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LINEAR AND CURVILINEAR BASEFLOW RECESSIONS

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ABSTRACT

The common form of equation for baseflow recessions, $Q_{t+1} = k.Q_t$, implies that recession is linear on a semi-log hydrograph. Examples are given of both linear and curvilinear recessions; the latter is common for small and relatively wet catchments. Relating baseflow discharge to the storage supplying the discharge gives more information about the water balance of a catchment than relating flow to flow on the preceding day. Calculation procedures for relating discharge to storage for both linear and curvilinear recessions are developed, with examples for both types.

INTRODUCTION

Between storms, runoff in many streams is maintained by baseflow draining into channels from a saturated groundwater zone. Baseflow discharge diminishes as the stored water is depleted, and a characteristic baseflow recession is a hydrological property of a catchment. An excellent review of studies of baseflow recessions, some dating back to 1842, was prepared by Hall (1968), and Toebes and Strang (1964) examined equations used to define recessions.

Most baseflow studies assume baseflow recession is approximately a straight line when the hydrograph is plotted in semi-logarithmic form (discharge on log scale, time on natural scale). Waugh (1970) characterised the low flows of some New Zealand rivers using this linear form of recession, and Klaassen and Pilgrim (1975) the recessions of 29 streams in New South Wales on the same basis. Some catchments, however, have recessions which are distinctly curvilinear on the semi-log hydrograph; this paper shows examples of both types.

Most past studies have related baseflow either to flow in the preceding time period or to the time elapsed since the start of recession. For water-balance studies, discharge must be related to the water in storage supplying the discharge. Both linear and curvilinear recessions can be used to relate discharge to storage and to provide information about the water balance of catchments.

LINEAR RECESSIONS

1 Background

Barnes (1939) introduced the following equation:

$$Q_t = Q_o . k^t \tag{1}$$

i.e.

$$k = \left(\frac{Q_t}{Q_o}\right)^{\frac{1}{t}} \tag{1a}$$

where Q_o = discharge at time zero,
Q_t = discharge t time units later, and
k = recession constant.

This equation commonly is used in studies of baseflow recessions, but other forms of this equation have been proposed. Singh and Stall (1971) used the following alternative:

$$Q_t = Q_o \,.\, 10^{\frac{-t}{K}} \qquad (2)$$

i.e.

$$K = 10^{\frac{-1}{K}} \qquad (2a)$$

where K = storage delay factor, in same units as t

The storage delay factor, K, is the time taken for discharge to reduce by a factor of 10. This is a reduction by one log cycle on the semi-log hydrograph, and therefore has a visual significance which the recession constant does not. Martin (1973) used the "half-flow period", the time required for flow in a simple linear recession to halve.

$$t_{0.5} = \frac{\log 0.5}{\log k} \qquad (3a)$$

$$\text{or} \quad k = 10^{-\frac{0.301}{t_{0.5}}} \qquad (3b)$$

The storage supplying baseflow is found by integrating all discharges throughout recession which, in theory, continues to infinity.

$$S_o = Q_o + k.Q_o + k^2.Q_o + k^3.Q_o \ldots \qquad (4a)$$

where S_o = storage at the start of recession

Multiplying both sides by k:

$$k.S_o = k.Q_o + k^2.Q_o + k^3.Q_o \ldots \qquad (4b)$$

Subtracting (4b) from (4a) —

$$S_o(1-k) = Q_o \qquad (4c)$$

$$\text{or} \quad S_o = \frac{Q_o}{1-k} \qquad (4d)$$

The term (1–k) is the fraction of storage that is discharged in any period. This is the physical significance of the linear recession constant.

2 Example of Linear Recession

Figure 1 shows data for three months of runoff from the 28 sq km Pullen Pullen Creek catchment on the western edge of Brisbane, southeastern Queensland, recorded at Moggill Road gauging station (no. 143024). The

