A MATHEMATICAL CATCHMENT MODEL FOR ESTIMATING RUN-OFF

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The development of mathematical catchment models from past studies of catchment water balance is reviewed. The structure and operation of a catchment model which uses daily rainfalls to estimate daily volumes of run-off is described. A steepest-ascent method for evaluating variables in the model is described and the results obtained with the model on five catchments in New Zealand are reported.

INTRODUCTION

There are different types of simulation methods used in the analysis of the rainfall/run-off process. Dooge (1968) classifies the types as:

(1) direct simulation (scale models, analogues, etc.),
(2) semi-direct simulation (network analysers, Hele-Shaw models, etc.),
(3) indirect simulation (desk calculators, digital computers, etc.).

Chow and Harbaugh (1965) and Amorocho (1963) have described laboratory models of catchment systems and Riley, Chadwick and Israelson (1967) described an electric analogue computer for direct simulation of a small catchment in southern Arizona.

In recent years, most interest in the simulation of catchment behaviour has been concentrated in the indirect type of simulation associated with mathematical catchment models. This paper deals with mathematical catchment models and reports some results obtained with such a model on the digital computer at Lincoln College. The terms 'model' and 'catchment model' in this paper refer only to mathematical models.

CONCEPT OF A CATCHMENT MODEL

There are two major parts of any catchment model which simulates the rainfall/run-off process. Firstly, there is a water-balance aspect which deals essentially with the disposition of losses

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from rainfall and is of prime importance in determining the volume of run-off. Secondly, there are the storage-delay effects of the catchment which determine the time-distribution of run-off.

Much attention has been given to the second aspect in the study of floods. Unit hydrograph models and run-off routing models have been developed much further than those aspects which deal with losses. In most if not all studies which deal only with flood aspects, attention is given only to storms where substantial rates of run-off occur, and each storm is considered as an isolated event.

In the traditional methods of flood analysis, the start of rise of the surface run-off hydrograph is taken as the point when initial loss has been satisfied and this is the factor which determines the amount of initial loss. By contrast, in catchment models, the potential of the catchment to absorb rain (i.e. the potential initial loss) is continuously simulated and it is when the potential loss is satisfied that determines when run-off starts, not the other way around.

Any complete catchment model must be capable of simulating those events when heavy rain occurs and little or no run-off results, as well as simulating flood events. The model must be capable of accounting for high loss rates with corresponding low rates of run-off as well as accounting for the low loss rates and high run-off rates of floods.

A complete catchment model is therefore proposed as one which simulates the continuous behaviour of a catchment, drying phases as well as run-off phases, rather than one which simulates only the catchment storage-delay effects which determine hydrograph shape.

DEVELOPMENT OF THE CATCHMENT MODEL

The model reported in this paper uses daily rainfall data and monthly evaporation data to determine daily volumes of surface run-off. The model has been described in previous papers (Boughton 1965, 1966, 1968), but a short review is given here to explain how the model was developed.

The basis of the model is a water balance between:

(1) input to the catchment as rainfall;

(2) output from the catchment as evapotranspiration loss, run-off, and deep percolation;

(3) change in the volume of water storage in the catchment.

The balance may be summarized in the equation:

$$P = E_t + Q + Q_{dp} + \Delta S \qquad \text{.... ....} \qquad (1)$$

where P = rainfall,
E_t = evapotranspiration loss,
Q = run-off,
Q_{dp} = deep percolation,
ΔS = change in storage.

The water-balance technique is not new to hydrology but it has developed very rapidly in recent years as complex catchment models have developed. Originally, water-balance concepts and models were very simple. Fig. 1 illustrates an early and simple approach where the catchment was represented as a single storage of fixed capacity. Evapotranspiration and deep percolation losses from the storage were commonly lumped together and estimated as a single percentage of the amount remaining in the system. This is the Antecedent Precipitation Index (API) approach, still in use for flood forecasting (e.g. Commonwealth of Australia, 1963).

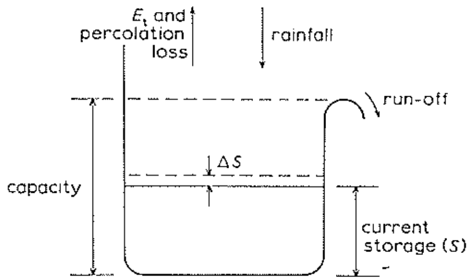

FIG. 1 — Antecedent precipitation index catchment model.

TABLE 1 — Antecedent precipitation index model. Capacity=4.00 inches, coefficient=0.9; starting from saturation, i.e., S=4.00.

Day	*Rainfall (inches)*	*Soil moisture level (inches)*	*Run-off (inches)*
1	—	3.60	—
2	—	3.24	—
3	—	2.92	—
4	0.05	2.63	—
5	—	2.41	—
6	0.50	2.67	—
7	2.00	4.00	0.40
8	1.00	4.00	0.60
9	0.10	3.70	—
10	—	3.33	—
11	—	3.00	—

In operation, this model involves multiplying the amount in storage on the previous day by a multiplying factor (value usually about 0.85 to 0.98; Linsley *et al.*, 1958), then adding the daily rainfall for the day considered. When the daily rainfall is sufficient to exceed the current moisture deficiency, run-off is assumed to occur. Table 1 illustrates a sample calculation using a catchment storage capacity of 4.0 inches and a value of 0.9 for the API factor. The storage capacity and the factor value are the two parameters which define the operation of the system, and these must be evaluated in order to use the model.

To improve the estimation of catchment moisture deficiency, Linsley *et al.* (1958: ch.8) introduced other factors into the model by using coaxial graphical correlations. The factors used to estimate run-off in this model were antecedent precipitation index as described above, week of the year (to account for seasonal effects), amount of storm rainfall and duration of storm rainfall. The correlations were established graphically from records of rainfall and run-off.

In many places deep percolation losses are small compared to evapotranspiration losses, commonly five percent or less. Some water-balance models, applicable to these conditions, have neglected deep percolation and have estimated output from the catchment as evapotranspiration and run-off only. In this case the water-balance equation becomes:

$$P = E_t + Q + \Delta S \qquad \ldots \quad \ldots \quad \ldots \quad (2)$$

Evapotranspiration is generally larger than either run-off or the change in storage, and is of principal concern in a water balance. Evapotranspiration losses include evaporation from the surface of vegetation, transpiration through leaves, and evaporation from the surface of the soil.

Transpiration through leaves accounts for most of the water lost by evapotranspiration, and this is undoubtably the most important single factor in determining moisture lost from a catchment. The rate at which a plant uses water under given meteorological conditions and when soil moisture is freely available is termed the potential transpiration rate. This has been widely studied in determining the water requirements of irrigated crops. From the results of these studies, potential transpiration (and potential evapotranspiration) can now be estimated with reasonable accuracy. However, in hydrological studies of water lost from a natural catchment, it is necessary to be able to calculate the transpiration rate at very low soil-moisture levels as the catchment dries out.

