test channels and other channel types – the Missouri River in particular (as it is a wide, sand-bed meandering river) – should be sufficient implication that the mechanism or mechanisms are common.

ACKNOWLEDGEMENTS

This study was undertaken while the author was a doctoral candidate in the Department of Geography, University of Canterbury. The assistance of Professor I. R. Wood is gratefully acknowledged.

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