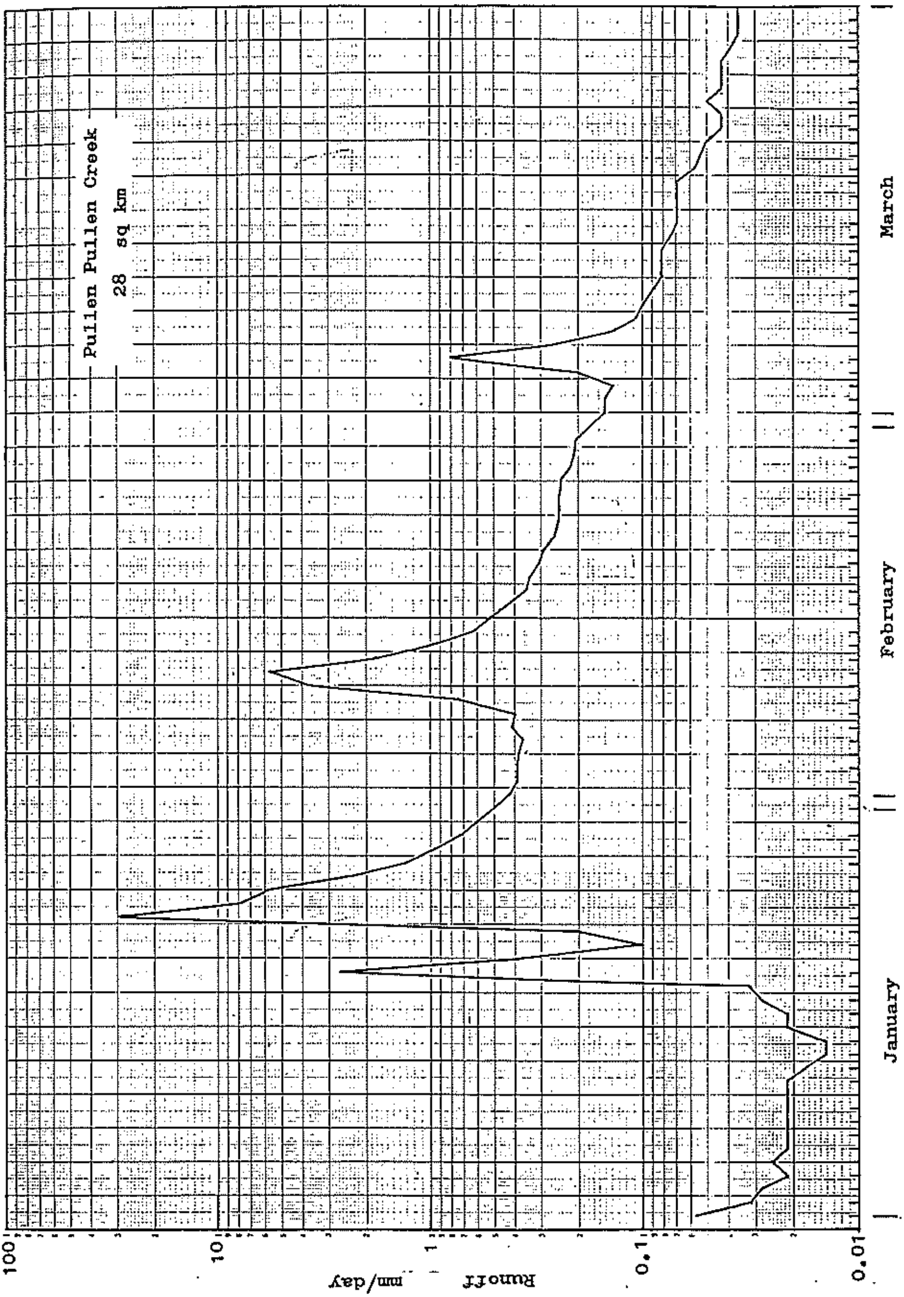

FIG. 1 — Baseflow recession curve for Pullen Creek, southeastern Queensland.

catchment is covered mostly with native forest, with scattered rural properties, and a small urban area, the suburb of Bellbowrie, close to the outlet of the catchment. In the rural part of the catchment, seven landholders have licences from the Queensland Water Resources Commission to pump water from the creek. Five licences are for irrigation of a total area of 18 ha. The recession period (Fig. 1), however, was probably not affected by pumping because of very wet conditions at the time.

Using the flow rates on 16 February (0.354 mm/day) and on 21 March (0.050 mm/day), and equation (1a), the daily recession constant k is found to be 0.9424. From equation (4a) the amount of storage supplying baseflow on 16 February is 6.2mm. The value of (1-k) is 0.0576, i.e. 5.76% of the water in storage is discharged each day.

The very low rates of flow on 13-14 January are the end of a preceding recession. Using equation (4a), the storage supplying baseflow on 14 January is 0.2mm, hence storage was recharged by 6.0mm during storms in late January and early February.

Rainfall in late January and early February totalled almost 300mm, and surface runoff was almost 70mm. Clearly, much of the rainfall was used to recharge soil moisture or was lost by evapotranspiration. By comparison, amounts of water entering and leaving baseflow storage in this catchment are very small.