There has been considerable argument about this matter. The most well-known proposals were those of Veihmeyer and Hendrickson (1955) who maintained that transpiration continued at the potential rate at all soil-moisture levels between field capacity and wilting point, and Thornthwaite and Mather (1955) on the other hand who proposed that transpiration decreased in a linear manner from potential rate at field capacity to zero at the wilting point.

More recently, it has been shown by Denmead and Shaw (1962) and Slatyer and Denmead (1963) that the ratio of actual to potential transpiration is not a single value depending on the soil-moisture level alone, but that it also depends on the prevailing potential rate. The relationship of actual to potential transpiration is shown in Fig. 2A. It can be seen in this figure that transpiration can continue at the potential rate while soil moisture is reduced almost to wilting point, provided that the prevailing potential rate is low. When the prevailing rate is high the actual transpiration rate is reduced below the potential rate when the soil-moisture level is only fractionally less than field capacity.

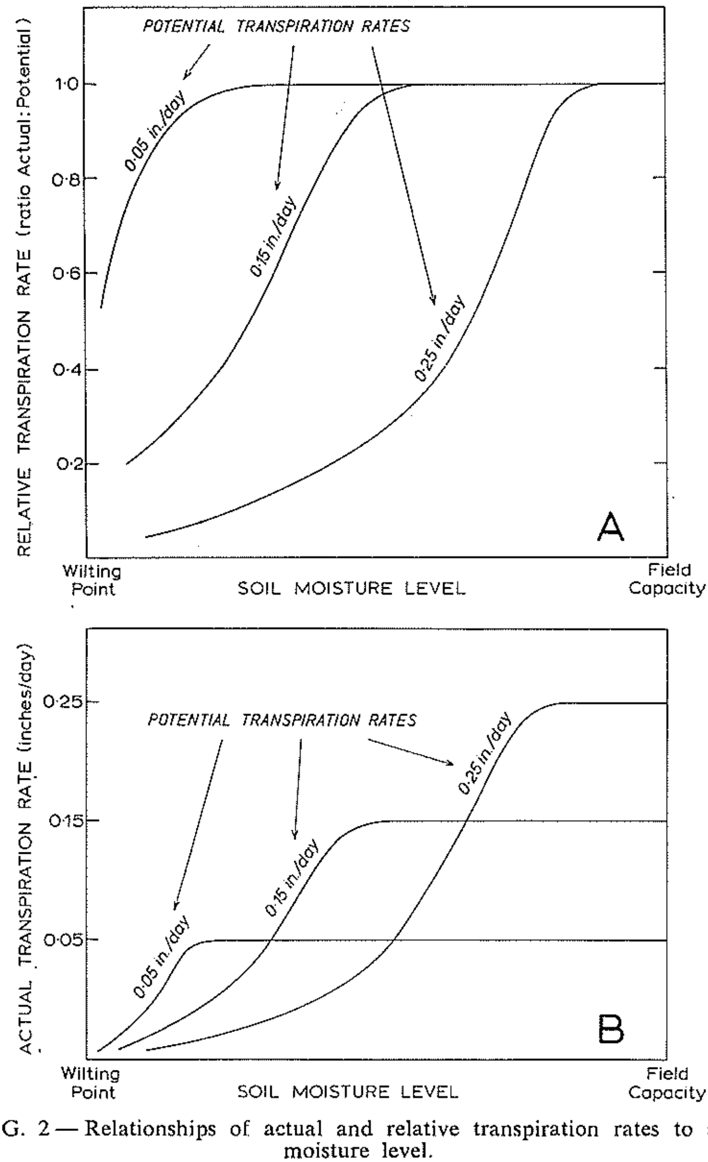

FIG. 2 — Relationships of actual and relative transpiration rates to soil moisture level.

For calculation of the evapotranspiration losses from natural catchments, the assumption has been made that the ratio of actual to potential loss at different soil-moisture levels would be of the same form as in Fig. 2.

Early water-balance models were generally based on the concept of a single moisture deficiency which must be satisfied before run-off begins. It has been appreciated for some time that losses are estimated better if considered in two parts — an initial loss at the start of each storm and a continuing loss throughout the storm (see Fig. 3).

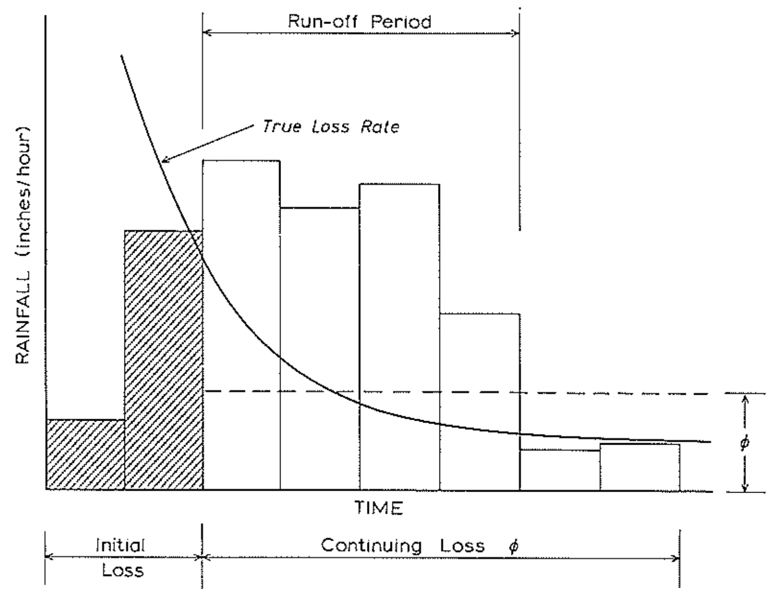

FIG. 3 — Initial loss and continuing loss used to estimate losses from rainfall.

The Bureau of Meteorology of the Commonwealth of Australia (1963) adopted a two-part model of losses of this sort for the flood-forecasting system of Macleay River Valley on the coast of New South Wales. In this model, initial loss is estimated using an antecedent precipitation index, and a constant value of five points per hour is used for the continuing loss during the supply period of the storm.

The use of an API instead of a water-balance calculation for determining the value of initial loss, and the use of a constant loss rate for all storms have been adopted by the Bureau because of the need to have both a fast method of estimating the volume of run-off while a storm is in progress and also to have a method not conducive to errors in calculation. The model is orientated to manual calculation. For research purposes these restrictions can be relaxed and more realistic — albeit more complex — methods used.

