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IS THE "100-YEAR FLOOD" INTERPRETED CORRECTLY?

K.J.A. Revfeim

Meteorological Service, Wellington, New Zealand

ABSTRACT

The mean time to first exceeding (return period) of a prescribed threshold (design flood) Q is not the same as the mean time to an attained value X, which cannot be predetermined. The mean size of maxima in any specified time interval has a 43% chance of being exceeded. It is shown how very large events can be qualified by a much smaller chance of being exceeded i.e. a more confident statement can be made about the severity of an extreme event.

INTRODUCTION

A flood described as a 50-year or 100-year event may be followed within a decade by a similar or more extreme event. Naturally this tests the credibility of those who use the term 'T-year event' as far as those directly affected by the floods are concerned. Uncertainty about the terminology possibly extends to some professionals who use the descriptions 50-, 100- and 500-year event. It therefore seems timely to clarify terms and give a necessary qualification commonly omitted from the usual description of floods.

The term 'T-year-event' is not a standard definition in the literature on extreme values. Benham (1950) stated that the probability of exceeding a discharge Q was the area beyond Q under the frequency distribution of annual flood maxima. If this probability is 1/100 then "Q is designated the 100-year flood". Beable and McKerchar (1982) define the T-year event, or design flood Q as the flood threshold which is exceeded on average once in T years. This is equivalent to the usual definition of 'return periods' viz. the average time to the first exceedence of the design flood Q.

Tomlinson (1980) makes a distinction between the 100-year event "that occurs on average once every 100 years" and the "100-year return period value" as defined above. While not giving specific methods for distinguishing them, Tomlinson does question the over-frequent use of descriptions such as '100-year events'.

Since Q is a threshold there is no justification in the *a posteriori* description of a particular flood as a T-year event. Benham (1950, p 123) emphasised that "a greater discharge (than the T-year event) may occur" within T years. Such "greater discharges" are not related to the frequency distribution of maxima in the same manner as prescribed thresholds.

However it is not usually size which is specified but a range of time periods for which expected maxima are predicted. If a particular event exceeds the 50-year prediction, but is less than the prediction for the next largest time value, say 100 years, that event may also be called a 50-year event.

A basic assumption of the historical theory is that "intervals between successive floods are all equal to one year" (Gumbel, 1941). The statistical model of this situation is the geometric distribution with probability pq^{k-1} of first exceeding Q in the k-th year where $q = G(Q)$ is the probability distribution function of the maximum event (flood) in any one year. The mean time (number of years) to the first exceedence is $1/(1-q)$ and hence by definition the return period is $T^* = 1/[1 - G(Q)]$

The usual results related to this definition are that the probability of exceeding Q in any one year is $1/T^*$ and that the risk of at least one T^* - year flood in L years is $R^* = 1 - (1-1/T^*)^L$ (Beable and McKerchar, 1982).

A more plausible model of random occurrence of events in time, with size of event being independent of occurrence, gives simpler formulae which are also valid for short term predictions (Revfeim, 1983a). If the events occur as a Poisson process at rate ρ and the sizes of underlying events have a probability distribution function $F(x)$ then the average time to first exceedence of Q is $T = 1/\rho\,[1-F(Q)]$.

T^* is greater than T for the probability distribution function $G(Q)$ of maxima in the random model (Revfeim, 1983a). The probabilities of exceeding Q in any one year, $1-e^{-1/T}$, and in L years, $1-e^{-L/T}$, are greater than for the Gumbel model. These latter related results give a simpler table of risks than shown in Beable and McKerchar (1982).

An attained value X is different from a threshold Q. The average time to first *attainment* of X in the random occurrence model is nearer to the time $X-\mu$ is first exceeded since μ, the mean event size, is also the mean excess (Revfeim, 1983a). The threshold Q is equivalent to $X-\gamma\mu$ where $\gamma = 0.5772$ is Euler's Constant.

The purpose of this note is to identify the sources of error in making predictions with any model of extreme values. Then, using parameters of the random occurrence model, the average T-year event is qualified by the probability that no greater event will occur in T years.

SOURCES OF PREDICTION ERROR

The first, and frequently overlooked, source of error is the adequacy of the chosen model to represent either the underlying stochastic process or the maxima for that process over any time segment. A common assumption is that samples from which maxima are recorded are of independent values. Also the traditional asymptotic theory of extremes demands large sample sizes for valid conclusions to be drawn.