CURVILINEAR RECESSIONS

1 Background

The simplest form of curvilinear recession equation which relates discharge to storage is:

$$Q = a.S^b \tag{5}$$

where a and b are constants

When b = 1.0, the equation reverts to the linear regression equation, and $a = (1-k)$.

To fit this equation to a baseflow recession, values must be found for constants a and b, and for storage S at some rate of discharge Q. As there are three unknowns, three points on the recession are needed to get unique values of a, b, and S.

Let Q_1 = baseflow discharge at the earliest point
S_1 = amount in storage at the earliest point
Q_2 = baseflow discharge at the second point
S_2 = amount in storage at the second point
d_{1-2} = baseflow runoff between the first and second points
(Note: $S_2 = S_1 - d_{1-2}$)
Q_3 = baseflow discharge at the third point
S_3 = amount in storage at the third point
d_{1-3} = baseflow runoff between the first and third points
(Note: $S_3 = S_1 - d_{1-3}$)

Equation (5) is converted to logarithmic form:

$$\log Q = \log a + b.\log S \qquad (6)$$

Taking the first two points on the recession:

$$\log Q_1 = \log a + b.\log S_1 \qquad (7a)$$
$$\log Q_2 = \log a + b.\log (S_1 - d_{1\text{-}2}) \qquad (7b)$$

Subtracting (7b) from (7a) and rearranging:

$$b = \frac{\log Q_1 - \log Q_2}{\log S_1 - \log (S_1 - d_{1\text{-}2})} \qquad (8)$$

In the same way, using the first and third points on the recession:

$$b = \frac{\log Q_1 - \log Q_3}{\log S_1 - \log (S_1 - d_{1\text{-}3})} \qquad (9)$$

Using (8) and (9) together:

$$\frac{\log Q_1 - \log Q_2}{\log S_1 - \log (S_1 - d_{1\text{-}2})} = \frac{\log Q_1 - \log Q_3}{\log S_1 - \log (S_1 - d_{1\text{-}3})} \qquad (10)$$

Rearranging (10) gives:

$$\frac{\log Q_1 - \log Q_2}{\log Q_1 - \log Q_3} = \frac{\log S_1 - \log (S_1 - d_{1\text{-}2})}{\log S_1 - \log (S_1 - d_{1\text{-}3})} \qquad (11)$$

Given values of Q_1, Q_2, Q_3 and $d_{1\text{-}2}$ and $d_{1\text{-}3}$, the only unknown in equation (11) is S_1. Equation (11) is solved by trial and error to find S_1; exponent b is found using equation (8), and equation (5) is then used to find the value of a.

2 Example of Curvilinear recession

Figure 2 shows three baseflow recessions for the 7 sq km Back Creek catchment, southeastern Queensland, based on daily discharges measured at Beechmont gauging station (no. 146014 — Queensland Water Resources Commission). Back Creek drains from Beechmont Plateau, which was formed from the eruption of Mount Warning volcano in the Tertiary and is capped by basalt. The original sub-tropical rainforest of the area has been replaced by pasture, used mainly for dairy farming. The plateau is some 500 to 600m above sea level and the gauging station is located at the northern end, where the creek drops from the plateau to the valley floor. Average annual rainfall is about 1350mm.

The first hydrograph portion (Fig. 2) covers January to September, 1972; the second May to November, 1974; and the third March to October 1975. The first hydrograph portion is used to fit the equation to the recession, and the other two portions to test the consistency of the fitted equation.

The following three points were chosen to fit the equation:

9th April — 4.13 mm/day
13th June — 0.943 mm/day
31st August — 0.314 mm/day

The volumes of discharge, d_{1-2} and d_{1-3}, are estimated by adding the baseflow between 9th April and 13th June, and between 9th April and 31st August. A small amount of surface runoff (7.1mm) in the period 9th–16th May must be subtracted from total runoff to get baseflow discharge. After allowing for surface runoff, the amounts of baseflow are:

$d_{1-2} = 134.1\text{mm}$
$d_{1-3} = 181.9\text{mm}$
Using equation (11) : $S_1 = 237.6\text{mm}$
Using equation (8) : $b = 1.776$
Using equation (5) : $a = 0.0002494$

The fitted equations are:

$$Q = 0.0002494\ S^{1.776} \tag{12}$$

and $$S = 107\ Q \tag{13}$$

The estimated baseflows using equations (12) and (13) are shown in Figure 2. Given the starting baseflow of a recession, the amount in storage is calculated from equation (13). Storage on the following day is found by subtracting starting discharge from starting storage, and the discharge on that day is found from equation (12). This procedure is repeated to calculate daily baseflow throughout the recession.