The continuing loss during storms (called 'loss rate' in flood-estimation procedures) is not the same for all storms. In flood-estimation and flood-forecasting studies, most attention is given to the very low values of loss rate which occur after prolonged rain and which are associated with peak floods. However, loss rates can have very high values — particularly when the soil is very dry. It is necessary to determine which factors influence the loss rate in order to simulate loss rates which take on different values at different times. Laurenson and Pilgrim (1963) give a good review of these factors and indicate that antecedent moisture conditions appear to be the most important.

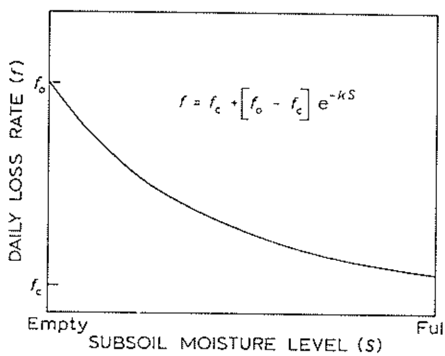

FIG. 4 — Relationship between subsoil moisture level and daily loss rate.

In the catchment model used in the study reported here, it was necessary to assume a relationship between antecedent moisture level and loss rate. The loss rate was assumed to be high when the soil-moisture level was low, and vice versa, and the relationship was assumed to be of an inverse exponential form as shown in Fig. 4. This relationship was described by an equation similar to Horton's infiltration equation:

$$f = f_c + (f_o - f_c) e^{-kS} \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (3)$$

where f = daily loss rate,
f_o = loss rate when soil is at wilting point,
f_c = a limiting value which the loss rate approaches at high soil-moisture levels,
k = an exponent which determines the shape of the curve bewteen f_o and f_c,
S = current soil-moisture level.

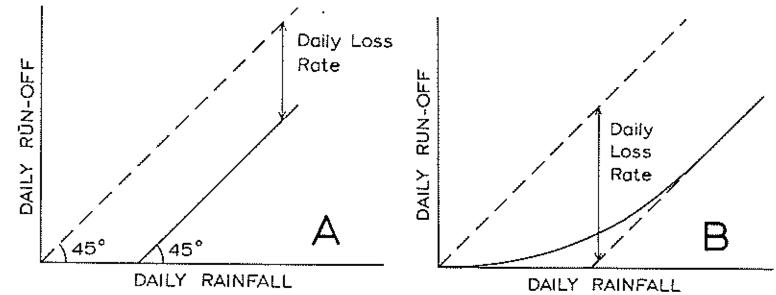

FIG. 5 — Relationship between daily rainfall and daily run-off for one daily loss rate.

The loss rate determined by this relationship is a potential value which would be satisfied if the rainfall was uniform throughout the day and exceeded the loss rate throughout the day. If this was so, the relationship of run-off to rainfall for a single value of loss rate would be as shown in Fig. 5A. The higher the daily rainfall, the greater the likelihood of this condition being met.

However, when the daily rainfall is small or when the loss rate is high compared to the rainfall, then the relationship is more likely to be of the form shown in Fig. 5B. For a range of different loss rates, the rainfall/run-off relationship would be of the form shown in Fig. 6. The algebraic equation used to define the relationship is:

$$Q = P - f \,.\, \tanh\,(P/f) \qquad (4)$$

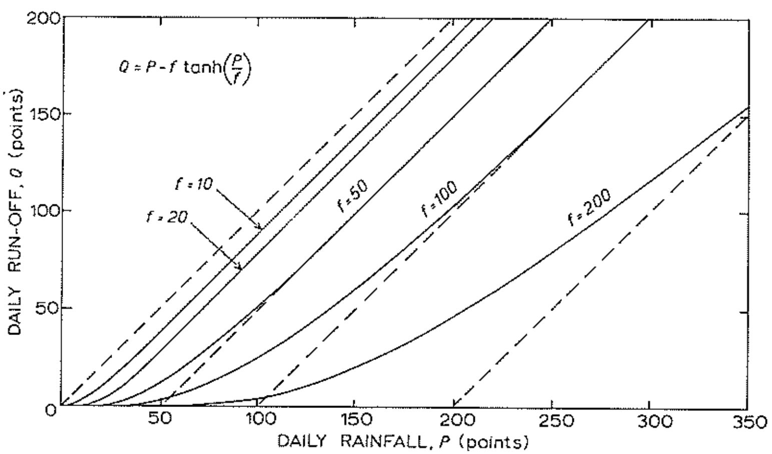

FIG. 6 — Relationship between daily rainfall and daily run-off for several daily loss rates.

STRUCTURE OF THE MODEL

The structure of the model was developed from the arguments and assumptions set out in the previous sections. The major elements of the model were chosen as:

(1) Interception store — where loss of water from the system is not restricted by soil-moisture level.
(2) Drainage store — to stimulate the capacity of depression storage.
(3) Upper soil store — which forms the surface-storage capacity of the catchment in conjunction with the interception and drainage stores.
(4) Subsoil store — which determines the daily loss rate.

The structure of the model is set out in Fig. 7.

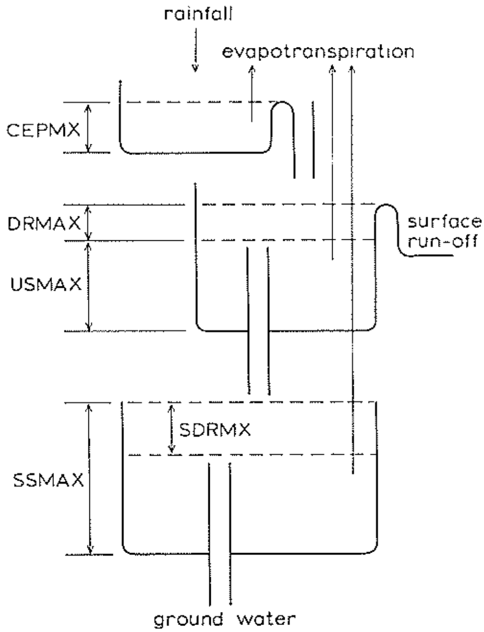

FIG. 7 — Structure of the catchment model.

OPERATION OF THE MODEL

The time unit used in operation of the model is one day, and there are three phases of water balance within each day — a wetting phase, a drying phase, and a drainage phase.

Rainfall is first added to the moisture stores and, if any run-off occurs, the results are printed out.

Losses by evapotranspiration are then calculated and the moisture stores are adjusted.