It is not easy to detect statistically significant correlation between consecutive rainfall events, or always to be able to identify distinct events. River flows above some arbitrary threshold are identifiable. Whether they should be measured by the peak flow-rate exceedence above the threshold, or maximum integrated exceedence over some time interval, say 30 minutes, is seldom discussed. Any correlation between exceedences decreases as the threshold is raised. For short interval observations, such as 1-minute or 5-minute rainfalls, there is the further problem of clustering of occurrences within an event.

The frequency distribution of rainfall amounts or flow exceedences has a decreasing 'tail' at the upper end. While the distribution may have an 'exponential tail' the maximum values are sensitive to the shape of the whole distribution. The arbitrary threshold sometimes used with river flows is simply a device to make the truncated distribution approximately exponential throughout. Intuition suggests that rainfall amounts in finite intervals have a non-zero mode at the lower tail i.e. the most frequent amount of rain is not nil. However the effect of this lower tail may be significant only for the shortest and longest intervals. This is because the upper tail is limited by the maximum rainfall intensity that can be sustained over such intervals. Testing the goodness of fit of distributions can often be difficult simply because the number of maxima observed is less than 50. A histogram of 50 or fewer observations would need to be grouped into, at most, five class intervals to give reliable frequencies, and the significance of goodness-of-fit is somewhat elusive with this number of classes. This does not seem to deter the use of traditional asymptotic extreme-value theory. A further possibility is that rainfalls and resulting river flows may be the result of a mixture of two or more rainfall processes (Revfeim 1983b).

The second, and often the only, error recognised in making prediction is the reliability of parameter estimates. In the random recurrence model the expected value of the maximum in T years is

$$X_T = \mu(\ln\rho + \gamma + \ln T) \qquad (1)$$

where μ is the mean size of events and ρ is the mean number of events per year (Revfeim, 1983a). Parameter estimates $\hat{\mu}$, $\hat{\rho}$ from a record of n-years maxima are substituted in equation (1) so that $\hat{X}_T$ has a variance due to these estimates. This variance component reduces as $n^{-1/2}$ with increasing length of record for estimating parameters. For instance, analysis of annual peak flows of the Ngaruroro River at Fernhill (1923–1977), Table 1, gives maximum likelihood parameter estimates (using Revfeim's (1983a) iterative procedure) of $\rho = 6.8$ events/year and $\mu = 403$ cumecs. These parameters are related to the location (mode u) and scale (α) parameters of the usual form of the Gumbel distribution $G(x) = \{\exp -e^{-\alpha(x-u)}\}$ by $\mu = 1/\alpha$ and $\rho = e^{\alpha u}$. The importance of the revised parametric setting is that ρ and μ have physical meanings which can be compared with an observed or intuitive understanding of the number of individual events (rainfalls or flow surges) underlying maxima and their expected magnitude.

The third and mostly ignored source of error is due to the variance of the stochastic process itself. X_T is the predicted mean maximum with variance $\mu^2\pi^2/6$, (Revfeim, 1983a) which is independent of T. This could be used to give standard error bands, but for descriptive purposes a predicted maximum with low probability of being exceeded may be preferable.

The fourth source of error is spatial variability, especially with regard to rainfall. Integration of rainfall by catchments reduces the effect of this variability in river flows. Stormwater drainage systems are sensitive to maximum precipitation in localised areas. Rainfall recorded at one point of the drainage system may not be representative of rainfall for the whole area.

CONFIDENCE IN PREDICTION

The probability distribution function of maxima for the random occurrence model is (Revfeim, 1983a)

$$G_T(x) = \exp(-\rho T e^{-x/\mu}). \qquad (2)$$

Hence, substituting X_T for x in (2), the probability of getting a larger value is $1-G_T(X_T) = 1 - \exp(-e^{-\gamma}) = 0.43$. This means there is a 3 in 7 chance of getting a maximum value larger than the T-year average maximum.