There are many minor irregularities in the three recessions (Fig. 2), but the equation fits the shape of each recession well. Minor irregularities may reflect evapotranspiration from riparian vegetation, rain falling onto the flowing water, or minor pumping from the stream.

The amount of storage at the start of the first recession was 237mm. This is large compared with the storage of 6mm in Pullen Pullen Creek which was used to illustrate linear recession. Storages are determined by the summation of measured baseflows plus extrapolation in the very small range of flows, hence they are more accurate than other values in a water balance such as areal rainfall. Consequently, storage values provide information about water balance which can be used to improve current models of catchment behaviour. The values of storage were about the maximum for each study catchment in the 10-year periods of record.

CONCLUSIONS

Relating baseflow to the storage which supplies discharge is much more useful than relating flow to flow on the preceding day, or to time elapsed from the start of recession. The discharge-storage relationships illustrated in this paper can provide detailed information about the amount of water

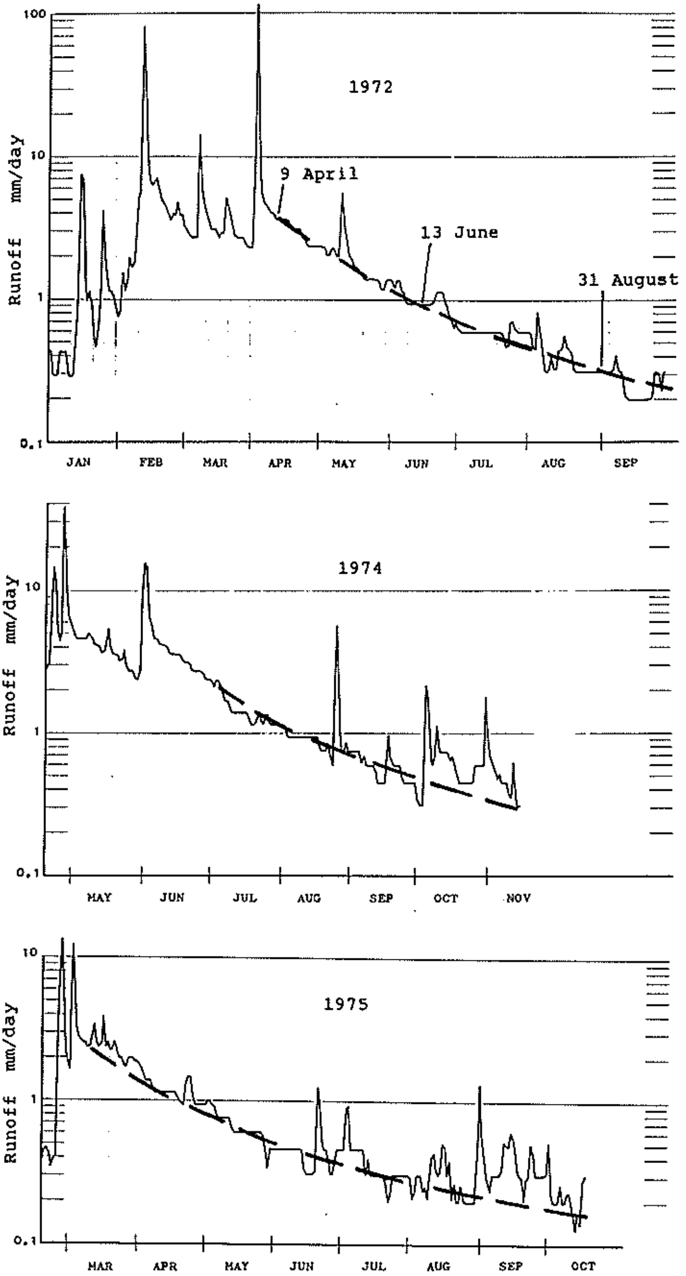

FIG. 2 — Baseflow recession curves for Back Creek, southeastern Queensland. Heavy dashed lines are baseflows estimated using equations (12) and (13).

entering and discharging from baseflow storage, which can be used to improve catchment water-balance models.

Curvilinear baseflow recessions can be handled as well as linear recessions although the calculations are slightly more cumbersome. Curvilinear recessions occur in small and relatively wet catchments. There is also evidence in other studies of curvilinear recession and procedures set out in this paper may facilitate their analysis without the need to make linear approximations.

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