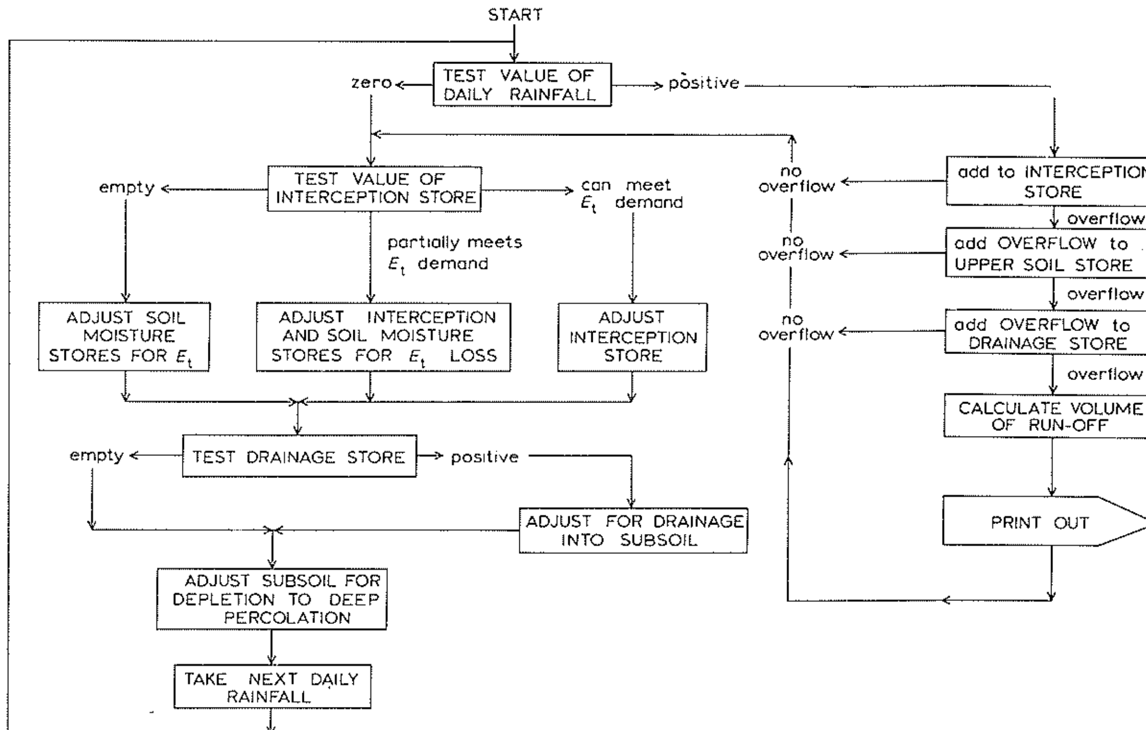

FIG. 8 — Flow chart of operating procedures.

Finally, any movements of water by drainage are made and the stores are again adjusted.

The next daily rainfall is then taken and the cycle of operation is repeated. The flow chart of daily adjustments is shown in Fig. 8. In practice, the whole sequence of calculations for operation of the model is performed by a digital computer. The Fortran programme for operation of the model on the IBM 1130 computer at Lincoln College is set out in Appendix 1.

VALUES OF VARIABLES

For operation of the model it is necessary to give values to the capacities of the moisture stores and to the variables of equation (3), which defines the daily loss rates. These capacities and variables are as follows:

Moisture Stores:

1. Interception (CEPMX)
2. Drainage (DRMAX)
3. Upper Soil (USMAX)
4. Subsoil (SSMAX)
5. Subsoil Drainage (SDRMX)

Infiltration Variables:

6. f_o
7. f_c
8. k

(see Appendix 1)

When the model was first developed, values of these eight variables were estimated from assumed depths and moisture-holding capacities of soil profiles, and published infiltration data. Some trial-and-error adjustments were made, particularly of the infiltration variables, in order to get agreement between calculated volumes of run-off and actual volumes of run-off when using rainfall and run-off records from gauged catchments. Generally a different set of values for the variables is used with each different catchment.

It was clear from the start that there was considerable freedom in the values which could be given to the variables. In order to make the selection of values less subjective, it was decided to incorporate a systematic sampling procedure with the model to find in the most objective way that set of values which gives best agreement between calculated and recorded run-off.

Each variable is continuous in the range of values that it can take, but — even if each was restricted to a small number of discrete values — testing of all possible combinations of the variables would not be possible. It is necessary to select some sampling procedure which would keep the number of trials within practical limits.

The methods generally available for this type of problem are reviewed by Hufschmidt (Maass *et al.*, 1962: ch. 10). These are:

(1) uniform-grid or factorial sampling,
(2) single-factor sampling,
(3) incremental or marginal analysis,
(4) steepest-ascent method.

Uniform-grid or factorial sampling can be a lengthy business even with as few as eight variables. If only 5 values for each of 8 variables were to be tested, there would be $5^8 = 387625$ trials to be made. It seems impracticable to reduce the number of variables and/or the number of values of each sufficiently to make this method useful. The other procedures available, which use the results of trials already made to search for better combinations of values, are superior.

Single-factor sampling and marginal-analysis sampling were initially tried, and the results obtained with these methods have already been reported (Boughton, 1968). These methods proved to be inefficient in that a large number of trials was need to find the optimum combination of values. The method of steepest ascent proved to be more efficient, and this is described in the following section.

STEEPEST-ASCENT METHOD

The model calculates daily volumes of run-off. To make a comparison of calculated run-off with actual run-off, the daily volumes are added to get monthly totals. The difference between calculated monthly totals and recorded monthly totals are squared and the squares of differences are added to get a sum of squares for the length of record which is available. The agreement between calculated and recorded run-off is measured by this sum of squares, and the objective of the steepest-ascent routine is to find the set of values of the variables in the model which gives a minimum value of the sum of squares.

Searching for a minimum value involves a steepest-descent search rather than an ascent search but the point is a minor one of wording. By stipulating the reciprocal of the sum of squares of differences as the objective, the search becomes one of ascent. The term 'steepest ascent' is now in common usage and is used here even though the minimum value of the sum of squares is given as the objective.

The method derives its name from the fact that sampling moves sequentially from lower to higher elevations on the response surface, and does this along the steepest and hence shortest path up the slope. A starting set of values of the variables is selected and the value of the ranking function (i.e. the sum of squares of differences) is calculated. A change is then made in the value of one variable and the effect of the change on the sum of squares is noted. The value of the variable is then returned to its original value.

The effect of change in value of each of the other variables is then determined in turn, testing one variable at a time. The relative change in the sum of squares produced by a variable determines the relative amount of change in value of that variable which should be made in order to progress along the path of steepest ascent.

An example of the method used is set out in Table 2 using data from the Makara 2 catchment. The starting values of the variables are shown in the top line of the table, and the resulting sum of squares is 393287.

The first round of adjustment is shown in the next seven lines. At each trial one variable is increased by 10% of its value and the resultant sum of squares calculated. When all variables have been tested separately, the rate of adjustment of each variable is determined in proportion to the amount of change it produced in the sum of squares. Also, the variable is either increased or decreased in value depending on whether the trial increase in value resulted in a decrease or an increase in the sum of squares.