Now consider a fraction of the time T/k but with the predicted mean increased by $\mu \ln k$. That is

$$x = X_{T/k} + \mu \ln k = \mu(\ln \rho + \gamma + \ln T)$$

which is the same size event as X_T but predicted to occur in the time T/k. The probability that this event is exceeded is $1-\{\exp(-e^{-\gamma})\}^{1/k}$ which is smaller than the probability of exceeding X_T. These probabilities, for a range of values of k, are given in Table 2. Judged *a posteriori* the same-sized event is a "100-year event" with 43% chance of being exceeded, a "50-year event" with a 24% chance of being exceeded, or a "10-year event" with a 5% chance of being exceeded. Qualification of the prediction by the chance of being exceeded gives more confidence in the description of the event. The chance of being exceeded might be called the significance level for that prediction.

EXAMPLES: NEW ZEALAND RIVERS

From the 46-year record of the Ngaruroro River at Fernhill (1923-77) (Beable and McKerchar, 1982), the estimated recurrence rate is ρ = 6.8 events/year and μ = 403 cumecs. From these parameters the 1897 flood peak of 4247 cumecs is the average maximum for a period of 3120 years. The 1897 peak is taken as the maximum for the 140-year period 1837–1977 and, from Table 1, there is only a 3% chance of this peak being exceeded in 156 (3120/20) years. That is, an event such as the 1897 flood is certainly exceptional in the 140-year observation period, exceeding the '3% significance level' of a 140-year extreme event.

Similar analysis of data for the Taeri River at Outram (1955-1979) (Beable and McKerchar, 1982) gives ρ = 3.1 events/year and μ = 284 cumecs. From these parameters the 1980 peak flood of 2600 cumecs is the average maximum for a time of 1740 years. There was a peak of 2179 cumecs in 1868 and the 1980 peak is taken as the maximum for 1868–1979 i.e. 112 years. Thus the 1980 event also exceeds the 5% significance level of a 112-year event.

The 28 year record to 1975 of 24-hr rainfalls at Avalon (Lower Hutt) gives parameter estimates ρ = 44 events/year and μ = 19 mm/event. The 265 mm rainfall of December 1976 is an average maximum in 190 years, but only a 19-year event at the 5% significance level. Analysis of the parallel record to 1975 of 24-hr rainfalls at Kelburn gives parameter estimates of ρ = 31 events/year and μ = 19 mm/event. Thus the second-largest recorded 24-hr rainfall of 153 mm, which occurred during the Hutt Valley floods of 1976, gives a time of only 58 years for which this is the average maximum i.e. the event was in no way exceptional. These analyses of sites about 15 km apart also indicate some spatial variability, although the one standard error range on ρ is about 7.

Similar analysis of 24-hr rainfalls at Invercargill Airport for 1941-83 gives ρ = 39 events/year and μ = 10mm/event. During the January 1984 floods, 135 mm of rainfall was recorded in 24 hrs and the time for which this is the average maximum is 10,000 years. Assuming that the 1941–83 data were interpreted correctly, one could be sure that the recent disaster was a 1000-year event at the 5% significance level or a 200-year event at the 1% significance level.

However, there is also the possibility that extremes can arise from a mixture of processes. The 24-hr rainfall on 26–27 January 84 was almost double the previous high of 74 mm in 1980 and it is quite possible that this event arose from a very rare combination of atmospheric processes. That is, the 135 mm extreme, and perhaps second or third highest maxima, may be realisations of this rare event. An extremely long record would be needed to identify a mixture of extreme-value distributions, let alone estimate parameters, with sufficient precision to be confident in any prediction. It is thus unwise to describe events as extreme occurrences of much more than double the length of the data record.

CONCLUSIONS

Attained flood levels or rainfall event sizes are conceptually different from prescribed thresholds which may be exceeded by several extreme events over a long period.

The description of extreme events can be statistically qualified so that one has more confidence in the reliability of that description. Rare occurrences of unforeseen events will give rise to 'outlying' data observations the

TABLE 1—Annual peak flows (m^3s^{-1}) of Ngaruroro river at Fernhill.

1920—	..	..	..	487	2531	484	289	1402	821	1798
1930—	..	821	1076	1597	437	1218	2265	283	3511	..
1940—	..	..	..	..	1402	..	..	680	821	1444
1950—	1246	626	767	949	906	793	1274	765	906	1075
1960—	706	482	1190	1126	404	1540	533	551	1339	..
1970—	562	909	396	750	1353	1250	522	771	..	..

TABLE 2—Probability P_k of exceeding X_T in time T/k

k	1	2	5	10	20	50
P_k	0.43	0.24	0.11	0.05	0.03	0.01

interpretation of which cannot be ignored in qualifying further predictions.

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