All variables are then changed in value together by amounts and in the direction determined from the trials. The adjustments continue while the sum of squares continues to decrease. When the sum of squares reaches a low point and the next trial results in an increase in the sum, the value of the variable which gave the lowest sum of squares is taken as the starting point for the second round of adjustments.

After two rounds of adjustments, the sum of squares for the Makara 2 catchment had fallen to 182445. Table 3 shows the comparison of monthly recorded and calculated run-off which produced this sum. Table 4 illustrates a sample of the daily values of run-off calculated by the model, compared with recorded values of run-off.

RESULTS

Data from five New Zealand catchments and six Australian catchments have been used with the model. The New Zealand catchments are Makara catchments Nos. 1, 2 and 10, and the

TABLE 2 — Steepest ascent adjustment, Makara 2 catchment.

Moisture stores				Infiltration				
USMAX	*DRMAX*	*SSMAX*	*SDRMX*	*FO*	*FC*	*K*	*Sum of squares*	*Change*
124	31	1050	80	1010	129	0.0037	393287	
1st set of trials								
126	31	1050	80	1010	129	0.0037	393623	+336
124	*34*	1050	80	1010	129	0.0037	406887	+13600
124	31	*1155*	80	1010	129	0.0037	375371	−17916
124	31	1050	*88*	1010	129	0.0037	393287	0
124	31	1050	80	*1111*	129	0.0037	399878	+6591
124	31	1050	80	1010	*142*	0.0037	420603	+27316
124	31	1050	80	1010	129	*0.0041*	375561	−17726
1st round of adjustments								
124	29	1155	80	974	110	0.0041	304446	
124	26	1260	80	938	93	0.0044	235982	
124	24	1365	80	900	75	0.0048	*208048*	
124	22	1470	80	866	68	0.0052	235435	
Best result after 1st round								
124	24	1365	80	900	75	0.0048	208048	
2nd set of trials								
136	24	1365	80	900	75	0.0048	203800	−4248
124	*26*	1365	80	900	75	0.0048	211198	+3150
124	24	*1500*	80	900	75	0.0048	219826	+11778
124	24	1365	*88*	900	75	0.0048	208048	0
124	24	1365	80	*990*	75	0.0048	204831	−3217
124	24	1365	80	900	*83*	0.0048	215401	+7353
124	24	1365	80	900	75	*0.0053*	219396	+11348
2nd round of adjustments								
129	23	1230	80	925	70	0.0043	184306	
132	22	1100	80	950	65	0.0039	*182445*	
135	21	970	80	975	60	0.0036	211464	

TABLE 3 — Comparison of recorded and calculated run-off. Makara 2 catchment.

Month	*Rec. run-off (pts)*	*Calc. run-off (pts)*	*Month*	*Rec. run-off (pts)*	*Calc. run-off (pts)*	*Month*	*Rec. run-off (pts)*	*Calc. run-off (pts)*
1957			**1958**			**1959**		
Jan	8	–	Jan	19*	130	Jan	–	–
Feb	–	–	Feb	153	120	Feb	–	–
Mar	–	–	Mar	10	5	Mar	–	–
Apr	29	4	Apr	–	0	Apr	2	1
May	3	75	May	278	268	May	303	139
Jun	396	500	Jun	127	76	Jun	–	–
Jul	1	15	Jul	25	1	Jul	33	15
Aug	–	14	Aug	25	3	Aug	–	–
Sep	–	104	Sep	63	22	Sep	187	180
Oct	2	24	Oct	15	13	Oct	156	121
Nov	56	108	Nov	–	–	Nov	–	–
Dec	1	5	Dec	19	8	Dec	–	–
1960			**1961**			**1962**		
Jan	–	–	Jan	–	4	Jan	75	99
Feb	2	7	Feb	–	–	Feb	–	–
Mar	–	–	Mar	–	1	Mar	–	–
Apr	–	–	Apr	–	–	Apr	108	160
May	14	–	May	20	1	May	82	84
Jun	192	96	Jun	108	35	Jun	90	22
Jul	177	29	Jul	244	212	Jul	158	156
Aug	84	64	Aug	179	57	Aug	94	17
Sep	98	21	Sep	309	174	Sep	21	4
Oct	80	70	Oct	–	–	Oct	140	65
Nov	–	–	Nov	–	–	Nov	5	–
Dec	1	11	Dec	–	–	Dec	30	35
1963			**1964**					
Jan	–	9	Jan	–	–			
Feb	6	17	Feb	–	–			
Mar	–	–	Mar	–	–			
Apr	3	15	Apr	–	–			
May	–	–	May	–	–			
Jun	135	153	Jun	97	82			
Jul	102	114	Jul	177	201			
Aug	326	390	Aug	25	–			
Sep	23	3	Sep	40	33			
Oct	–	–	Oct	–	–			
Nov	–	–	Nov	–	–			
Dec	–	–	Dec	–	–			

* Published run-off data apparently in error. This value should be 113 points. See footnote to Table 4.

TABLE 4 — Comparison of recorded and calculated run-off, Makara 2 catchment 1958.

Day	January Rain-fall (pts)	January Rec. run-off (pts)	January Calc. run-off (pts)	February Rain-fall (pts)	February Rec. run-off (pts)	February Calc. run-off (pts)	March Rain-fall (pts)	March Rec. run-off (pts)	March Calc. run-off (pts)
1	83	1	1	–	–	–	–	–	–
2	–	–	–	1	–	–	–	–	–
3	–	–	–	–	–	–	1	–	–
4	13	–	–	–	–	–	–	–	–
5	256	16*	129	–	–	–	–	–	–
6	4	2	–	–	–	–	–	–	–
7	–	–	–	–	–	–	–	–	–
8	–	–	–	30	–	–	–	–	–
9	–	–	–	–	–	–	–	–	–
10	–	–	–	–	–	–	1	–	–
11	–	–	–	–	–	–	42	–	–
12	–	–	–	–	–	–	–	–	–
13	–	–	–	–	–	–	26	–	–
14	–	–	–	–	–	–	7	–	–
15	–	–	–	–	–	–	32	–	–
16	30	–	–	–	–	–	8	–	–
17	3	–	–	28	–	–	3	–	–
18	–	–	–	55	–	–	–	–	–
19	–	–	–	4	–	–	–	–	–
20	–	–	–	–	–	–	–	–	–
21	–	–	–	3	–	–	–	–	–
22	–	–	–	43	–	–	8	–	–
23	–	–	–	–	–	–	16	–	–
24	50	–	–	174	10	11	–	–	–
25	2	–	–	235	143	109	–	–	–
26	–	–	–	–	–	–	113	10	5
27	4	–	–	–	–	–	9	–	–
28	2	–	–	–	–	–	3	–	–
29	–	–	–	–	–	–	8	–	–
30	–	–	–	–	–	–	–	–	–
31	1	–	–	–	–	–	–	–	–
Total	448	19*	130	573	153	120	277	10	5

* A review made recently shows that published data are apparently in error. Run-off on 5 January was approximately 110 points, monthly run-off approximately 113 points.

TABLE 5 — Comparison of recorded and calculated run-off, Makara 1 catchment.

Month	*Rec. run-off (pts)*	*Calc. run-off (pts)*	*Month*	*Rec. run-off (pts)*	*Calc. run-off (pts)*	*Month*	*Rec. run-off (pts)*	*Calc. run-off (pts)*
1957			**1958**			**1959**		
Jan	2	–	Jan	108	67	Jan	–	–
Feb	–	–	Feb	58	64	Feb	1	–
Mar	–	–	Mar	1	1	Mar	–	–
Apr	–	3	Apr	–	–	Apr	–	–
May	73	56	May	193	186	May	127	61
Jun	308	516	Jun	57	15	Jun	–	–
Jul	29	17	Jul	19	–	Jul	9	2
Aug	18	2	Aug	16	–	Aug	–	–
Sep	52	52	Sep	36	6	Sep	103	93
Oct	90	2	Oct	3	8	Oct	91	68
Nov	49	28	Nov	–	–	Nov	–	–
Dec	9	–	Dec	7	–	Dec	–	–
1960			**1961**			**1962**		
Jan	–	–	Jan	–	1	Jan	3	73
Feb	–	1	Feb	–	–	Feb	1	–
Mar	2	–	Mar	2	–	Mar	–	–
Apr	–	–	Apr	–	–	Apr	27	112
May	1	–	May	1	–	May	35	25
Jun	110	41	Jun	60	14	Jun	30	5
Jul	79	4	Jul	172	85	Jul	104	86
Aug	100	13	Aug	93	18	Aug	8	1
Sep	59	7	Sep	191	113	Sep	2	–
Oct	5	27	Oct	–	–	Oct	56	37
Nov	–	–	Nov	–	–	Nov	1	–
Dec	–	4	Dec	–	–	Dec	4	24
1963			**1964**					
Jan	–	3	Jan	–	–			
Feb	–	5	Feb	–	–			
Mar	–	–	Mar	–	–			
Apr	–	6	Apr	–	–			
May	–	–	May	–	–			
Jun	61	58	Jun	2	37			
Jul	52	43	Jul	74	107			
Aug	228	291	Aug	–	–			
Sep	8	–	Sep	9	12			
Oct	–	–	Oct	–	–			
Nov	–	–	Nov	–	–			
Dec	–	–	Dec	–	–			

TABLE 6 — Comparison of recorded and calculated run-off, Makara 10 catchment.

Month	*Rec. run-off (pts)*	*Calc. run-off (pts)*	*Month*	*Rec. run-off (pts)*	*Calc. run-off (pts)*	*Month*	*Rec. run-off (pts)*	*Calc. run-off (pts)*
1960			**1961**			**1962**		
Jan	–	–	Jan	15	–	Jan	125	181
Feb	–	13	Feb	14	–	Feb	30	–
Mar	11	–	Mar	22	–	Mar	42	–
Apr	4	–	Apr	20	–	Apr	128	68
May	5	–	May	32	–	May	124	143
Jun	73	89	Jun	20	24	Jun	167	99
Jul	142	99	Jul	200	296	Jul	195	176
Aug	231	134	Aug	195	197	Aug	135	75
Sep	70	64	Sep	259	283	Sep	56	2
Oct	68	92	Oct	16	–	Oct	109	76
Nov	21	–	Nov	16	–	Nov	51	–
Dec	11	1	Dec	22	–	Dec	47	41
1963			**1964**					
Jan	43	14	Jan	23	–			
Feb	41	20	Feb	18	–			
Mar	13	–	Mar	39	–			
Apr	33	2	Apr	27	–			
May	17	–	May	32	–			
Jun	113	108	Jun	104	17			
Jul	101	100	Jul	288	194			
Aug	332	457	Aug	36	–			
Sep	71	37	Sep	185	142			
Oct	13	–	Oct	20	–			
Nov	17	–	Nov	18	–			
Dec	14	–	Dec	28	–			

TABLE 7 — Comparison of recorded and calculated run-off, Hoon Hay catchment. I.O.O. indicates recording instruments out of order (no records are available for 1964).

Month	*Rec. run-off (pts)*	*Calc. run-off (pts)*	*Month*	*Rec. run-off (pts)*	*Calc. run-off (pts)*	*Month*	*Rec. run-off (pts)*	*Calc. run-off (pts)*
1963			**1965**			**1966**		
Jan	–	–	Jan	–	–	Jan	–	–
Feb	–	6	Feb	–	–	Feb	–	–
Mar	–	–	Mar	12	29	Mar	–	10
Apr	29	55	Apr	11	5	Apr	–	–
May	–	–	May	10	1	May	4	3
Jun	17	91	Jun	67	42	Jun	2	–
Jul	314	378	Jul	200	157	Jul	3	–
Aug	76	82	Aug	211	117	Aug	21	54
Sep	55	–	Sep	117	80	Sep	10	–
Oct	2	–	Oct	4	–	Oct	–	–
Nov	23	–	Nov	27	44	Nov	7	36
Dec	I.O.O.		Dec	1	–	Dec	–	–

TABLE 8 — Comparison of recorded and calculated run-off, Cashmere catchment. I.O.O. indicates recording instruments out of order (no records are available for 1964).

Month	*Rec. run-off (pts)*	*Calc. run-off (pts)*	Month	*Rec. run-off (pts)*	*Calc. run-off (pts)*	Month	*Rec. run-off (pts)*	*Calc. run-off (pts)*
1963			**1965**			**1966**		
Jan	–	–	Jan	–	–	Jan	–	–
Feb	1	19	Feb	–	–	Feb	–	–
Mar	2	–	Mar	63	67	Mar	3	14
Apr	50	114	Apr	22	–	Apr	1	–
May	5	–	May	27	2	May	17	–
Jun	63	115	Jun	152	113	Jun	36	1
Jul	391	407	Jul	247	211	Jul	27	–
Aug	147	121	Aug	I.O.O.		Aug	32	7
Sep	57	44	Sep	I.O.O.		Sep	25	–
Oct	4	–	Oct	I.O.O.		Oct	5	–
Nov	5	–	Nov	43	47	Nov	19	48
Dec	I.O.O.		Dec	5	–	Dec	1	–

Cashmere and Hoon Hay catchments on the Port Hills near Christchurch. The comparison of monthly recorded and calculated run-offs for these is shown in Tables 5 to 8. Results obtained from the six Australian catchments are set out in a previous paper (Boughton, 1968).

In the Northern Territory of Australia, Kingston and Fenwick (Northern Territory Administration, 1968) have adopted the model for design of a reservoir to augment the Darwin water supply. A ground-water store was added to the model, and values of the parameters were found by trial and error to give agreement between calculated run-off and recorded run-off. Monte Carlo methods were then used to generate 500 years of daily rainfalls, and these were used as an input to the model to produce monthly flows for design of the dam.

In the United Kingdom, the model has been used by the Devon River Authority to provide information on run-off for a Select Parliamentary Committee (Herbert, pers. comm., 1968), and in Australia the model is being used for analysis of data from the experimental catchments of the Forest Research Institute in the Cotter River basin near Canberra (Thistlethwaite, pers. comm., 1967).

STATE OF THE ART

The development of catchment models has been very rapid, paralleling the development in high-speed digital computers. It is only recently that the pattern of development has become clear and some sort of methodology established. It is now possible to set out the basic steps in development of a catchment model in order to see how the present approach can be improved.

The three main parts of the development of a catchment model are:

(1) setting up the structure and operating rules of the model,
(2) evaluating the variables in the model,
(3) interpreting the results.

Structure and Operating Rules

The structure and operating rules of any catchment model can be designed either as a composite of basic physical processes or as devices of calculation to reproduce certain observable effects.

It is difficult to construct a model wholly from previously established physical processes with values of components established from actual measured characteristics of the catchment represented. The principal problems are firstly the variability of processes such as infiltration, depression storage, and interflow, over the area of a catchment, and secondly the lack of established procedures for taking field measurements in order to simulate these processes. It is simpler to adopt methods which will reproduce observed effects without worrying about representing physical processes.

One obstacle to the development of models based on physical processes is the lack of suitable catchments with records other than rainfall and run-off. In order to model the processes realistically, records are needed of soil-moisture levels, sources of partial-area run-off, transmission loss in the channel system, and so on. Measurements of some of these factors are planned under the IHD experimental-catchment programme in New Zealand.

Evaluating the Variables

There are two basic approaches to the selection of values of the variables:

(1) values derived from prior knowledge or assumptions about soil depth, water-holding capacity, etc., with physical meaning attributed to component parts of the catchment model;
(2) the use of some trial-and-error approach which searches for values of the variables to give best agreement between calculated run-off and recorded run-off.

Available reports indicate that most attention is currently being given to the second approach. However, some assumptions must be made about, for instance, the range of values to be considered, and the incremental steps to be taken in trial-and-error search. Consequently, the pattern that is developing in catchment-model studies is that values of the variables are evaluated using some objective optimizing procedure such as the steepest ascent but with ranges of values, starting values, incremental steps, and so on, set down from prior considerations.

Dawdy and O'Donnell (1965) have introduced the concept of sensitivity analysis into the evaluation of variables. This will undoubtably become part of future methods of evaluation as it offers promise of improving the efficiency of current sampling methods.

Interpreting the Results

Catchment models are satisfactory for establishing a rainfall/run-off relationship where records of run-off are available — even short records. Past run-off patterns can then be established where long rainfall records are available or, alternatively, simulated rainfall sequences can be generated using Monte Carlo methods to generate rainfall inputs.

A bigger problem arises when it is necessary to give physical meaning to the results or structure of the model, or to interpret results and structure in terms of a real phenomenon such as a change in land use. Where models are to be used for evaluating land-use changes, one set of values of the model parameters is used to simulate 'before-treatment' catchment behaviour and a second set of values is used to represent the 'after-treatment' condition. The difference in the simulated outputs from the model, using first one set of values and then the other, is assumed to be the effect caused by the treatment.

The Stanford Watershed Model is being used in this way to investigate a variety of applications ranging from the response of catchments to weather modifications, to the hydrological changes caused by urban development (Crawford and Linsley, 1966).

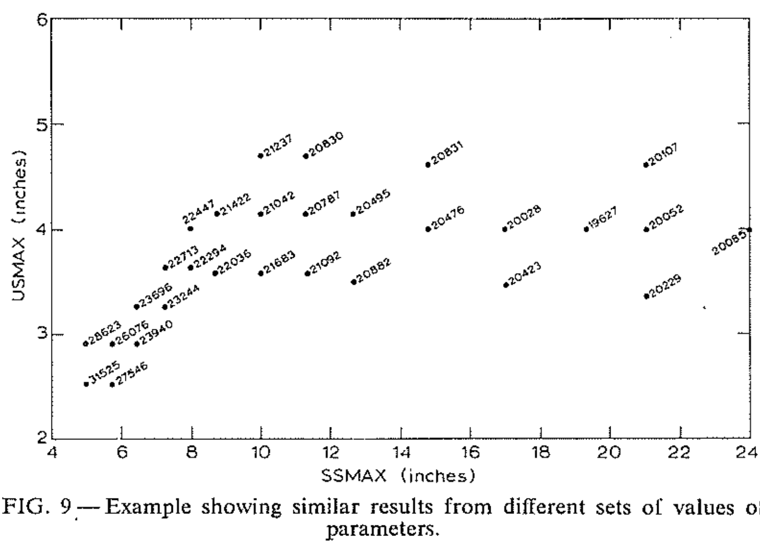

FIG. 9 — Example showing similar results from different sets of values of parameters.

However, it has been demonstrated (Boughton, 1968) that catchment models can contain compensating parameters where, for example, change in the value of a moisture-storage capacity can be compensated for by change in value of the infiltration parameters to give a very similar output.

Fig. 9 illustrates the results obtained on an Australian catchment with marginal-analysis sampling. For each combination of the moisture-storage capacities USMAX and SSMAX, the optimum combination of values for the infiltration parameters was found to give best agreement between recorded and calculated run-off. Values of the sum of squares of differences between monthly totals of recorded and calculated run-off are shown in the figure.

Changes in value of the subsoil moisture capacity from 10 to 20 inches and the upper soil capacity from 3 to 5 inches can be compensated for by change in the infiltration parameters such that there is little change in the sum of squares. The opportunities for misinterpretation, where small variations in output are sought among wide natural fluctuations, are obvious.

SUMMARY

The catchment model described in this paper seems adequate for the practical purpose of establishing a rainfall/run-off relationship where run-off records are available. The model is suitable for extending short run-off records, and the level of accuracy which can be obtained is now established from the results presented.

There is less certainty about evaluating changes in land use and similar applications where physical meaning must be given to the structure of the model or to results obtained. One problem is that changes in run-off produced by changes in land use are small compared to natural fluctuations. The order of magnitude of errors of current catchment models is still about the same as the differences caused by change in land use.

Before these models can be used to account with accuracy for the effects of change in land use, it seems necessary to incorporate several other factors such as the variation in evapotranspiration loss caused by differences in slope and aspect over a catchment (necessary to account for partial-area run-off) and the effects of interflow and transmission loss (to account for the differences in behaviour between catchments of different size).

Although catchment models are still in their infancy, there is little doubt that they have great potential as a tool for hydrological analysis.

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APPENDIX 1

Computer Programme of Catchment Model

Explanation of Variable Names

RAIN(J)	Value of daily rainfall. J defines the day of the month.
EVAP(M)	Daily evapotranspiration rate. M defines the month of the year concerned.
CEP	Moisture level of the Interception store.
CEPMX	Capacity of the Interception store.
US	Moisture level of the Upper Soil store.
USMAX	Capacity of the Upper Soil store.
DR	Moisture level of the Drainage store.
DRMAX	Capacity of the Drainage store.
SS	Moisture level of the Subsoil store.
SSMAX	Capacity of the Subsoil store.
SDRMX	Drainage component of the Subsoil store.
FO	Daily infiltration rate at subsoil moisture level zero.
FC	Minimum daily infiltration rate.
AAK	Exponent *k* in infiltration equation.
AAC	Multiplying factor for depleting subsoil moisture by drainage.
H	Maximum limit of evapotranspiration rate (taken as 35 points per day).
PCUS	Percentage of evapotranspiration loss from the Upper Soil store.

Computer Programme

Written in Fortran IV for the IBM 1130 computer at Lincoln College.

```
      DIMENSION RAIN(31),EVAP(120),RNFF(120),FEBET(12)
  220 RUNMN=0.0
      READ(2,1000)CEPMX,USMAX,DRMAX,SSMAX,SDRMX
      IF(SSMAX)772,772,773
  773 WRITE(3,1300)CEPMX,USMAX,DRMAX,SSMAX,SDRMX
      READ(2,1000)CEP,US,DR,SS
      WRITE(3,1300)CEP,US,DR,SS
      READ(2,1010)FO,FC,AAK,AAC
      WRITE(3,1310)FO,FC,AAK,AAC
      READ(2,1000)H,PCUS
      WRITE(3,1300)H,PCUS
      READ(2,1001) (FEBET(J),J=1,12)
      READ(2,1002)NM,MSTRT,NXM,INIT
      WRITE(3,1302)NM,MSTRT,NXM,INIT
      IF(MSTRT)502,502,503
  503 READ(2,1003) (EVAP(J),J=1,MSTRT)
      WRITE(3,1303) (EVAP(J),J=1,MSTRT)
  502 MST=MSTRT+1
      READ(2,1003) (EVAP(J),J=MST,NM)
      WRITE(3,1303) (EVAP(J),J=MST,NM)
      READ(2,1009) (RNFF(J),J=MST,NM)
      WRITE(3,1309) (RNFF(J),J=MST,NM)
      SPILL=0.0
      SQARE=0.0
      SUM=CEPMX+USMAX+DRMAX
      DEF=0.0
      DEFMX=0.0
      MST=12-MSTRT
      DO 102 M=1,NM
      READ(2,1004) (RAIN(J),J=1,31)
      MST=MST+1
```

```
      IF(MST - 13)501,500,500
  500 INIT=INIT+1
      MST=1
  501 RUNMN=0.0
      DO 101 J=1,31
      IF(RAIN(J))101,11,12
   12 AA=CEP
      AB=US
      AC=DR
      AD=SS
      CEP=CEP+RAIN(J)
      IF(CEPMX - CEP)13,11,11
   13 EX=CEP - CEPMX
      CEP=CEPMX
      US=US+EX
      IF(USMAX - US)14,11,11
   14 EX=US - USMAX
      US=USMAX
      DR=DR+EX
      IF(DRMAX - DR)15,11,11
   15 EX=DR - DRMAX
      IF(SSMAX - SS)301,301,302
  301 F=FC+0.01
      GO TO 263
  302 IF(SS) 303,303,304
  303 F=FO
      GO TO 263
  304 F=FC+(FO - FC) /EXP(AAK*SS)
  263 A=EX/F
      A=EXP(A)
      B=1.0/A
      RUN=EX - F*(A - B) /(A+B)
      RUNMN=RUNMN+RUN
      DR=DR - RUN
      SSINC=F
      IF(SSMAX - SS - F)226,227,227
  226 SPILL=SS+F - SSMAX
  227 WRITE(3,2003)J,FEBET(MST),INIT,RUN
 2003 FORMAT(1X,I3,1X,A3,I5,F7.1)
      SPILL=0.0
      DEF=0.0
      DEFMX=0.0
   11 CEP=CEP - EVAP(M)
      IF(CEP)16,117,117
   16 EP=ABS(CEP)
      CEP=0.0
      US=US - PCUS*FUNCT(US,USMAX,H,EP)/100.0
      SS=SS - (100.0 - PCUS) *FUNCT(SS.SSMAX,H,EP) /100.0
  117 IF(DR) 18,92,19
   18 DR=0.0
      GO TO 92
   19 IF(SSMAX - SS)305,305,306
  305 F=FC+0.01
      GO TO 310
  306 IF(SS) 307,307,308
  307 F=FO
      GO TO 310
  308 F=FC+(FO - FC) /EXP(AAK*SS)
  310 IF(DR - F)121,121,122
```

```
 121  SS=SS+DR
      DR=0.0
      GO TO 123
 122  SS=SS+F
      DR=DR-F
 123  IF(SSMAX-SS)91,92,92
  91  SS=SSMAX
  92  IF(SS-SDRMX)120,120,93
  93  SS=SS*AAC
 120  DEF=SUM-CEP-US-DR
      IF(DEFMX-DEF)22,101,101
  22  DEFMX=DEF
 101  CONTINUE
      WRITE(3,1007)RUNMX,SS,CEP,US,DR,DEFMX
      IF(NXM)35,35,36
  35  IF(RNFF(M))102,37,37
  37  YP=RUNMN-RNFF(M)
      SQARE=SQARE+YP*YP
  36  NXM=NXM-1
 102  CONTINUE
      WRITE(3,1007)SQARE
      GO TO 220
1007  FORMAT(1X,6F9.0)
1000  FORMAT(5F5.0)
1001  FORMAT(12(A3,1X))
1002  FORMAT(4I5)
1003  FORMAT(12F5.1)
1004  FORMAT(16F4.0/15F4.0)
1009  FORMAT(12F5.0)
1010  FORMAT(2F5.02F5.4)
1300  FORMAT(1X,5F5.0)
1302  FORMAT(1X,4I5)
1303  FORMAT(1X,12F6.0)
1309  FORMAT(1X,12F6.0)
1310  FORMAT(1X,2F6.0,2F6.4)
 772  CALL EXIT
      END

      FUNCTION FUNCT(SMLEV,SMMAX,H,ET)
      POINT=H*SMLEV/SMMAX
      IF(POINT-ET)1800,1800,1801
1800  FUNCT=POINT
      RETURN
1801  FUNCT=ET
      RETURN
      END
```